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Research Article

Approaches to Multiple Attribute Decision-Making with Fuzzy Number Intuitionistic Fuzzy Information and Their Application to English Teaching Quality Evaluation

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Many experts and scholars focus on the Maclaurin symmetric mean (MSM) operator, which can reflect the interrelationship among the multi-input arguments. It has been generalized to different fuzzy environments and put into use in various actual decision problems. The fuzzy number intuitionistic fuzzy numbers (FNIFNs) could well depict the uncertainties and fuzziness during the English teaching quality evaluation. And the English teaching quality evaluation is frequently viewed as the multiple attribute decision-making (MADM) issue. We expand the MSM equation with FNIFNs to propose the fuzzy number intuitionistic fuzzy MSM (FNIFMSM) equation and fuzzy number intuitionistic fuzzy weighted MSM (FNIFWMSM) equation in this study. A few MADM tools are developed with FNIFWMSM equation. Finally, taking English teaching quality evaluation as an example, this paper illustrates the depicted approach.

1. Introduction

In 1965, Zadeh [1] established a novel fuzzy set (FS) to deal with decision information in the fuzzy new domain [2–7]. To extend novel FS, the intuitionistic fuzzy sets (IFSs) [8, 9] were developed. Subsequently, FS and its related extension knowledges are exploited into the more and more decision domains [10–17]. Iakovidis and Papageorgiou [18] defined the cognitive maps for medical decision making under IFSs. Li [19] built the GOWA operator to MADM using IFSs. Su et al. [20] proposed the interactive method for dynamic IF-MAGDM. Tan [21] constructed the Choquet integral-based TOPSIS method for IF-MADM. Wu and Zhang [22] built the IF-MADM based on weighted entropy. Yu [23] defined the generalized prioritized geometric operators under IFSs. Yu et al. [24] defined the derivatives and differentials for multiplicative IFSs. Zhao et al. [25] defined the interactive intuitionistic fuzzy algorithms for multilevel programming problems. Arya and Yadav [26] defined the intuitionistic fuzzy super-efficiency slack-based measure. Büyüközkan et al. [27] selected the transportation schemes with the integrated intuitionistic fuzzy Choquet integral method. De and Sana [28] defined the (p, q, r, l) method for random demand with Bonferroni mean under IFSs. Garg [29] proposed the improved cosine similarity measure for IFSs. Joshi et al. [17] defined the Jensen-alpha-Norm dissimilarity measure for IFSs. Li et al. [30] defined the time-preference and VIKOR-based dynamic method for IF-MADM. The authors in [31] built the intuitionistic fuzzy MABAC method based on cumulative prospect theory for MAGDM. Niroomand [32] defined the multiobjective-based direct solution method for linear programming along with intuitionistic fuzzy parameters. Zhao et al. [33] perfected TODIM for IF-MAGDM on the strength of cumulative prospect theory. Furthermore, Liu and Yuan [34] built the fuzzy number IFSs (FNIFSs) to combine the IFSs with the triangular fuzzy sets (TFSs). Li et al. [35] developed the entropy and similarity measure under FNIFSs. Wang [36] built the geometric operators under FNIFSs. Verma [37] defined the GFNIFWBM operator under FNIFSs.

Nevertheless, all the functions and tools proposed by the above scholars do not take into account the relationship between parameters [38–41]. To conquer these shortcomings, the crucial purpose of the article is to connect the FNIFSs with MSM operator [42–47] to build several novel fused formulas under FNIFSs.

Consequently, the rest work would be depicted. Several basic concepts of FNIFSs and MSM formulas would be depicted in the second section. The MSM formulas with FNIFSs would be constructed in the third section. An instance about English teaching quality evaluation is given in the fourth section. The conclusions reached will be depicted the last section.

2. Preliminaries

In this section, we introduced the concept of fuzzy number intuitionistic fuzzy sets (FNIFSs) [32] and the Maclaurin symmetric mean (MSM) operator [44].

2.1. Fuzzy Number Intuitionistic Fuzzy Set. Liu and Yuan [34] gave the definition of FNIFS, and the membership and nonmembership are given in the form of TFNs.

Definition 1 (see [34]). Supposed $E = \{e_1, e_2, \ldots, e_n\}$ is a fixed set, B is a FNIFS on E and its expression form is given as follows:

$$B = \in \{\langle e, T_B(e), F_B(e)\rangle e \in E\}. \quad (1)$$

$T_B(e)$ and $F_B(e)$ are two TFNs between 0 and 1, and $T_B(e) = (X(e), Y(e), Z(e)), e \longrightarrow [0,1]$, $F_B(e) = (A(e), S(e), D(e)), E \longrightarrow [0,1]$, and $0 \le Z(e) + D(e) \le 1$, $\forall e \in E$.

Let $T_B(e) = (X(e), Y(e), Z(e))$, $F_B(e) = (A(e), S(e), D(e))$, so $Q(e) = \langle (X(e), Y(e), Z(e)), (A(e), S(e), D(e))\rangle$, $Q(e)$ is viewed as a FNIFN.

Definition 2 (see [36, 48]). $Q(e_i) = \langle (X(e_i), Y(e_i), Z(e_i)), (A(e_i), S(e_i), D(e_i))\rangle$ and $Q(e_j) = \langle (X(e_j), Y(e_j), Z(e_j)), (A(e_j), S(e_j), D(e_j))\rangle$ are two FNIFNs.

(1) $$Q(e_i) \oplus Q(e_j) = \left\{\begin{pmatrix} X(e_i) + X(e_j) - X(e_i)X(e_j), \\ Y(e_i) + Y(e_j) - Y(e_i)Y(e_j), \\ Z(e_i) + Z(e_j) - Z(e_i)Z(e_j) \end{pmatrix}, \begin{pmatrix} A(e_i)A(e_j), \\ S(e_i)S(e_j), \\ D(e_i)D(e_j) \end{pmatrix}\right\}$$

(2) $$Q(e_i) \otimes Q(e_j) = \left\{\left(\begin{pmatrix} X(e_i)X(e_j), \\ Y(e_i)Y(e_j), \\ Z(e_i)Z(e_j) \end{pmatrix}\right), \left(\begin{pmatrix} A(e_i) + A(e_j) - A(e_i)A(e_j), \\ S(e_i) + S(e_j) - S(e_i)S(e_j), \\ D(e_i) + D(e_j) - D(e_i)D(e_j) \end{pmatrix}\right)\right\}$$

(3) $$\lambda Q(e_i) = \left\{\begin{pmatrix} 1 - (1 - X(e_i))^\lambda, \\ 1 - (1 - Y(e_i))^\lambda, \\ 1 - (1 - Z(e_i))^\lambda \end{pmatrix}, \begin{pmatrix} (A(e_i))^\lambda, \\ (S(e_i))^\lambda, \\ (D(e_i))^\lambda \end{pmatrix}\right\}, \quad \lambda \ge 0$$

(4) $$(Q(e_i))^\lambda = \left\{\begin{pmatrix} (X(e_i))^\lambda, \\ (Y(e_i))^\lambda, \\ (Z(e_i))^\lambda \end{pmatrix}, \begin{pmatrix} 1 - (1 - A(e_i))^\lambda, \\ 1 - (1 - S(e_i))^\lambda, \\ 1 - (1 - D(e_i))^\lambda \end{pmatrix}\right\}, \quad \lambda \ge 0$$

Definition 3 (see [36, 48]). Let $Q(e) = \langle (X(e), Y(e), Z(e)), (A(e), S(e), D(e))\rangle$ be a given FNIFN, a score function of a FNIFN $Q(e)$ can be depicted as follows:

$$\mathrm{SF}(Q(e)) = \frac{X(e) + 2Y(e) + Z(e)}{4} - \frac{A(e) + 2S(e) + D(e)}{4}, \quad \mathrm{SF}(Q(e)) \in [-1, 1]. \quad (2)$$

Definition 4 (see [36, 48]). Let $Q(e) = \langle (X(e), Y(e), Z(e)), (A(e), S(e), D(e))\rangle$ be a given FNIFN, an accuracy function of a FNIFN $Q(e)$ can be defined as follows:

$$\mathrm{AH}(Q(e)) = \frac{X(e) + 2Y(e) + Z(e)}{4} + \frac{A(e) + 2S(e) + D(e)}{4}, \quad \mathrm{AH}(Q(e)) \in [-1, 1]. \quad (3)$$

Based on the $\mathrm{SF}(Q(e))$ and $\mathrm{AH}(Q(e))$, next, let us look at the size comparison of the two FNIFNs.

Definition 5 (see [36, 48]). Let $Q(e_1)$ and $Q(e_2)$ be two FNIFNs, then if $\mathrm{SF}(Q(e_1)) < \mathrm{SF}(Q(e_2))$, then $Q(e_1) < Q(e_2)$; if $\mathrm{SF}(Q(e_1)) = \mathrm{SF}(Q(e_2))$, then

(1) If $\mathrm{AH}(Q(e_1)) = \mathrm{AH}(Q(e_2))$, then $Q(e_1) = Q(e_2)$

(2) If $\mathrm{AH}(Q(e_1)) < \mathrm{AH}(Q(e_2))$, then $Q(e_1) < Q(e_2)$

2.2. MSM Operators. Maclaurin [44] proposed the MSM formula.

Definition 6 (see [44]). Let $g_m\,(m = 1, 2, \ldots, k)$ be is a real number greater than 0, and $n = (1, 2, \ldots, k)$. If

$$\mathrm{MSM}^{(n)}(g_1, g_2, \ldots, g_k) = \left(\sum_{1 \le m_1 < \cdots < m_n \le m_k} \frac{\prod_{l=1}^{n} c_{m_l}}{C_k^n}\right)^{(1/n)}, \quad (4)$$

then we call $\mathrm{MSM}^{(n)}$ the MSM formula, where $(m_1, m_2, \ldots, m_n)$ traverses all the k-tuple combinations of $(1, 2, \ldots, n)$ and C_n^k is the binomial coefficient.

3. FNIFMSM and FNIFWMSM Operators

3.1. The FNIFMSM Operator. Here, we are going to expand MSM to coalesce all FNIFNs and establish the fuzzy number intuitionistic fuzzy MSM (FNIFMSM) operators.

Definition 7. Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle$, $m = 1, 2, \ldots, k$, be a set of given FNIFNs. The FNIFMSM operator could be depicted as follows:

$$\text{FNIFMSM}^{(n)}(Q(e_1), Q(e_2), \ldots, Q(e_k)) = \left(\frac{\oplus_{1\le m_1 < \ldots < m_n \le m_k}\left(\otimes_{l=1}^{n} Z\left(x_{m_l}\right)\right)}{C_k^n}\right)^{(1/n)}. \tag{5}$$

Theorem 1. *$Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle$, $m = 1, 2, \ldots, k$, be a suite of given FNIFNs. The coalesced data obtained from FNIFMSM equations are still an FNIFN.*

$$\begin{aligned}
\text{FNIFMSM}^{(n)}(Q(e_1), Q(e_2), \ldots, Q(e_k)) &= \left(\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n} Q\left(e_{m_l}\right)\right)}{C_k^n}\right)^{(1/n)} \\
&= \left\{\begin{pmatrix}
\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n} X\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}, \\
\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n} Y\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}, \\
\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n} Z\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}
\end{pmatrix},\right. \\
&\qquad \left.\begin{pmatrix}
1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-A\left(e_{m_j}\right)\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}, \\
1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-S\left(e_{m_j}\right)\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}, \\
1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-D\left(e_{m_j}\right)\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}
\end{pmatrix}\right\}.
\end{aligned} \tag{6}$$

Proof. According to Definition 2, we can derive

$$\underset{l=1}{\overset{n}{\otimes}} Q\left(e_{m_l}\right) = \left\{\begin{array}{c}
\left(\prod_{l=1}^{n} X\left(e_{m_l}\right), \prod_{l=1}^{n} Y\left(e_{m_l}\right), \prod_{l=1}^{n} Z\left(e_{m_l}\right)\right), \\
\left(1-\prod_{l=1}^{n}\left(1-A\left(e_{m_l}\right)\right), 1-\prod_{l=1}^{n}\left(1-S\left(e_{m_l}\right)\right), 1-\prod_{l=1}^{n}\left(1-D\left(e_{m_l}\right)\right)\right)
\end{array}\right\}. \tag{7}$$

Thus,

$$\underset{1\le m_1<\cdots<m_n\le m_k}{\oplus}\left(\overset{n}{\underset{l=1}{\otimes}} Q\left(e_{m_l}\right)\right)$$

$$=\left\{\begin{pmatrix} 1-\prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}X\left(e_{m_l}\right)\right), \\ 1-\prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}Y\left(e_{m_l}\right)\right), \\ 1-\prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}Z\left(e_{m_l}\right)\right) \end{pmatrix}, \begin{pmatrix} \prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}\left(1-A\left(e_{m_l}\right)\right)\right), \\ \prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}\left(1-S\left(e_{m_l}\right)\right)\right), \\ \prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}\left(1-D\left(e_{m_l}\right)\right)\right) \end{pmatrix}\right\}. \tag{8}$$

Thereafter,

$$\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{j=1}^{k} Q\left(e_{m_l}\right)\right)}{C_n^k}$$

$$=\left\{\begin{pmatrix} 1-\prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}X\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}, \\ 1-\prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}Y\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}, \\ 1-\prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}Z\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)} \end{pmatrix}, \begin{pmatrix} \prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}\left(1-A\left(e_{m_l}\right)\right)\right)^{\left(1/C_k^n\right)}, \\ \prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}\left(1-S\left(e_{m_l}\right)\right)\right)^{\left(1/C_k^n\right)}, \\ \prod\limits_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod\limits_{l=1}^{n}\left(1-D\left(e_{m_l}\right)\right)\right)^{\left(1/C_k^n\right)} \end{pmatrix}\right\}. \tag{9}$$

Therefore,

$$\mathrm{FNIFMSM}^{(n)}\left(Q(e_1),Q(e_2),\ldots,Q(e_k)\right)=\left(\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n}Q\left(e_{m_l}\right)\right)}{C_k^n}\right)^{(1/n)}$$
$$=\left\{\begin{pmatrix}\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}X\left(e_{m_l}\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\ \left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}Y\left(e_{m_l}\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\ \left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}Z\left(e_{m_l}\right)\right)^{(1/C_k^n)}\right)^{(1/n)}\end{pmatrix},\right.$$
$$\left.\begin{pmatrix}1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-A\left(e_{m_l}\right)\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\ 1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-S\left(e_{m_l}\right)\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\ 1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-D\left(e_{m_l}\right)\right)\right)^{(1/C_k^n)}\right)^{(1/n)}\end{pmatrix}\right\}. \tag{10}$$

Hence, (6) is kept.

Then, we need to prove that equation (6) is still an FNIFN. We need to prove two following conditions:

① $(X(e),Y(e),Z(e))\subseteq[0,1]$, $(A(e),S(e),D(e))\subseteq[0,1]$

② $0\le Z(e)+D(e)\le 1$ □

Proof. ① Since $0\le X(e_{m_l})\le 1$, we get

$$0\le\prod_{l=1}^{n}X\left(e_{m_l}\right)\le 1,$$
$$0\le\left(1-\prod_{l=1}^{n}X\left(e_{m_l}\right)\right)^{(1/C_k^n)}\le 1. \tag{11}$$

Then,

$$0\le\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}X\left(e_{m_l}\right)\right)^{(1/C_k^n)}\le 1. \tag{12}$$

Thus,

$$0\le\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}X\left(e_{m_l}\right)\right)^{(1/C_k^n)}\right)^{(1/n)}\le 1. \tag{13}$$

That is to say, $X(e)=(1-\prod_{1\le m_1<\cdots<m_n\le m_k}(1-\prod_{l=1}^{n}X(e_{m_l}))^{(1/C_k^n)})^{(1/n)}\subseteq[0,1]$; similarly, we can get $(Y(e),Z(e))\subseteq[0,1]$ and $(A(e),S(e),D(e))\subseteq[0,1]$, so ① is maintained.

② For $Z(e_{m_l})+D(e_{m_l})\le 1$, we can derive $Z(e_{m_l})\le 1-D(e_{m_l})$; thus,

$$
\begin{aligned}
0 &\le Z(e)+D(e)\\
&=\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}X\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}+1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}D\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/k)}\\
&\le\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}D\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}+1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}D\left(e_{m_l}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/k)}\\
&=1.
\end{aligned}
\tag{14}
$$

□

Example 1. Let $Q(e_1)=\langle(0.1,0.2,0.3),(0.2,0.5,0.6)\rangle$, $Q(e_2)=\langle(0.2,0.3,0.3),(0.2,0.5,0.5)\rangle$, and $Q(e_3)=\langle(0.4,0.5,0.5),(0.3,0.4,0.4)\rangle$ be three FNIFNs and suppose $n=2$, then according to (6), we have

$$
\begin{aligned}
&\mathrm{FNIFMSM}^{(n)}\left(Q(e_1),Q(e_2),\ldots,Q(e_k)\right)=\left(\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n}Q\left(e_{m_l}\right)\right)}{C_k^n}\right)^{(1/n)}\\
&=\left\{\begin{array}{c}
\left(\begin{array}{c}
\left(1-((1-0.1\times0.2)\times(1-0.1\times0.4)\times(1-0.2\times0.4))^{\left(1/C_3^2\right)}\right)^{(1/2)},\\
\left(1-((1-0.2\times0.3)\times(1-0.2\times0.5)\times(1-0.3\times0.5))^{\left(1/C_3^2\right)}\right)^{(1/2)},\\
\left(1-((1-0.3\times0.3)\times(1-0.3\times0.5)\times(1-0.3\times0.5))^{\left(1/C_3^2\right)}\right)^{(1/2)}
\end{array}\right),\\
\left(\begin{array}{c}
1-\left(1-((1-(1-0.2)\times(1-0.2))\times(1-(1-0.2)\times(1-0.3))\times(1-(1-0.2)\times(1-0.3)))^{\left(1/C_3^2\right)}\right)^{(1/2)},\\
1-\left(1-((1-(1-0.5)\times(1-0.5))\times(1-(1-0.5)\times(1-0.4))\times(1-(1-0.5)\times(1-0.4)))^{\left(1/C_3^2\right)}\right)^{(1/2)},\\
1-\left(1-((1-(1-0.6)\times(1-0.5))\times(1-(1-0.6)\times(1-0.4))\times(1-(1-0.5)\times(1-0.4)))^{\left(1/C_3^2\right)}\right)^{(1/2)}
\end{array}\right)
\end{array}\right\}\\
&=\langle(0.2168,0.3227,0.3612),(0.2329,0.4674,0.5023)\rangle.
\end{aligned}
\tag{15}
$$

Next, we explore some properties about the FNIFMSM formula.

Property 1 (idempotency). If $Q(e_m)=\langle(X(e_m),Y(e_m),Z(e_m)),(A(e_m),S(e_m),D(e_m))\rangle, m=1,2,\ldots,k$ are equal, then

$$\mathrm{FNIFMSM}^{(n)}\left(Q(e_1),Q(e_2),\ldots,Q(e_k)\right)=Q(e). \tag{16}$$

Property 2 (monotonicity). Let $Q(e_m)=\langle(X(e_m),Y(e_m),Z(e_m)),(A(e_m),S(e_m),D(e_m))\rangle, m=1,2,\ldots,k$, and $Q(f_m)=\langle(X(f_m),Y(f_m),Z(f_m)),(A(f_m),S(f_m),D(f_m))\rangle, m=1,2,\ldots,k$, be two sets of given FNIFNs. If $X(e_m)\le X(f_m), Y(e_m)\le Y(f_m), Z(e_m)\le Z(f_m)$, $A(e_m)\ge A(f_m), S(e_m)\ge S(f_m)$, and $D(e_m)\ge D(f_m)$ hold for all m, then

$$
\begin{aligned}
&\mathrm{FNIFMSM}^{(n)}\left(Q(e_1),Q(e_2),\ldots,Q(e_k)\right)\\
&\quad\le\mathrm{FNIFMSM}^{(n)}\left(Q(f_1),Q(f_2),\ldots,Q(f_k)\right).
\end{aligned}
\tag{17}
$$

Property 3 (boundedness). Let $Q(e_m)=\langle(X(e_m),Y(e_m),Z(e_m)),(A(e_m),S(e_m),D(e_m))\rangle, m=1,2,\ldots,k$, be a set of given FNIFNs. If $Q(e)^+=\left\{\begin{array}{c}(\max(X(e_m)),\max(Y(e_m)),\max(Z(e_m))),\\(\min(A(e_m)),\min(S(e_m)),\min(D(e_m)))\end{array}\right\}(m=1,2,\ldots,k)$ and $Q(e)^-=\{(\min(X(e_m)),\min(Y(e_m)),\min(Z(e_m))),(\max(A(e_m)),\max(S(e_m)),\max(D(e_m)))\}$ $(m=1,2,\ldots,k)$, then

$$Q(e)^-\le\mathrm{FNIFMSM}^{(n)}\left(Q\left(e_q\right),Q(e_2),\ldots,Q(e_n)\right)\le Q(e)^+. \tag{18}$$

Property 4 (commutativity). Let $Q(e_m)\,(m = 1, 2, \ldots, k)$ be a set of given FNIFNs and $Q(e'_m)\,(m = 1, 2, \ldots, k)$ be any permutation of $Q(e_m)\,(m = 1, 2, \ldots, k)$, then

$$\begin{aligned}&\text{FNIFMSM}^{(n)}\left(Q(e_1), Q(e_2), \ldots, Q(e_m)\right)\\&\quad = \text{FNIFMSM}^{(n)}\left(Q(e'_1), Q(e'_2), \ldots, (e'_m)\right).\end{aligned} \tag{19}$$

3.2. The FNIFWMSM Operator. In real-life MADM, it is crucial to fully take attribute weights into account. We shall build the fuzzy number intuitionistic fuzzy weighted MSM (FNIFWMSM) formula.

Definition 8. Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle$ $(m = 1, 2, \ldots, k)$ be a set of given FNIFNs with weight vector $\xi_m = (\xi_1, \xi_2, \ldots, \xi_k)^T$ and $\xi_m \in [0, 1]$, $\sum_{m=1}^{k} \xi_m = 1$. If

$$\begin{aligned}&\text{FNIFWMSM}_{k\xi}^{(n)}\left(Q(e_1), Q(e_2), \ldots, Q(e_k)\right)\\&\quad = \left(\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n}\left(k\xi_{m_l}\otimes Q\left(e_{m_l}\right)\right)\right)}{C_k^n}\right)^{(1/n)},\end{aligned} \tag{20}$$

then we called $\text{FNIFWMSM}_{k\xi}^{(n)}$ the fuzzy number intuitionistic fuzzy weighted MSM (FNIFWMSM) formula.

Theorem 2. *Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle$ $(i = 1, 2, \ldots, n)$ be a set of given FNIFNs. The coalesced data obtained from the FNIFWMSM formula are still a FNIFN.*

$$\begin{aligned}\text{FNIFWMSM}_{k\xi}^{(n)}\left(Q(e_1), Q(e_2), \ldots, Q(e_k)\right) &= \left(\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n}\left(k\xi_{m_l}\otimes Z\left(e_{m_l}\right)\right)\right)}{C_k^n}\right)^{(1/n)}\\
&= \left\{\begin{pmatrix}\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-X\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\
\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Y\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\
\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Z\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}\right)^{(1/n)}\end{pmatrix},\right.\\
&\left.\begin{pmatrix}1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(A\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\
1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(S\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}\right)^{(1/n)},\\
1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(D\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}\right)^{(1/n)}\end{pmatrix}\right\}.\end{aligned} \tag{21}$$

Proof. According to Definition 2, we could derive

$$k\xi_{m_l} \otimes Z(e_{m_l}) = \left\{ \begin{array}{c} \left(1-\left(1-X(e_{m_l})\right)^{k\xi_{m_l}}, 1-\left(1-Y(e_{m_l})\right)^{k\xi_{m_l}}, 1-\left(1-Z(e_{m_l})\right)^{k\xi_{m_l}}\right), \\ \left(\left(A(e_{m_l})\right)^{\xi_{m_l}}, \left(S(e_{m_l})\right)^{\xi_{m_l}}, \left(D(e_{m_l})\right)^{\xi_{m_l}}\right) \end{array} \right\}. \tag{22}$$

Thus,

$$\begin{aligned} &\otimes_{l=1}^{n}\left(k\xi_{m_l} \otimes Q(e_{m_l})\right) \\ &= \left\{ \begin{array}{c} \left(\prod_{l=1}^{n}\left(1-\left(1-X(e_{m_l})\right)^{k\xi_{m_l}}\right), \prod_{l=1}^{n}\left(1-\left(1-Y(e_{m_l})\right)^{k\xi_{m_l}}\right), \prod_{l=1}^{n}\left(1-\left(1-Z(e_{m_l})\right)^{k\xi_{m_l}}\right)\right), \\ \left(1-\prod_{l=1}^{n}\left(1-\left(A(e_{m_l})\right)^{k\xi_{m_l}}\right), 1-\prod_{l=1}^{n}\left(1-\left(S(e_{m_l})\right)^{k\xi_{m_l}}\right), 1-\prod_{l=1}^{n}\left(1-\left(D(e_{m_l})\right)^{k\xi_{m_l}}\right)\right) \end{array} \right\}. \end{aligned} \tag{23}$$

Thereafter,

$$\begin{aligned} &\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n}\left(k\xi_{m_l} \otimes Q(e_{m_l})\right)\right) \\ &= \left\{ \left(\begin{array}{c} 1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-X(e_{m_l})\right)^{k\xi_{m_l}}\right)\right), \\ 1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Y(e_{m_l})\right)^{k\xi_{m_l}}\right)\right), \\ 1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Z(e_{m_l})\right)^{k\xi_{m_l}}\right)\right) \end{array} \right), \left(\begin{array}{c} \prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(A(e_{m_l})\right)^{k\xi_{m_l}}\right)\right), \\ \prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(S(e_{m_l})\right)^{k\xi_{m_l}}\right)\right), \\ \prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(D(e_{m_l})\right)^{k\xi_{m_l}}\right)\right) \end{array} \right) \right\}. \end{aligned} \tag{24}$$

Furthermore,

$$\begin{aligned} &\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n}\left(k\xi_{m_l} \otimes Q(e_{m_l})\right)\right)}{C_k^n} \\ &= \left\{ \left(\begin{array}{c} 1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-X(e_{m_l})\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}, \\ 1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Y(e_{m_l})\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}, \\ 1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Z(e_{m_l})\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)} \end{array} \right), \left(\begin{array}{c} \prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(A(e_{m_l})\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}, \\ \prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(S(e_{m_l})\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)}, \\ \prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(D(e_{m_l})\right)^{k\xi_{m_l}}\right)\right)^{(1/C_k^n)} \end{array} \right) \right\}. \end{aligned} \tag{25}$$

Therefore,

$$\begin{aligned}
&\mathrm{FNIFWMSM}_{k\xi}^{(n)}\left(Q\left(e_1\right),Q\left(e_2\right),\ldots,Q\left(e_k\right)\right)=\left(\frac{\oplus_{1\le m_1<\cdots<m_n\le m_k}\left(\otimes_{l=1}^{n}\left(k\xi_{m_l}\otimes Q\left(e_{m_l}\right)\right)\right)}{C_k^n}\right)^{(1/n)}\\
&=\left\{\left(\begin{array}{l}\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-X\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)},\\
\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Y\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)},\\
\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-Z\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}\end{array}\right),\right.\\
&\left.\left(\begin{array}{l}1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(A\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)},\\
1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(S\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)},\\
1-\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(D\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}\end{array}\right)\right\}.
\end{aligned}\tag{26}$$

Hence, (21) is kept.

Then, we could prove that equation (21) is an FNIFN. We need to prove two following conditions:

① $(X(e),Y(e),Z(e))\subseteq[0,1]$, $(A(e),S(e),D(e))\subseteq[0,1]$

② $0\le Z(e)+D(e)\le 1$ □

Proof. ① Since $0\le X(e_{m_l})\le 1$, we get

$$\begin{aligned}
&0\le\left(1-X\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\le 1,\\
&0\le\prod_{l=1}^{n}\left(1-\left(1-X\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\le 1.
\end{aligned}\tag{27}$$

Then,

$$0\le\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-X\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\le 1.\tag{28}$$

Thus,

$$0\le\left(1-\prod_{1\le m_1<\cdots<m_n\le m_k}\left(1-\prod_{l=1}^{n}\left(1-\left(1-X\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)}\le 1.\tag{29}$$

That means $X(e)=(1-\prod_{1\le m_1<\cdots<m_n\le m_k}(1-\prod_{l=1}^{n}(1-(1-X(e_{m_l}))^{k\xi_{m_l}}))^{(1/C_k^n)})^{(1/n)}\subseteq[0,1]$; similarly, we can get $(Y(e),Z(e))\subseteq[0,1]$, and $(A(e),S(e),D(e))\subseteq[0,1]$, so ① is maintained.

② For $Z(e_{m_l}) + D(e_{m_l}) \leq 1$, we can derive $D(e_{m_l}) \leq 1 - Z(e_{m_l})$; thus,

$$\begin{aligned} 0 \leq Z(e) + D(e) = & \left(1 - \prod_{1 \leq m_1 < \cdots < m_n \leq m_k} \left(1 - \prod_{l=1}^{n} \left(1 - \left(1 - Z\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)} \\ & + 1 - \left(1 - \prod_{1 \leq m_1 < \cdots < m_n \leq m_k} \left(1 - \prod_{l=1}^{n} \left(1 - \left(D\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/k)} \\ \leq & \left(1 - \prod_{1 \leq m_1 < \cdots < m_n \leq m_k} \left(1 - \prod_{l=1}^{n} \left(1 - \left(1 - Z\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)} \\ & + 1 - \left(1 - \prod_{1 \leq m_1 < \cdots < m_n \leq m_k} \left(1 - \prod_{l=1}^{n} \left(1 - \left(1 - Z\left(e_{m_l}\right)\right)^{k\xi_{m_l}}\right)\right)^{\left(1/C_k^n\right)}\right)^{(1/n)} = 1. \end{aligned} \tag{30}$$

□

Example 2. Let $Q(e_1) = \langle (0.1, 0.2, 0.3), (0.2, 0.5, 0.6) \rangle$, $Q(e_2) = \langle (0.2, 0.3, 0.3), (0.2, 0.5, 0.5) \rangle$, and $Q(e_3) = \langle (0.4, 0.5, 0.5), (0.3, 0.4, 0.4) \rangle$ be three given FNIFNs and suppose $n = 2$ and $\xi = (0.2, 0.3, 0.5)$, then according to (21), we have

$$\begin{aligned} & \text{FNIFWMSM}_{k\xi}^{(n)}\left(Q(e_1), Q(e_2), \ldots, Q(e_k)\right) = \left(\frac{\oplus_{1 \leq m_1 < \cdots < m_n \leq m_k}\left(\otimes_{l=1}^{n}\left(k\xi_{m_l} \otimes Q\left(e_{m_l}\right)\right)\right)}{C_k^n}\right)^{(1/n)} \\ & = \left\{ \begin{array}{l} \left(\begin{array}{l} \left(1 - \left(\begin{array}{c} \left(1 - \left(1 - (1-0.1)^{0.6}\right) \times \left(1 - (1-0.2)^{0.9}\right)\right) \times \left(1 - \left(1 - (1-0.1)^{0.6}\right) \times \left(1 - (1-0.4)^{1.5}\right)\right) \\ \times \left(1 - \left(1 - (1-0.2)^{0.9}\right) \times \left(1 - (1-0.4)^{1.5}\right)\right) \end{array}\right)^{\left(1/C_3^2\right)}\right)^{(1/2)}, \\ \left(1 - \left(\begin{array}{c} \left(1 - \left(1 - (1-0.2)^{0.6}\right) \times \left(1 - (1-0.3)^{0.9}\right)\right) \times \left(1 - \left(1 - (1-0.2)^{0.6}\right) \times \left(1 - (1-0.5)^{1.5}\right)\right) \\ \times \left(1 - \left(1 - (1-0.3)^{0.9}\right) \times \left(1 - (1-0.5)^{1.5}\right)\right) \end{array}\right)^{\left(1/C_3^2\right)}\right)^{(1/2)}, \\ \left(1 - \left(\begin{array}{c} \left(1 - \left(1 - (1-0.3)^{0.6}\right) \times \left(1 - (1-0.3)^{0.9}\right)\right) \times \left(1 - \left(1 - (1-0.3)^{0.6}\right) \times \left(1 - (1-0.5)^{1.5}\right)\right) \\ \times \left(1 - \left(1 - (1-0.3)^{0.9}\right) \times \left(1 - (1-0.5)^{1.5}\right)\right) \end{array}\right)^{\left(1/C_3^2\right)}\right)^{(1/2)} \end{array} \right), \\ \left(\begin{array}{l} 1 - \left(1 - \left(\begin{array}{c} \left(1 - \left(1 - 0.2^{0.6}\right) \times \left(1 - 0.2^{0.9}\right)\right) \times \left(1 - \left(1 - 0.2^{0.6}\right) \times \left(1 - 0.3^{1.5}\right)\right) \\ \times \left(1 - \left(1 - 0.2^{0.9}\right) \times \left(1 - 0.3^{1.5}\right)\right) \end{array}\right)^{\left(1/C_3^2\right)}\right)^{(1/2)}, \\ 1 - \left(1 - \left(\begin{array}{c} \left(1 - \left(1 - 0.5^{0.6}\right) \times \left(1 - 0.5^{0.9}\right)\right) \times \left(1 - \left(1 - 0.5^{0.6}\right) \times \left(1 - 0.4^{1.5}\right)\right) \\ \times \left(1 - \left(1 - 0.5^{0.9}\right) \times \left(1 - 0.4^{1.5}\right)\right) \end{array}\right)^{\left(1/C_3^2\right)}\right)^{(1/2)}, \\ 1 - \left(1 - \left(\begin{array}{c} \left(1 - \left(1 - 0.6^{0.6}\right) \times \left(1 - 0.5^{0.9}\right)\right) \times \left(1 - \left(1 - 0.6^{0.6}\right) \times \left(1 - 0.4^{1.5}\right)\right) \\ \times \left(1 - \left(1 - 0.5^{0.9}\right) \times \left(1 - 0.4^{1.5}\right)\right) \end{array}\right)^{\left(1/C_3^2\right)}\right)^{(1/2)} \end{array} \right) \end{array} \right\} \\ & = \langle (0.2187, 0.3157, 0.3462), (0.2589, 0.4932, 0.5226) \rangle. \end{aligned} \tag{31}$$

Then, we will discuss some properties of FNIFWMSM operator.

Property 5 (idempotency). If $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle$ $(m = 1, 2, \ldots, k)$ are equal, then

$$\text{FNIFWMSM}_{k\xi}^{(n)}(Q(e_1), Q(e_2), \ldots, Q(e_k)) = Q(e). \quad (32)$$

Property 6 (monotonicity). Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle, m = 1, 2, \ldots, k$, and $Q(f_m) = \langle (X(f_m), Y(f_m), Z(f_m)), (A(f_m), S(f_m), D(f_m))\rangle, m = 1, 2, \ldots, k$, be two sets of given FNIFNs. If $X(e_m) \leq X(f_m), Y(e_m) \leq Y(f_m), Z(e_m) \leq Z(f_m), A(e_m) \geq A(f_m), S(e_m) \geq S(f_m), D(e_m) \geq D(f_m)$ hold for all m, then

$$\begin{aligned} &\text{FNIFWMSM}_{k\xi}^{(n)}(Q(e_1), Q(e_2), \ldots, Q(e_k)) \\ &\quad \leq \text{FNIFWMSM}_{k\xi}^{(n)}(Q(f_1), Q(f_2), \ldots, Q(f_k)). \end{aligned} \quad (33)$$

Property 7 (boundedness). Let $Q(e_m) = \langle (X(e_m), Y(e_m), Z(e_m)), (A(e_m), S(e_m), D(e_m))\rangle, m = 1, 2, \ldots, k$, be a set of given FNIFNs. If $Q(e)^+ = \left\{ \begin{matrix} (\max(X(e_m)), \max(Y(e_m)), \max(Z(e_m))), \\ (\min(A(e_m)), \min(S(e_m)), \min(D(e_m))) \end{matrix} \right\}$ $(m = 1, 2, \ldots, k)$ and $Q(e)^- = \left\{ \begin{matrix} (\min(X(e_m)), \min(Y(e_m)), \min(Z(e_m))), \\ (\max(A(e_m)), \max(S(e_m)), \max(D(e_m))) \end{matrix} \right\}$ $(m = 1, 2, \ldots, k)$, then

$$Q(e)^- \leq \text{FNIFWMSM}_{k\xi}^{n}\left(Q\left(e_q\right), Q(e_2), \ldots, Q(e_k)\right) \leq Q(e)^+. \quad (34)$$

Property 8 (commutativity). Let $Q(e_m)$ $(m = 1, 2, \ldots, k)$ be a set of FNIFNs and $Q(e_m')$ $(m = 1, 2, \ldots, k)$ be any permutation of $Q(e_m)$ $(m = 1, 2, \ldots, k)$, then

$$\begin{aligned} &\text{FNIFWMSM}_{k\xi}^{n}(Q(e_1), Q(e_2), \ldots, Q(e_m)) \\ &\quad = \text{FNIFWMSM}_{k\xi}^{n}(Q(e_1'), Q(e_2'), \ldots, (e_m')). \end{aligned} \quad (35)$$

4. Numerical Example

The quality of higher education has always been an important issue of general concern. It is also a hot issue that has been widely concerned by all sectors of society since the popularization of higher education in China. At the end of the last century, colleges and universities began to expand the enrollment scale beyond the norm, among which the enrollment of higher vocational colleges increased the most. According to the national long-term development plan and outline for education in 2010–2020, by 2020, the number of students in higher vocational colleges should account for nearly half of the total number of students of all universities and colleges. The education of higher vocational colleges has become a key link in the system of cultivating talents in today's society, and its importance is self-evident. With the rapid growth of enrollment in higher vocational colleges, the hardware facilities well equipped and the urgency of improving teaching quality is becoming increasingly prominent. The Ministry of Education has also issued a series of documents requiring higher vocational colleges to strengthen the quality awareness, especially to strengthen the construction of the teaching quality evaluation system of all disciplines in higher vocational colleges. At the same time, due to the stable and even reduced number of new high school graduates in recent years, general colleges and higher vocational colleges have launched fierce competition in enrollment. If higher vocational colleges want to survive and develop, they must attach great importance to the teaching quality, the construction of teaching staff, the creation of a good learning atmosphere for students, and the improvement of vocational skills. With innovative education concept, higher vocational colleges should improve the system construction of all institutions of the college, build a teaching mechanism with social demand as the guide, employment as the goal, and training high-quality and high skilled vocational talents as the responsibility, and establish the development direction for colleges as well. English teaching plays an indispensable role in higher vocational education. In recent years, various relevant documents issued by the Ministry of Education have mentioned the urgency and importance of colleges and universities for the cultivation of high-quality, multiskilled international talents. With the deep promotion of global economic integration, talents who master English pragmatic ability and are familiar with professional English and professional skills have shown great advantages in learning advanced foreign knowledge and understanding advanced foreign concepts and skills and other international aspects. It can be seen that the English education of higher vocational colleges is undertaking more and more responsibilities for cultivating high-quality and high skilled talents in the new era and new development environment, and the improvement of the English teaching quality in higher vocational colleges requires the improvement of the teaching quality evaluation system to better monitor and guide. A point in case about the selection of the excellent college English teachers with FNIFNs would be utilized to illustrate the above methods. We shall give 5 college English teachers H_i $(i = 1, 2, 3, 4, 5)$ to choose. The experts select four attributes to evaluate these college English teachers: ① J_1 represents teaching content; ② J_2 means teachers' specialization degree; ③ J_3 represents teachers' artistic accomplishment; and ④ J_4 means the teachers' appreciation ability. Several college English teachers shall be depicted with FNIFNs by the DMs on the strength of 4 criterions (whose weighting vector $\xi = (0.15, 0.40, 0.25, 0.20)$); the FNIFN decision matrix is depicted in Table 1.

Then, we shall use the developed method to select the excellent college English teachers.

Step 1. According to Table 1, we can fuse all FNIFNs r_{ij} by FNIFWMSM; use $U = (1, 2, 3, 4)$ and

Table 1: The FNIFN DM.

	J_1	J_2	J_3	J_4
H_1	$\langle(0.1, 0.2, 0.3), (0.2, 0.5, 0.6)\rangle$	$\langle(0.2, 0.3, 0.3), (0.2, 0.5, 0.5)\rangle$	$\langle(0.4, 0.5, 0.5), (0.3, 0.4, 0.4)\rangle$	$\langle(0.2, 0.3, 0.3), (0.4, 0.4, 0.6)\rangle$
H_2	$\langle(0.5, 0.7, 0.7), (0.1, 0.2, 0.2)\rangle$	$\langle(0.3, 0.4, 0.5), (0.1, 0.2, 0.4)\rangle$	$\langle(0.4, 0.4, 0.6), (0.2, 0.2, 0.3)\rangle$	$\langle(0.5, 0.6, 0.6), (0.2, 0.3, 0.3)\rangle$
H_3	$\langle(0.4, 0.5, 0.6), (0.2, 0.3, 0.3)\rangle$	$\langle(0.6, 0.6, 0.7), (0.1, 0.1, 0.2)\rangle$	$\langle(0.5, 0.6, 0.6), (0.1, 0.2, 0.2)\rangle$	$\langle(0.6, 0.6, 0.7), (0.1, 0.1, 0.2)\rangle$
H_4	$\langle(0.2, 0.2, 0.4), (0.3, 0.3, 0.4)\rangle$	$\langle(0.1, 0.3, 0.4), (0.2, 0.3, 0.5)\rangle$	$\langle(0.4, 0.6, 0.6), (0.1, 0.2, 0.3)\rangle$	$\langle(0.1, 0.4, 0.5), (0.2, 0.3, 0.4)\rangle$
H_5	$\langle(0.1, 0.1, 0.4), (0.3, 0.3, 0.4)\rangle$	$\langle(0.4, 0.5, 0.5), (0.3, 0.3, 0.4)\rangle$	$\langle(0.3, 0.4, 0.4), (0.4, 0.5, 0.5)\rangle$	$\langle(0.2, 0.2, 0.4), (0.1, 0.2, 0.2)\rangle$

Table 2: The coalesced values by the FNIFWMSM operators.

Aggregation operator	College English teachers	Operation results
	H_1	<(0.2207, 0.3191, 0.3385), (0.3055, 0.4679, 0.5362)>
	H_2	<(0.3999, 0.4978, 0.5760), (0.1830, 0.2536, 0.3227)>
FNIFWMSM	H_3	<(0.5107, 0.5571, 0.6255), (0.1795, 0.1996, 0.2503)>
	H_4	<(0.1770, 0.3659, 0.4599), (0.2181, 0.2979, 0.4215)>
	H_5	<(0.2518, 0.3017, 0.4110), (0.2902, 0.3487, 0.3941)>

Table 3: The SF of the college English teachers.

	FNIFWMSM
H_1	−0.1450
H_2	0.2397
H_3	0.3554
H_4	0.0333
H_5	−0.0289

Table 4: Order of the college English teachers.

	Order
FNIFWMSM	$H_3 > H_2 > H_4 > H_5 > H_1$

FNIFWDMSM operator to calculate global FNIFNs $H_i\,(i = 1, 2, 3, 4, 5)$ of the college English teachers H_i; the coalesced values are shown in Table 2 $(n = 2)$.

Step 2. The SF of college English teachers is calculated in Table 3.

Step 3. According to T3, the order of the college English teachers is depicted in Table 4. Note that ">" means "preferred to." The best college English teachers are H_3.

5. Conclusion

As we all know, the demand for English majors is obviously on the rise, which puts forward new and higher requirements for application-oriented undergraduate colleges to train compound English majors. However, from the perspective of teaching quality evaluation of English majors in application-oriented undergraduate colleges, the results are not optimistic. Therefore, it is an important task for higher education research in China to explore the problems existing in the process of teaching quality evaluation for English majors in application-oriented undergraduate colleges and how to better train qualified and versatile talents for English majors to adapt to the economic and social development in the new era. We study the MADM issues with FNIFNs and utilize the MSM formula to build several MSM fused formulas with FNIFNs: FNIFMSM operator and FNIFWMSM formula. The characteristic of these two operators is also deliberated. The FNIFWMSM formula is utilized to cope with the MADM issues with FNIFNs. Finally, a point in case for English teaching quality evaluation is employed to depict the raised method. Later, the adhibition and expansion of the built formulas of FNIFNs would be debated in the other MADM direction [49–56].

Data Availability

The data used to support the findings of this study are included in the article.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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