

Research Article

Equilibrium and Welfare Analysis in Second-Price Auctions with Resale and Costly Entry

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This study investigates the effects of resale allowance on entry strategies, seller's expected revenue, and social welfare in a second-price auction with two-dimensional private information on values and participation costs. We characterize the perfect Bayesian equilibrium in cutoff strategies and identify sufficient conditions under which the equilibrium is unique. Our analysis suggests that resale allowance leads the low-value bidder to become more aggressive on entry, while high-value bidder has a lower incentive to enter. Furthermore, the allowance of resale can increase the social welfare under a sufficient condition, and its effect on expected revenue is ambiguous.

1. Introduction

When bidders participate in an auction, they often incur participation costs. For instance, sellers may charge an entry fee or require registration or pre-qualification for an auction. It may be costly for bidders to prepare bids, travel to the auction site, or acquire information about the auction rules and the values of the object to be auctioned. In the presence of such participation costs, not all potential bidders are willing to participate in an auction. In this case, to participate or not, as well as how to bid, should be modeled collectively. If the bidder with the highest value is excluded from the auction, the auction outcome is necessarily inefficient ex post. This sort of inefficiency may create a motive for post-auction resale and such resale opportunity affects significantly, as we shall show, bidders' entry strategies, social welfare, and expected revenue.

In this paper, we study the effects of a resale opportunity in second-price auctions when bidders are independently and privately informed about their participation costs and their valuations. Introducing post-auction resale into models with costly entry enriches the analysis, yet it complicates matters so that the existence and uniqueness of entry equilibrium are no longer obvious. The additional difficulty

emerges since, with resale, the option value for staying out is also positive and varies with types. To make our analysis tractable, we apply a simple setting where the values, following a binomial distribution, can only be either high (v_h) or low (v_l). The resale opportunity in the auction can have important implications for policy design and empirical studies on auctions. We believe that our research can bring insights on the study of second-price auctions with participation costs and resale under more general distributions on valuations.

First, we characterize the perfect Bayesian equilibrium in cutoff strategies. For each case, we show that there is always an "intuitive" equilibrium in which the higher-value bidder participates in the auction more frequently than the lower-value bidder does. While the symmetric entry equilibrium is unique in the no-resale benchmark, the uniqueness of the symmetric equilibrium cannot be established when resale is allowed. We also identify sufficient conditions that assure the uniqueness of the symmetric entry equilibrium in the resale case. Then, we compare the symmetric entry equilibrium when resale is allowed to the equilibrium when resale is banned. Our first finding is that, with resale, the entry cutoff is higher for low-value bidders and lower for high-value bidders. This suggests that when resale is allowed,

low-value bidders become more aggressive on entry and high-value bidders are less likely to enter. Finally, we investigate the effects of a resale opportunity on seller's expected revenue and social welfare. Our findings suggest that the opportunity of resale can increase the social welfare under a sufficient condition, and its effect on expected revenue is ambiguous, which has been well-documented in other auctions.

Few studies incorporate resale with costly entry. Che et al. [1] investigate the effect of resale allowance on entry strategies in a second-price auction with two bidders whose entries are sequential and costly. They show that there exists a unique threshold such that if the reseller's bargaining power is greater (less) than the threshold, resale allowance causes the leading bidder (the following bidder) to have a higher (lower) incentive on entry. Celik and Yilankaya [2] consider a setting in which valuations are private, but each bidder's cost is identical and commonly known. There may be resale due to asymmetric cutoff equilibria, and the equilibria under resale are more asymmetric. Akyol [3] shows that resale induces more symmetric equilibria, higher revenues for the seller, and higher social welfare when valuations are commonly known, but costs are private. Akyol [4] considers second-price auctions with participation costs and investigates the revenue effects of a resale possibility. When values are drawn from a uniform distribution, resale increases (decreases) entry of the lower-cost (higher-cost) bidder and decreases the original seller's expected revenue. These research studies on the equilibrium of second-price auctions apply single-dimensional framework, where either the valuations or the participation costs are common knowledge. The closest paper to ours is by Xu et al. [5] who study the effect of resale allowance on entry strategies in a second-price auctions with two-dimensional private information on values and participation costs. They show that a symmetric equilibrium exists and is unique under some conditions. However, owing to the existence of incomplete multidimensional information, only some numerical examples can be derived for the effect of resale on revenue and efficiency.

The study proceeds as follows. Section 2 presents the model, after which Section 3 describes the equilibrium for the resale and no-resale cases. This section also compares the equilibria between the two cases. Sections 4 and 5 present the results for the seller's revenue and social welfare, respectively. Lastly, we conclude the paper. Appendix contains all proofs omitted from the main body of the study.

2. The Setup

We consider an environment with one seller and two potential bidders. The seller is risk neutral and has an indivisible object to be auctioned. The seller values the object at zero. Each bidder i 's value is $v_i \in \{v_l, v_h\}$ with $0 < v_l < v_h \leq 1$, and they are risk neutral and independent. It is assumed that $\forall i$, $\Pr(v_i = v_h) = \alpha$ and $\Pr(v_i = v_l) = 1 - \alpha$.

In order to participate in the auction, bidder i must incur a nonrefundable participation cost c_i which is a private information and drawn from a distribution function $G(\cdot)$ with support $[0, 1]$. The corresponding

density for participation costs is $g(c)$. When a bidder is indifferent between participating in the seller's auction or not, we assume that he participates for illustration convenience. When a bidder submits a bid, he knows who else will submit a bid. Bidders do not know the participation decisions of other bidders when they make their own decisions.

The auction format is the usual second-price sealed-bid auction. A bidder with the highest bid wins the auction and pays the second highest bid. We investigate two cases. In the benchmark case, which we call the no-resale case, the game ends after the auction ends. In the second case, which we call the resale case, after the auction takes place, the winner of the object, if any, has the opportunity to sell the object to the other bidder by a standard Nash bargaining game. The bargaining power parameters of the reseller and the buyer are λ and $1 - \lambda$, respectively, where $\lambda \in (0, 1)$. Furthermore, we assume that the resale stage is costless.

Since bidders are risk neutral, they participate in the auction if and only if the expected payoff they can get from participating is greater than or equal to his expected revenue from staying out of the initial auction; we naturally restrict ourselves to equilibria in which each bidder uses a cutoff entry strategy. That is, if a bidder's participation cost is below a certain threshold level, he participates in the auction. Otherwise, he chooses not to participate in the auction. In both cases, we look for (Perfect Bayesian Nash) equilibria where a bidder's strategy consists of an entry decision and bidding behavior. More precisely, the individually rational action set for any type of bidder is $\{\text{No}\} \cup [0, 1]$, where " $\{\text{No}\}$ " denotes not participating in the auction. Bidder i incurs the participation cost if and only if his action is different from " $\{\text{No}\}$ ". Then, bidder i 's strategy, denoted by $b_i(v_i, c_i)$, can be expressed as follows:

$$b_i(v_i, c_i) = \begin{cases} b_i(v_i, m), & \text{if } c_i \leq c'_i, \\ \text{No}, & \text{otherwise,} \end{cases} \quad (1)$$

where c'_i denotes bidder i 's entry cutoff and m denotes the number of bidders participating in the auction. For the game described above, each bidder's action is to choose a cutoff and decides how to bid when he participates. Thus, an (Bayesian-Nash) equilibrium of the sealed-bid second-price auction with participation costs is composed of bidders' cutoff strategies as well as the corresponding bidding strategies.

3. Equilibrium Characterization

In this section, we characterize the equilibrium in the no-resale benchmark case and in the case when resale is allowed. We will focus on the symmetric entry equilibrium characterized by a pair of entry thresholds, (c_l^*, c_h^*) in the case when resale is banned and $(\tilde{c}_l, \tilde{c}_h)$ in the case when resale is allowed, so that a bidder with a type (c, v_k) enters the auction if and only if $c \leq c_k^*$ in the no-resale benchmark and $c \leq \tilde{c}_k$ in the case with resale, $k \in \{l, h\}$; in other words, bidders with the same value will follow the same entry cutoff in a symmetric entry equilibrium.

Definition 1. When two bidders have the same distribution on participation costs, an equilibrium $c^* = (c_1^*, c_2^*)$ is a symmetric equilibrium if $c_1^* = c_2^* = (c_l^*, c_h^*)$; otherwise, it is an asymmetric equilibrium.

3.1. No-Resale Case. Note that, once a bidder enters the auction, he can observe who has also entered the auction and thus update his belief about others' valuation distributions. If we observe that bidder i participates in the auction, it can be inferred that bidder i 's cost is lower than or equal to c_i^* . Then, by Bayes' rule, for bidders who enter the auction, $\Pr(v_i = v_h) = (\alpha G(c_h^*)/\alpha G(c_h^*) + (1 - \alpha)G(c_l^*))$ and $\Pr(v_i = v_l) = ((1 - \alpha)G(c_l^*)/\alpha G(c_h^*) + (1 - \alpha)G(c_l^*))$.

We focus on the symmetric equilibrium in which all bidders use the same cutoff strategy $c_i^* = (c^*(v_l), c^*(v_h))$ or (c_l^*, c_h^*) for notational convenience. We also know $0 < c_i^* < 1$, since bidder i will not enter if his cost is higher than his valuation ($c_i^* \leq v_i < 1$). In the first bidding stage, upon entry, it is a weakly dominant strategy for each bidder to bid his value, which is shown by Tan and Yilankaya [6], Xu et al. [5], Cao and Tian [7], Celik and Yilankaya [2], and Cao et al. [8]. In the entrance decision making stage, a bidder participates if and only if $c_i \leq c_i^*$. When $c_i = c_i^*(v_i)$, the expected revenue from participating is equal to his participation cost. For any bidder i , the expected revenues from participating are given by the following equations for $v_i = v_l$ and $v_i = v_h$, respectively.

For bidder i with value v_l , he has positive surplus only when he is the only bidder submitting a bid. In this case, he can bid 0 and pay nothing. Thus, low-value bidder, with cost c_l^* , has an expected payoff of

$$-c_l^* + v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)], \quad (2)$$

from participating in the auction. If he does not enter, his payoff is 0.

For bidder i with value v_h , the expected payoff can be divided into two parts. The first part is the payoff when he is the only bidder submitting a bid. In this case, he can bid 0 and obtain the object being auctioned. The second part is the expected payoff when there is another bidder submitting bid. Thus, high-value bidder, with cost c_h^* , has an expected payoff of

$$\begin{aligned} & -c_h^* + v_h \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)] + (v_h - v_l)(1 - \alpha)G(c_l^*) \\ & = -c_h^* + v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)] + (v_h - v_l) \times [1 - \alpha G(c_h^*)], \end{aligned} \quad (3)$$

from participating in the auction. If he does not enter, his payoff is 0.

In equilibrium, bidder i with c_i^* must be indifferent between entering and not entering. Therefore, the equilibrium (c_l^*, c_h^*) is a solution of the following equation system:

$$(P1) \begin{cases} c_l^* = v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)], \\ c_h^* = v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)] + (v_h - v_l) \times [1 - \alpha G(c_h^*)]. \end{cases} \quad (4)$$

We then have the following result.

Proposition 1. Suppose that each bidder's participation cost is independently drawn from a distribution function $G(\cdot)$ with support $[0, 1]$. In the no-resale benchmark case, there always exists a unique symmetric equilibrium (c_l^*, c_h^*) where each bidder uses the same cutoff strategy determined by (P1).

3.2. Resale Case. We now augment the model just analyzed by allowing a resale stage where the auction winner may resell the item. Similar to the benchmark case without resale, we will focus on symmetric entry equilibria characterized by entry thresholds (\hat{c}_l, \hat{c}_h) . In reality, resale can be conducted in different formats, such as bargaining, optimal auction, and monopoly pricing. In this study, we follow Gupta and Lebrun [9], Pagnozzi [10], Cheng [11], and Zhang and Wang [12] to assume that the resale stage is conducted in a standard Nash bargaining game. The bargaining power parameters of the reseller and the buyer are λ and $1 - \lambda$, respectively, where $\lambda \in (0, 1)$.

To characterize symmetric entry equilibria, we start our analysis from the last stage. Given the special features in our model, it is easily seen that, in equilibrium, the initial auction winner can only possibly benefit from resale when his value is v_l . So, resale can only be initiated when low-value bidder wins the initial auction, and the potential buyer participating in resale must be a high-value bidder who stays out of the initial auction.

Next, we consider the auction stage. Note that, if bidder i enters the auction while the other does not enter, bidder i bids zero. If both bidders participate in the auction, upon entry, the auction format is the standard second-price auction. In that case, there is an equilibrium in which both bidders bid their values. Absent costs are shown by Hafalir and Krishna [13] and Virág [14]. Since participation costs are sunk costs, they do not affect the analysis. Thus, "bid-your-value" constitutes a robust equilibrium independent of the value distributions. When there are more than two potential bidders, "bid-your-value" may not be an equilibrium. In this case, the low-value bidder would bid the "adjusted value," that is, the expected value from a possible resale to the high-value bidder, which is larger than his true value. This is shown by Xu et al. [5] and Celik and Yilankaya [2].

Finally, we look at entry behavior in the initial auction. When there is a resale opportunity after the auction ends, the lower-value bidder will still get the object only if no other bidder enters the auction, and if possible, he can sell it in the resale stage at a price $\lambda v_l + (1 - \lambda)v_h$. Thus, the low-value bidder with cost \hat{c}_l has an expected payoff of

$$\begin{aligned} & -\hat{c}_l + v_l(1 - \alpha)[1 - G(\hat{c}_l)] + [\lambda v_l + (1 - \lambda)v_h]\alpha[1 - G(\hat{c}_h)] \\ & = -\hat{c}_l + v_l \times [1 - \alpha G(\hat{c}_h) - (1 - \alpha)G(\hat{c}_l)] + \alpha(1 - \lambda)(v_h - v_l)[1 - G(\hat{c}_h)], \end{aligned} \quad (5)$$

from participating. If he does not enter, he will get 0 with certainty, regardless of the other bidder's entry decision.

For bidder i with value v_h , he will get the object with certainty if he enters the auction; the expected payoff can be divided into two parts. The first part is the payoff when he is the only bidder submitting a bid. In this case, he can bid 0

and obtain the object being auctioned. The second part is the expected payoff when there is other bidder submitting bids. In this case, bidder will bid truth value. Thus, the high-value bidder with cost \widehat{c}_h has an expected payoff of

$$-\widehat{c}_h + v_h \times [1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] + (v_h - v_l)(1 - \alpha)G(\widehat{c}_l), \quad (6)$$

from participating. Note that if he does not enter, he will get the object in the resale stage if low-value bidder enters the auction in the first stage, but will have to pay $\lambda v_l + (1 - \lambda)v_h$. Thus, his expected payoff is

$$[v_h - \lambda v_l - (1 - \lambda)v_h](1 - \alpha)G(\widehat{c}_l) = \lambda(v_h - v_l)(1 - \alpha)G(\widehat{c}_l). \quad (7)$$

In equilibrium, bidder i with \widehat{c}_i must be indifferent between entering and not entering. Therefore, the equilibrium $(\widehat{c}_l, \widehat{c}_h)$ is a solution of the following equation system:

$$(P2) \begin{cases} \widehat{c}_l = v_l \times [1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] + \alpha(1 - \lambda)(v_h - v_l)[1 - G(\widehat{c}_h)], \\ \widehat{c}_h = v_h \times [1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] + (1 - \lambda)(v_h - v_l)(1 - \alpha)G(\widehat{c}_l). \end{cases} \quad (8)$$

Proposition 2. *Assume that each bidder's participation cost is independently drawn from a distribution function $G(\cdot)$ with positive density $g(c_i)$ over $(0,1)$. In the resale case, (i) there always exists a symmetric equilibrium where each bidder uses the same cutoff strategy $(\widehat{c}_l, \widehat{c}_h)$, and (ii) if $\sup_{[0,1]} g(c_i) < (1/v_h - v_l)$, then the symmetric equilibrium is unique.*

The condition in Proposition 2 can be easily satisfied, for instance, when participation costs are more dispersed. As an illustrative example when $G(\cdot)$ follows a uniform distribution on $[0, 1]$, in this case, independent of the distribution of the valuations, the equilibrium is unique.

In general, it is very difficult to obtain closed-form solutions for the entry equilibrium in both the no-resale and resale cases; nonetheless, we can establish the following comparison results.

$$TS(c_h^*, c_l^*) = v_l \{ 2\alpha(1 - \alpha)G(c_l^*)[1 - G(c_h^*)] + (1 - \alpha)^2 [1 - (1 - G(c_l^*))^2] \} \\ + v_h \{ 2\alpha(1 - \alpha)G(c_h^*) + \alpha^2 [1 - (1 - G(c_h^*))^2] \} - \sum_{k=0}^2 C_2^k \alpha^k (1 - \alpha)^{2-k} \left[k \int_0^{c_h^*} cdG(c) + (2 - k) \int_0^{c_l^*} cdG(c) \right]. \quad (9)$$

The first term refers to the expected surplus if the good is allocated to a low-value bidder with v_l . The second term is the expected surplus if the good is allocated to a high-value

Proposition 3. *For any equilibrium (c_l^*, c_h^*) in the benchmark case of no resale and corresponding equilibrium $(\widehat{c}_l, \widehat{c}_h)$ in the resale case, we must have $c_l^* < \widehat{c}_l$, $c_h^* > \widehat{c}_h$. Resale allowance leads the low-value bidders to become more aggressive on entry and high-value bidders have a lower incentive to enter.*

Resale affects entry cutoffs for both types. Our comparison in Proposition 3 suggests that when resale is allowed, the entry cutoff for the high-value bidder becomes lower, implying that he has a lower incentive to enter; because the resale opportunity is available and he might be able to obtain the item from the post-auction resale, the bidder would prefer to directly attend the resale market to avoid the participation cost in the auction. On the contrary, the resale allowance would also encourage bidders to enter the auction, i.e., low-value bidders become more aggressive on entry, because the possibility of reselling the object to the other bidder may generate a higher expected payoff.

4. Social Welfare

We next investigate the effect of resale on social welfare. There is a line of thought that resale will ensure full efficiency if there is any inefficiency in the allocation of an auction. However, Hafalir and Krishna [15] show that this may not be the case when there is uncertainty regarding values. That is, resale may not induce fully efficient outcomes. To examine the welfare effect of resale, we consider the social surplus as a function of two entry cutoffs. This surplus function is constructed under the assumption that, once the bidders enter in or stay out of the initial auction according to these entry cutoffs, they follow the equilibrium bidding and resale strategies described above.

First, we consider the no-resale case; the total surplus (TS) when bidders conform to cutoff strategies (c_h^*, c_l^*) is the sum of the payoffs of the bidders and the seller, minus the expected participation costs of the bidders. That is,

bidder with v_h . The last term measures the expected costs of participation. Simplifying it, we obtain

$$TS(c_h^*, c_l^*) = v_l [2(1 - \alpha)G(c_l^*) - 2\alpha(1 - \alpha)G(c_l^*)G(c_h^*) - (1 - \alpha)^2 G(c_l^*)^2] \\ + v_h [2\alpha G(c_h^*) - \alpha^2 G(c_h^*)^2] - 2\alpha \int_0^{c_h^*} cdG(c) - 2(1 - \alpha) \int_0^{c_l^*} cdG(c). \quad (10)$$

Next, we consider the case when resale is allowed. Similarly, for any $(\widehat{c}_h, \widehat{c}_l)$, we have

$$TS^R(\widehat{c}_h, \widehat{c}_l) = 2\alpha(1 - \alpha)[\lambda v_l + (1 - \lambda)v_h]G(\widehat{c}_l)[1 - G(\widehat{c}_h)] + v_l(1 - \alpha)^2[1 - (1 - G(\widehat{c}_l))^2] + v_h\{2\alpha(1 - \alpha)G(\widehat{c}_h) + \alpha^2[1 - (1 - G(\widehat{c}_h))^2]\} - \sum_{k=0}^2 C_2^k \alpha^k (1 - \alpha)^{2-k} \left[k \int_0^{\widehat{c}_h} cdG(c) + (2 - k) \int_0^{\widehat{c}_l} cdG(c) \right]. \quad (11)$$

Compared to $TS(c_h^*, c_l^*)$, the first two terms refer to the expected surplus if the initial auction allocates the good to a

low-cutoff bidder with v_l , which is calculated by taking the possibility of resale into account. Simplifying it, we obtain

$$TS^R(\widehat{c}_h, \widehat{c}_l) = v_l[2(1 - \alpha)G(\widehat{c}_l) - 2\alpha(1 - \alpha)G(\widehat{c}_l)G(\widehat{c}_h) - (1 - \alpha)^2G(\widehat{c}_l)^2] + v_h[2\alpha G(\widehat{c}_h) - \alpha^2G(\widehat{c}_h)^2] + (1 - \lambda)(v_h - v_l)2\alpha(1 - \alpha)G(\widehat{c}_l)[1 - G(\widehat{c}_h)] - 2\alpha \int_0^{\widehat{c}_h} cdG(c) - 2(1 - \alpha) \int_0^{\widehat{c}_l} cdG(c). \quad (12)$$

For fixed cutoffs, the possibility of resale increases total welfare:

$$TS^R(c_h^*, c_l^*) - TS(c_h^*, c_l^*) = (1 - \lambda)(v_h - v_l)2\alpha(1 - \alpha)G(c_l^*)[1 - G(c_h^*)] > 0. \quad (13)$$

This difference is simply the surplus gain of transferring the object from a low-value bidder to a high-value bidder in the resale phase. Consider an equilibrium in the benchmark case of no-resale with symmetric cutoffs c_l^* and c_h^* . As we just observed, if bidders were to use the same cutoffs when resale is allowed, then the surplus would be higher, i.e., $TS^R(c_l^*, c_h^*) > TS(c_l^*, c_h^*)$. However, the possibility of resale may also change equilibrium participation behavior of the bidders. Therefore, we need to know how the value of function TS^R changes as we move from the no-resale equilibrium cutoffs (c_l^*, c_h^*) to resale equilibrium cutoffs $(\widehat{c}_l, \widehat{c}_h)$. With our next result, we provide a sufficient condition for the social surplus to increase when resale is allowed.

Proposition 4. *Assume that each bidder's participation cost is independently drawn from a distribution function $G(\cdot)$ with support $[0, 1]$ and $\lambda < (1/2)$. The social welfare is higher in the resale case compared with that in the no-resale case.*

Proposition 4 provides a sufficient condition on the bargaining power parameter λ only. Hence, it is independent of the distribution of participation costs and the magnitude of the valuations.

Remark 1. Taking the derivative of the social surplus function with respect to λ , it is easy to know $(\partial T S^R / \partial \lambda) < 0$. Thus, the social surplus is decreasing in λ .

5. Seller's Expected Revenue

In this section, we examine how the seller's expected revenue is affected by resale. The seller obviously benefits when both bidders enter more frequently. However, in our setup, if one bidder enters more frequently, then the other enters less frequently, as shown in Proposition 3. Therefore, comparing the seller's revenue between the resale and no-resale case is not a trivial question, furthermore, we may face multiple equilibria in the resale case. Thus, a comparison becomes more difficult, and we need to consider all equilibria and the expected revenues they induce.

Here, $SS(c_l^*, c_h^*)$ denotes the original seller's expected revenue in the no-resale case; the seller's expected revenue is $SS(c_l^*, c_h^*) = (1 - \alpha)^2 v_l G(c_l^*)^2 + 2\alpha(1 - \alpha)v_l G(c_h^*)G(c_l^*) + \alpha^2 v_h G(c_h^*)^2$. (14)

The first term is the expected payment when two low-value bidders participate in the auction; the second term is the expected payment when both low-value bidder and high-value bidder participate in the auction; the third term is the expected payment when two high-value bidders participate in the auction.

In the resale case, the seller's expected surplus is the total surplus minus the bidders' surplus. The bidders' surplus is $BS^R(\widehat{c}_h, \widehat{c}_l)$ with resale, so we can utilize $SS^R(\widehat{c}_h, \widehat{c}_l) = TS^R(\widehat{c}_h, \widehat{c}_l) - BS^R(\widehat{c}_h, \widehat{c}_l)$ to compute the seller's expected surplus in the resale case and

$$BS^R = \sum_{k=0}^2 C_2^k \alpha^k (1-\alpha)^{2-k} \left\{ k \int_0^{\widehat{c}_h} [\widehat{c}_h + \lambda(v_h - v_l)(1-\alpha)G(\widehat{c}_l) - c] dG(c) + (2-k) \int_0^{\widehat{c}_l} (\widehat{c}_l - c) dG(c) \right\}, \quad (15)$$

where $\int_0^{\widehat{c}_h} [\widehat{c}_h + \lambda(v_h - v_l)(1-\alpha)G(\widehat{c}_l) - c] dG(c)$ is the high-value bidder's expected payoff, which is calculated by taking

the possibility of resale into account. $\int_0^{\widehat{c}_l} (\widehat{c}_l - c) dG(c)$ is the low-value bidder's expected payoff. Simplifying it, we obtain

$$\begin{aligned} BS^R(\widehat{c}_h, \widehat{c}_l) &= 2\alpha G(\widehat{c}_h) \widehat{c}_h + 2(1-\alpha)G(\widehat{c}_l) \widehat{c}_l + 2\alpha\lambda(v_h - v_l)(1-\alpha)G(\widehat{c}_l)G(\widehat{c}_h) \\ &\quad - 2\alpha \int_0^{\widehat{c}_h} c dG(c) - 2(1-\alpha) \int_0^{\widehat{c}_l} c dG(c). \end{aligned} \quad (16)$$

Thus,

$$\begin{aligned} SS^R(\widehat{c}_h, \widehat{c}_l) &= TS^R(\widehat{c}_h, \widehat{c}_l) - BS^R(\widehat{c}_h, \widehat{c}_l) \\ &= \alpha^2 v_h G(\widehat{c}_h)^2 + 2\alpha(1-\alpha)v_l G(\widehat{c}_h)G(\widehat{c}_l) + (1-\alpha)^2 v_l G(\widehat{c}_l)^2. \end{aligned} \quad (17)$$

Comparing (14) and (17), we can also utilize $SS(\widehat{c}_h, \widehat{c}_l)$ to denote the seller's expected surplus in the resale case, since the initial seller obtains revenue only if both bidders enter the auction. That is, the seller's expected surplus only depends on the bidders' cutoffs when both bidders enter the auction. According to the derivatives of this seller's expected surplus function with respect to its two arguments, it is easy to know the seller's expected surplus is increasing in equilibrium cutoffs for the set of points. We need to know how the value of function SS changes as we move from the no-resale equilibrium cutoffs (c_l^*, c_h^*) to resale equilibrium cutoffs $(\widehat{c}_l, \widehat{c}_h)$. As implied in Propositions 3, $c_l^* < \widehat{c}_l$, $c_h^* > \widehat{c}_h$, resale induces the competition effect (low-value bidders bid more aggressively) and displacement effect (high-value bidders are replaced by low-value bidders). While the competition effect pushes up expected revenue, the displacement effect works in the other direction. Thus, the net impact of resale on expected revenue depends on which effect dominates, and either effect may dominate in our setting.

Proposition 5. *Assume that each bidder's participation cost is independently drawn from a distribution function $G(\cdot)$ with support $[0, 1]$ and $0 < v_l < v_h \leq 1$. The effect of resale allowance on the original seller's expected revenue is ambiguous.*

As an illustrative example when $G(\cdot)$ follows a uniform distribution on $[0, 1]$, we show the unclear resale allowance.

Example 1. Assume that the private costs are uniformly distributed on the unit interval (i.e., $c \in [0, 1]$ and $G(c) = c$). If $\alpha = 0.5$, $v_h = 0.8$, and $v_l = 0.4$, it is easy to compute that no-resale yields $c_l^* = 0.2439$, $c_h^* = 0.5366$, and $SS(c_h^*, c_l^*) = 0.1146$; when $\lambda = 0.7$, resale allowance gives $\widehat{c}_l = 0.2739$, $\widehat{c}_h = 0.5049$, and $SS^R(\widehat{c}_h, \widehat{c}_l) = 0.088$. The seller is worse off with resale. However, if $\lambda = 0.3$, we obtain $\widehat{c}_l = 0.3041$, $\widehat{c}_h = 0.515$, and $SS^R(\widehat{c}_h, \widehat{c}_l) = 0.1186$ with resale. The seller's revenue increases after resale is allowed.

That resale has an ambiguous effect on expected revenue which has been well-documented in other auction settings with resale (e.g., [16, 18]); it can be explained intuitively in our setting based on the competition effect and the displacement effect induced by resale opportunities. This contradicts earlier findings that resale increases the seller's expected revenue, as demonstrated by Hafalir and Krishna [15], Akyol [3], and Garratt and Georganas [17].

6. Conclusion and Further Study

In this paper, we study the effects of resale allowance in second-price auctions with two-dimensional private information on values and participation costs. The values can only be either high (v_h) or low (v_l) while the participation costs are drawn from any general distributions. We demonstrate that the symmetric entry equilibrium is characterized by entry cutoffs, and we identify conditions under which such an equilibrium is unique in the resale case. Our comparison shows that high-value bidders have a lower incentive to enter, and they would prefer to directly attend the resale market to avoid the participation cost. However, low-value bidders become more aggressive on entry because the possibility of reselling the object to the other bidders may generate a higher expected payoff.

We assume that there are no participation costs at the resale stage; our analysis suggests that the opportunity of resale can increase the social welfare under a sufficient condition, and its effect on expected revenue is ambiguous. The implication is that a market regulator, whose objective is to maximize the social surplus or expected revenue, should exercise caution in suggesting whether or not resale should be permitted in auction settings similar to what is under our consideration.

This study gives a try to integrate and analyze both entry and resale in the second-price auction model with two-dimensional private information, but our analysis relies on several key assumptions; relaxation of those assumptions to

allow for a more general analysis on entry and resale is left for future research [18].

Appendix

A. Proofs

Proof of Proposition 1. The existence and uniqueness of a symmetric equilibrium in the no-resale case can be established by focusing on the following two equations:

$$c_l^* = v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)], \quad (\text{A.1})$$

$$c_h^* = v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)] + (v_h - v_l)[1 - \alpha G(c_h^*)], \quad (\text{A.2})$$

with $c_l^* < c_h^* \leq v_h$. Equation (A.1) defines c_l^* as a decreasing function of c_h^* , denoted by $c_l^* = f(c_h^*)$, since $f'(c_h^*) = (dc_l^*/dc_h^*) = -(v_l \alpha g(c_h^*)/1 + v_l(1 - \alpha)g(c_l^*)) < 0$. We know that $f(c_h^*)$ has a fixed point c^s which is determined

by $c^s = v_l \times [1 - G(c^s)]$. Since $f(c_h^*)$ is monotonically decreasing, we have $c_l^* < c^s$ and $c_h^* > c^s$.

We insert $c_l^* = f(c_h^*)$ into equation (A.2), and let $\phi(c_h^*) = c_l^* + (v_h - v_l)[1 - \alpha G(c_h^*)] - c_h^*$ with $c_l^* < c_h^* \leq v_h$; we have

$$\begin{aligned} \phi(c^s) &= c^s + (v_h - v_l)[1 - \alpha G(c^s)] - c^s \\ &= (v_h - v_l)[1 - \alpha G(c^s)] > 0, \\ \phi(v_h) &= v_l \times [1 - \alpha G(v_h) - (1 - \alpha)G(f(v_h))] + (v_h - v_l)[1 - \alpha G(v_h)] - v_h \\ &= v_l \times [1 - \alpha G(v_h) - (1 - \alpha)G(f(v_h))] - v_l[1 - \alpha G(v_h)] + v_h[1 - \alpha G(v_h)] - v_h < 0. \end{aligned} \quad (\text{A.3})$$

Thus, when all bidders have the same distribution function, by the mean value theorem, there is a $c_h^* \in (c^s, v_h]$ such that $\phi(c_h^*) = 0$ so that there exists a symmetric equilibrium at which $c_l^* = f(c_h^*) < c^s < c_h^* \leq v_h$.

Next, we prove the uniqueness of a symmetric equilibrium. Taking the derivative with respect to c_h^* , we have

$$\phi'(c_h^*) = f'(c_h^*) - (v_h - v_l)\alpha g(c_h^*) - 1 < 0, \quad (\text{A.4})$$

which indicates that $\phi(c_h^*)$ is a monotonically decreasing function; the symmetric equilibrium is unique in the no-resale case. \square

Proof of Proposition 2. The existence and uniqueness of a symmetric equilibrium in the resale case can be established by focusing on the following two equations:

$$\hat{c}_l = v_l \times [1 - \alpha G(\hat{c}_h) - (1 - \alpha)G(\hat{c}_l)] + \alpha(1 - \lambda)(v_h - v_l)[1 - G(\hat{c}_h)], \quad (\text{A.5})$$

$$\hat{c}_h = v_h \times [1 - \alpha G(\hat{c}_h) - (1 - \alpha)G(\hat{c}_l)] + (1 - \lambda)(v_h - v_l)(1 - \alpha)G(\hat{c}_l). \quad (\text{A.6})$$

First, we prove that $\hat{c}_l < \hat{c}_h$. In equilibrium, we have

$$\begin{aligned} \hat{c}_h - \hat{c}_l &= (v_h - v_l)[1 - \alpha G(\hat{c}_h)] - (1 - \lambda)(v_h - v_l)(1 - \alpha)G(\hat{c}_l) - \alpha(1 - \lambda)(v_h - v_l)[1 - G(\hat{c}_h)] \\ &= (v_h - v_l)[1 - \alpha + \alpha\lambda(1 - G(\hat{c}_h)) - (1 - \lambda)(1 - \alpha)G(\hat{c}_l)] \\ &= (v_h - v_l)[1 - \alpha + \alpha\lambda(1 - G(\hat{c}_h)) - (1 - \lambda)(1 - \alpha)G(\hat{c}_l)] \\ &= (v_h - v_l)[(1 - \alpha)(1 - (1 - \lambda)G(\hat{c}_l)) + \alpha\lambda(1 - G(\hat{c}_h))] > 0. \end{aligned} \quad (\text{A.7})$$

Next, we prove the existence of a symmetric equilibrium with $\hat{c}_l < \hat{c}_h \leq v_h$. Equation (A.5) defines \hat{c}_l as a decreasing function of \hat{c}_h , denoted by $\hat{c}_l = f(\hat{c}_h)$, since

$$f'(\hat{c}_h) = \frac{\partial \hat{c}_l}{\partial \hat{c}_h} = -\frac{v_l \alpha g(\hat{c}_h) + \alpha(1 - \lambda)(v_h - v_l)g(\hat{c}_h)}{1 + v_l(1 - \alpha)g(\hat{c}_l)} < 0. \quad (\text{A.8})$$

$f(\widehat{c}_h)$ has a fixed point c^s which is determined by $c^s = v_l \times [1 - G(c^s)]$. Since $f(\widehat{c}_h)$ is monotonically decreasing, we have $\widehat{c}_l < c^s$ and $\widehat{c}_h > c^s$.

We insert $\widehat{c}_l = f(\widehat{c}_h)$ into equation (A.6), and let $\phi(\widehat{c}_h) = v_h \times [1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] + (1 - \lambda)(v_h - v_l)(1 - \alpha)G(\widehat{c}_l) - \widehat{c}_h$ with $\widehat{c}_l < \widehat{c}_h \leq v_h$; we have

$$\begin{aligned}
\phi(c_s) &= v_h \times [1 - G(c_s)] + (1 - \lambda)(v_h - v_l)(1 - \alpha)G(c_s) - [v_l + \alpha(1 - \lambda)(v_h - v_l)] \times [1 - G(c_s)] \\
&= (v_h - v_l)[1 - G(c_s)] + (1 - \lambda)(v_h - v_l)[G(c_s) - \alpha] \geq (v_h - v_l)(1 - \alpha) > 0, \\
\phi(v_h) &= v_h \times [1 - \alpha G(v_h) - (1 - \alpha)G(f(v_h))] + (v_h - v_l)(1 - \alpha)G(f(v_h)) - v_h \\
&= v_h \times [1 - \alpha G(v_h)] - v_l(1 - \alpha)G(f(v_h)) - v_h \\
&= -v_h \alpha G(v_h) - v_l(1 - \alpha)G(f(v_h)) < 0.
\end{aligned} \tag{A.9}$$

Thus, when all bidders have the same distribution function, by the mean value theorem, there is a $\widehat{c}_h \in (c^s, v_h]$ such that $\phi(\widehat{c}_h) = 0$ so that there always exists a symmetric equilibrium at which $\widehat{c}_l = f(\widehat{c}_h) < c^s < \widehat{c}_h \leq v_h$.

Then, we prove the uniqueness of a symmetric equilibrium. Taking the derivative with respect to \widehat{c}_h , we have

$$\begin{aligned}
\phi'(\widehat{c}_h) &= -(1 - \alpha)g(\widehat{c}_l)f'(\widehat{c}_h)[v_h - (1 - \lambda)(v_h - v_l)] - v_h \alpha g(\widehat{c}_h) - 1, \\
&= \frac{(1 - \alpha)g(\widehat{c}_l)\alpha g(\widehat{c}_h)[v_h - (1 - \lambda)(v_h - v_l)][v_l + (1 - \lambda)(v_h - v_l)]}{1 + v_l(1 - \alpha)g(\widehat{c}_l)} - v_h \alpha g(\widehat{c}_h) - 1, \\
&= \frac{(1 - \alpha)g(\widehat{c}_l)\alpha g(\widehat{c}_h)\lambda(1 - \lambda)(v_h - v_l)^2 - v_h \alpha g(\widehat{c}_h) - v_l(1 - \alpha)g(\widehat{c}_l) - 1}{1 + v_l(1 - \alpha)g(\widehat{c}_l)}.
\end{aligned} \tag{A.10}$$

When $\max_{[0,1]} g(c)(v_h - v_l) < 1$, $(1 - \alpha)g(\widehat{c}_l)\alpha g(\widehat{c}_h)\lambda(1 - \lambda)(v_h - v_l)^2 - 1 < \alpha(1 - \alpha)\lambda(1 - \lambda) - 1 < 0$, $\phi'(\widehat{c}_h) < 0$. Thus, when $\max_{[0,1]} g(c)(v_h - v_l) < 1$, $\phi'(\widehat{c}_h) < 0$, $\phi(\widehat{c}_h)$ is a decreasing function of \widehat{c}_h ; the symmetric equilibrium is unique in the resale case. \square

Proof of Proposition 3. The equilibrium cutoff (c_l^*, c_h^*) in no-resale setting and $(\widehat{c}_l, \widehat{c}_h)$ in resale case should satisfy four equations simultaneously as follows:

$$c_l^* = v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)], \tag{A.11}$$

$$c_h^* = v_l \times [1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)] + (v_h - v_l) \times [1 - \alpha G(c_h^*)], \tag{A.12}$$

$$\widehat{c}_l = v_l \times [1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] + \alpha(1 - \lambda)(v_h - v_l)[1 - G(\widehat{c}_h)], \tag{A.13}$$

$$\widehat{c}_h = v_h \times [1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] + (1 - \lambda)(v_h - v_l)(1 - \alpha)G(\widehat{c}_l), \tag{A.14}$$

where we define $A^* = 1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)$ and $\widehat{A} = 1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)$. We will proceed by ruling out all the other cases:

- (1) assume that $c_l^* \leq \widehat{c}_l$ and $c_h^* \leq \widehat{c}_h$. If this is true, by using equations (A.12) and (A.14), we immediately have $c_h^* - \widehat{c}_h = v_l A^* + (v_h - v_l)[1 - \alpha G(c_h^*)] - v_l \widehat{A} - (v_h - v_l)$

$[1 - \alpha G(\widehat{c}_h)] + \lambda(v_h - v_l)(1 - \alpha)G(\widehat{c}_l) \geq \lambda(v_h - v_l)(1 - \alpha)G(\widehat{c}_l) > 0$. This implies that $c_h^* > \widehat{c}_h$, a contradiction.

- (2) assume that $c_l^* \geq \widehat{c}_l$ and $c_h^* \geq \widehat{c}_h$. If this is true, we immediately have $A \geq A^*$; by using equations (A.11) and (A.13), we get $\widehat{c}_l - c_l^* = v_l A - v_l A^* + \alpha(1 - \lambda)(v_h - v_l)[1 - G(\widehat{c}_h)] \geq \alpha(1 - \lambda)(v_h - v_l)[1 - G(\widehat{c}_h)] > 0$. This implies that $\widehat{c}_l > c_l^*$, a contradiction.
- (3) assume that $c_l^* \geq \widehat{c}_l$ and $c_h^* \leq \widehat{c}_h$. If this is true, by using equations (A.11) and (A.13), we have $\widehat{c}_l - c_l^* \geq [\lambda v_l + (1 - \lambda)v_h]\alpha[1 - G(\widehat{c}_h)] + v_l(1 - \alpha)[1 - G(c_l^*)] - v_l$

$$[1 - \alpha G(c_h^*) - (1 - \alpha)G(c_l^*)] = v_l \alpha [G(c_h^*) - G(\widehat{c}_h)] + (v_h - v_l)(1 - \lambda)\alpha[1 - G(\widehat{c}_h)] > 0.$$

This implies that $\widehat{c}_l > c_l^*$, a contradiction. Then, by using equations (A.12) and (A.14), we get $c_h^* - \widehat{c}_h \geq v_h [1 - \alpha G(c_h^*) - (1 - \alpha)G(\widehat{c}_l)] + (v_h - v_l)(1 - \alpha)G(\widehat{c}_l) - v_h [1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] - (v_h - v_l)(1 - \lambda)(1 - \alpha)G(\widehat{c}_l) = v_h \alpha [G(\widehat{c}_h) - G(c_h^*)] + \lambda(v_h - v_l)(1 - \alpha)G(\widehat{c}_l) > 0$; this implies that $c_h^* > \widehat{c}_h$, a contradiction.

Thus, $c_l^* < \widehat{c}_l$ and $c_h^* > \widehat{c}_h$ have been proved. \square

Proof of Proposition 4. The social surplus in the resale case is

$$\begin{aligned} TS^R &= v_l [2(1 - \alpha)G(\widehat{c}_l) - 2\alpha(1 - \alpha)G(\widehat{c}_l)G(\widehat{c}_h) - (1 - \alpha)^2 G(\widehat{c}_l)^2] + v_h [2\alpha G(\widehat{c}_h) - \alpha^2 G(\widehat{c}_h)^2] \\ &\quad + (1 - \lambda)(v_h - v_l)2\alpha(1 - \alpha)G(\widehat{c}_l)[1 - G(\widehat{c}_h)] - 2\alpha \int_0^{\widehat{c}_h} cdG(c) - 2(1 - \alpha) \int_0^{\widehat{c}_l} cdG(c). \end{aligned} \quad (A.15)$$

The derivatives of this social surplus function with respect to its two arguments can be written by referring to functions \widehat{c}_h and \widehat{c}_l that we just defined:

$$\begin{aligned} \frac{\partial TS^R}{\partial \widehat{c}_h} &= -v_l 2\alpha(1 - \alpha)G(\widehat{c}_l)g(\widehat{c}_h) + v_h 2\alpha g(\widehat{c}_h)[1 - \alpha G(\widehat{c}_h)] \\ &\quad - (1 - \lambda)(v_h - v_l)2\alpha(1 - \alpha)G(\widehat{c}_l)g(\widehat{c}_h) - 2\alpha g(\widehat{c}_h)\widehat{c}_h, \\ &= 2\alpha g(\widehat{c}_h)[v_h - v_h \alpha G(\widehat{c}_h) - v_l(1 - \alpha)G(\widehat{c}_l) - (1 - \lambda)(v_h - v_l)(1 - \alpha)G(\widehat{c}_l)] \\ &\quad - 2\alpha g(\widehat{c}_h)[v_h - v_h \alpha G(\widehat{c}_h) - v_h(1 - \alpha)G(\widehat{c}_l) + (1 - \lambda)(v_h - v_l)(1 - \alpha)G(\widehat{c}_l)], \\ &= 2\alpha g(\widehat{c}_h)(2\lambda - 1)(v_h - v_l)(1 - \alpha)G(\widehat{c}_l), \end{aligned} \quad (A.16)$$

$$\begin{aligned} \frac{\partial TS^R}{\partial \widehat{c}_l} &= 2v_l(1 - \alpha)g(\widehat{c}_l)[1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)], \\ &\quad + (1 - \lambda)(v_h - v_l)2\alpha(1 - \alpha)g(\widehat{c}_l)[1 - G(\widehat{c}_h)] - 2(1 - \alpha)G(\widehat{c}_l)\widehat{c}_l \\ &= 2v_l(1 - \alpha)g(\widehat{c}_l)[1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] + (1 - \lambda)(v_h - v_l)2\alpha(1 - \alpha)g(\widehat{c}_l)[1 - G(\widehat{c}_h)] \\ &\quad - 2(1 - \alpha)g(\widehat{c}_l)v_l[1 - \alpha G(\widehat{c}_h) - (1 - \alpha)G(\widehat{c}_l)] - 2(1 - \alpha)g(\widehat{c}_l)\alpha(1 - \lambda)(v_h - v_l)[1 - G(\widehat{c}_h)] = 0. \end{aligned}$$

These first-order conditions imply that the social surplus is decreasing in \widehat{c}_h when $\lambda < (1/2)$ and increasing in \widehat{c}_h when $\lambda > (1/2)$. Since $c_h^* > \widehat{c}_h$, when $\lambda < (1/2)$, we have

$TS^R(\widehat{c}_h, \widehat{c}_l) > TS^R(c_h^*, c_l^*)$. Thus, $TS^R(\widehat{c}_h, \widehat{c}_l) > TS^R(c_h^*, c_l^*) > TS(c_h^*, c_l^*)$; the social welfare is higher in the resale case compared with that in the no-resale case. \square

Proof of Equation 4. In the resale case, the seller's expected surplus is the total surplus minus the bidders' surplus; we have

$$\begin{aligned} SS^R(\hat{c}_h, \hat{c}_l) &= TS^R(\hat{c}_h, \hat{c}_l) - BS^R(\hat{c}_h, \hat{c}_l), \\ &= v_h [2\alpha G(\hat{c}_h) - \alpha^2 G(\hat{c}_h)^2] + v_l [2(1-\alpha)G(\hat{c}_l) - 2\alpha(1-\alpha)G(\hat{c}_l)G(\hat{c}_h) - (1-\alpha)^2 G(\hat{c}_l)^2] \\ &\quad + (1-\lambda)(v_h - v_l)2\alpha(1-\alpha)G(\hat{c}_l)[1 - G(\hat{c}_h)] - 2\alpha G(\hat{c}_h)\hat{c}_h \\ &\quad - 2(1-\alpha)G(\hat{c}_l)\hat{c}_l - 2\alpha\lambda(v_h - v_l)(1-\alpha)G(\hat{c}_l)G(\hat{c}_h), \end{aligned} \quad (\text{A.17})$$

where

$$\begin{aligned} &2\alpha G(\hat{c}_h)\hat{c}_h + 2(1-\alpha)G(\hat{c}_l)\hat{c}_l \\ &= v_h [2\alpha G(\hat{c}_h) - 2\alpha^2 G(\hat{c}_h)^2] + v_l [2(1-\alpha)G(\hat{c}_l) - 4\alpha(1-\alpha)G(\hat{c}_h)G(\hat{c}_l) - 2(1-\alpha)^2 G(\hat{c}_l)^2] \\ &\quad - 2\alpha\lambda(v_h - v_l)(1-\alpha)G(\hat{c}_h)G(\hat{c}_l) + 2\alpha(1-\alpha)(1-\lambda)(v_h - v_l)G(\hat{c}_l)[1 - G(\hat{c}_h)]. \end{aligned} \quad (\text{A.18})$$

From equation (A.17) and equation (A.18), we obtain

$$SS^R(\hat{c}_h, \hat{c}_l) = \alpha^2 v_h G(\hat{c}_h)^2 + 2\alpha(1-\alpha)v_l G(\hat{c}_h)G(\hat{c}_l) + (1-\alpha)^2 v_l G(\hat{c}_l)^2. \quad (\text{A.19})$$

Data Availability

The data that support the findings of this study can be obtained from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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