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QUADRATIC PAIRS IN CHARACTERISTIC 2 AND THE WITT CANCELLATION THEOREM

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We define the orthogonal sum of quadratic pairs and we show that there is no Witt cancellation theorem for this operation in characteristic 2.

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1. Introduction. Quadratic pairs on central simple algebras were defined in [5]. They play the same role for quadratic forms as involutions for symmetric or skew-symmetric bilinear forms. In particular, they can be used to define twisted orthogonal groups in characteristic 2. In this paper, a notion of orthogonal sum of quadratic pairs is introduced on the model of Dejaiffe's orthogonal sum of involutions [2]. Moreover, an example is given to show that there is no cancellation for this operation.

2. Orthogonal sum of quadratic pairs

Definition 2.1. Let A be a central simple algebra of degree n over a field F of characteristic 2. A quadratic pair on A is a pair (σ, f), where σ is a symplectic involution on A and $f : \operatorname{Sym}(A,\sigma) \to F$ is a linear map satisfying the following condition:

$$f(x+\sigma(x)) = \operatorname{Trd}_A(x) \tag{2.1}$$

for all $\in A$. In this case, n is always even.

We recall from [2] that a Morita equivalence $((A,\sigma),(B,\tau),M,N,f,g,\nu)$ between two algebras with involutions of the first kind (A,σ) and (B,τ) is given by

(i) an A-B bimodule M;
(ii) a B-A bimodule N;
(iii) two bimodule homomorphisms $f : M \otimes_B N \to A$ and $g : N \otimes_A M \to B$ which are associative in the sense that $f(m\otimes n)\cdot m' = m\cdot g(n\otimes m')$ and $g(n\otimes m)\cdot n' = n\cdot f(m\otimes n')$, for all $m,m' \in M$ and $n,n' \in N$;
(iv) a bijective F-linear map $\nu : M \to N$ such that $\nu(amb) = \tau(b)\nu(m)\sigma(a)$ for all $a\in A$, $m \in M$, $b\in B$, and ν^{-1} is its inverse.

If $((A,\sigma),(A',\sigma'),M,N,g,h,\nu)$ is a Morita equivalence of two algebras with symplectic involutions and (σ,f) and (σ',f') are quadratic pairs, respectively, on A and A', then we define the *orthogonal sum* of (A,σ,f) and (A',σ',f') as follows:

$$(A,\sigma,f)\oplus(A',\sigma',f')=(S,^*,f''), \tag{2.2}$$

where

$$S=\begin{pmatrix}A & M\\ N & A'\end{pmatrix},\qquad {}^*\begin{pmatrix}a & m\\ n & a'\end{pmatrix}=\begin{pmatrix}\sigma(a) & \nu^{-1}(n)\\ \nu(m) & \sigma'(a')\end{pmatrix}. \tag{2.3}$$

We have

$$\operatorname{Sym}(S,^*)=\left\{\begin{pmatrix}a & m\\ n & a'\end{pmatrix}\;\middle|\;\begin{matrix}\sigma(a)=a\\ \sigma'(a')=a'\\ n=\nu(m)\end{matrix}\right\} \tag{2.4}$$

and $f'':\operatorname{Sym}(S,^*)\to F$ defined by

$$f''\begin{pmatrix}a & m\\ \nu(m) & a'\end{pmatrix}=f(a)+f'(a'). \tag{2.5}$$

Proposition 2.2. *The orthogonal sum $(S,^*,f'')$ is an algebra with quadratic pair.*

Proof. It is clear that the involution * is symplectic, and we have, for all

$$x=\begin{pmatrix}a & m\\ n & a'\end{pmatrix}\in S, \tag{2.6}$$

that

$$\begin{aligned}f''(x+x^*)&=f''\begin{pmatrix}a+\sigma(a) & m+\nu^{-1}(n)\\ n+\nu(m) & a'+\sigma'(a')\end{pmatrix}\\ &=f(a+\sigma(a))+f'(a'+\sigma'(a'))\\ &=\operatorname{Trd}_A(a)+\operatorname{Trd}_{A'}(a')=\operatorname{Trd}_S(x).\end{aligned} \tag{2.7}$$

□

A particular case of this definition is the situation where $M=N=A=A'$. If A is a central simple algebra over a field of characteristic 2, we consider the two algebras with quadratic pairs (A,σ,f) and (A,σ',f'), where σ and σ' are symplectic involutions on A. Then we have $\sigma'=\operatorname{Int}(s)\circ\sigma$ with $s\in\operatorname{Alt}(A,\sigma)$. For $\lambda\in F^*$, we define on $M_2(A)$ the involution θ_λ by

$$\theta_\lambda\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}\sigma(a) & \lambda^{-1}\sigma(c)s^{-1}\\ \lambda s\sigma(b) & \sigma'(d)\end{pmatrix}. \tag{2.8}$$

The map ν is defined by $\nu(x) = \lambda s\sigma(x)$, for all $x \in A$, and we define the map $g : \mathrm{Sym}(\theta_\lambda) \to F$ by

$$g\begin{pmatrix} a & b \\ c & d \end{pmatrix} = f(a) + f'(d). \tag{2.9}$$

It is clear that $(M_2(A), \theta_\lambda, g)$ is an algebra with quadratic pair. We recall that $(A, \sigma, f) \simeq (A, \sigma', f')$ if and only if there exists $v \in A^*$ such that $\sigma' = \mathrm{Int}(v) \circ \sigma \circ \mathrm{Int}(v)^{-1} = \mathrm{Int}(v\sigma(v)) \circ \sigma$ and $f' = f \circ \mathrm{Int}(v^{-1})$.

3. Generalized quadratic forms. Let V be a right vector space on a central division F-algebra with involution $(D, *)$. A generalized quadratic form on V is a pair (ψ, Q) consisting of a hermitian form ψ and a map $Q : V \to D/\mathrm{Alt}(D, *)$ such that

(1) $Q(x+y) = Q(x) + Q(y) + [\psi(x,y)]$;
(2) $Q(x\lambda) = \lambda^* Q(x)\lambda$;
(3) $\psi(x,x) = Q(x) + Q(x)^*$.

This notion is due to Gross [4]. The space (V, ψ, Q) is called a quadratic space.

Let D be a central division algebra over F with an involution $*$ of any kind, V a D-vector space, and (ψ, Q) a generalized quadratic form. Then we have an F-linear map $\varphi_\psi : V \otimes_D {}^*V \to \mathrm{End}_D(V) = A$ such that

$$\varphi_\psi(v \otimes {}^*w)(x) = v \cdot \psi(w, x) \tag{3.1}$$

for $v, w, x \in V$. Here ${}^*V$ is the left D-vector space

$${}^*V = \{{}^*v \mid v \in V\} \tag{3.2}$$

with structure

$${}^*v + {}^*w = {}^*(v+w), \qquad \alpha \cdot {}^*v = {}^*(v \cdot \alpha^*), \tag{3.3}$$

for all $v, w \in V$ and $\alpha \in D$.

In fact, φ_ψ is one-to-one, by [5, page 54, Theorem 5.1]. If σ is the adjoint involution on $\mathrm{End}_D(V)$ with respect to ψ, then we have

$$\sigma(\varphi_\psi(v \otimes {}^*w)) = \varphi_\psi(w \otimes {}^*v) \tag{3.4}$$

for $v, w \in V$. Moreover, $\mathrm{Trd}_{\mathrm{End}_D(V)}(\varphi_\psi(v \otimes {}^*w)) = \mathrm{Trd}_D(\psi(w, v))$ for $v, w \in V$.

In [3], we established a relation between quadratic pairs and generalized quadratic forms.

Theorem 3.1. *To every generalized quadratic form (V, ψ, Q), a quadratic pair (σ, f) can be associated on $\mathrm{End}_D(V)$, where σ is the adjoint involution to ψ and f is defined by*

(1) $f(vd\otimes {}^*v)=\mathrm{Trd}_D(dQ(v))$ *for all* $v\in V$ *and* $d\in \mathrm{Sym}(D,{}^*)$;
(2) $f(x\otimes {}^*y+y\otimes {}^*x)=\mathrm{Trd}_D(\psi(x,y))$ *for all* $x,y\in V$.

The pair (σ,f) *is called the adjoint quadratic pair.*

From [3], we recall the following result.

Theorem 3.2. *Every quadratic pair on* $\mathrm{End}_D(V)$ *is associated to a unique generalized quadratic form up to a scalar.*

We now have the following theorem.

Theorem 3.3. *The quadratic pair associated to the orthogonal sum of two generalized quadratic forms is the orthogonal sum of the associated quadratic pairs.*

Proof. Let (V,ψ,Q) and (W,ψ',Q') be two generalized quadratic forms. We can construct two algebras with quadratic pairs: $(\mathrm{End}_D(V),\sigma_\psi,f_Q)$ and $(\mathrm{End}_D(W),\sigma_{\psi'},f_{Q'})$. We know that $\mathrm{Hom}_D(V,W)$ is an $\mathrm{End}_D(W)$-$\mathrm{End}_D(V)$ bimodule and $\mathrm{Hom}_D(W,V)$ is an $\mathrm{End}_D(V)$-$\mathrm{End}_D(W)$ bimodule. Let

$$\nu:\mathrm{Hom}_D(W,V)\longrightarrow \mathrm{Hom}_D(V,W) \tag{3.5}$$

be defined by the condition

$$\psi(h(w),v)=\psi'(w,\nu(h)(v))\quad \forall h\in \mathrm{Hom}_D(W,V). \tag{3.6}$$

We can easily verify that

$$\big((\mathrm{End}_D(V),\sigma_\psi),(\mathrm{End}_D(W),\sigma_{\psi'}),\mathrm{Hom}(W,V),\mathrm{Hom}_D(V,W),g,h,\nu\big) \tag{3.7}$$

is a Morita equivalence (with the same notation as in Section 2), and

$$\mathrm{End}_D(V\oplus W)\simeq \begin{pmatrix}\mathrm{End}_D(V) & \mathrm{Hom}_D(W,V)\\ \mathrm{Hom}_D(V,W) & \mathrm{End}_D(W)\end{pmatrix}. \tag{3.8}$$

Using this isomorphism, we deduce that the quadratic pair $(\sigma_{\psi\oplus\psi'},f_{Q\oplus Q'})$ corresponds to the orthogonal sum of quadratic pairs (σ_ψ,f_Q) and $(\sigma_{\psi'},f_{Q'})$. In fact, for

$$\begin{pmatrix} f & h\\ \ell & g\end{pmatrix}\in\begin{pmatrix}\mathrm{End}_D(V) & \mathrm{Hom}_D(W,V)\\ \mathrm{Hom}_D(V,W) & \mathrm{End}_D(W)\end{pmatrix}, \tag{3.9}$$

we want to show that

$$(\sigma_\psi\oplus\sigma_{\psi'})\begin{pmatrix} f & h\\ \ell & g\end{pmatrix}=\begin{pmatrix}\sigma_\psi(f) & \nu^{-1}(\ell)\\ \nu(h) & \sigma_{\psi'}(g)\end{pmatrix}=\sigma_{\psi\oplus\psi'}\begin{pmatrix} f & h\\ l & g\end{pmatrix}, \tag{3.10}$$

that is, if we have

$$\begin{aligned}\begin{pmatrix} f & h \\ \ell & g \end{pmatrix} : V \oplus W &\longrightarrow V \oplus W \\ (x,y) &\longmapsto \big(f(x)+h(y), \ell(x)+g(y)\big),\end{aligned} \tag{3.11}$$

then we have to show that

$$\begin{aligned}(\psi\oplus\psi')&\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} f & h \\ \ell & g \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) \\ &= (\psi\oplus\psi')\left(\begin{pmatrix} \sigma_\psi(f) & \nu^{-1}(\ell) \\ \nu(h) & \sigma_{\psi'}(g) \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right).\end{aligned} \tag{3.12}$$

We have

$$\begin{aligned}(\psi\oplus\psi')&\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} f & h \\ \ell & g \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) \\ &= (\psi\oplus\psi')\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} f(x_2)+f(y_2) \\ \ell(x_2)+g(y_2) \end{pmatrix}\right) \\ &= \psi\big(x_1, f(x_2)+h(y_2)\big)+\psi'\big(y_1, \ell(x_2)+g(y_2)\big).\end{aligned} \tag{3.13}$$

On the other hand,

$$\begin{aligned}(\psi\oplus\psi')&\left(\begin{pmatrix} \sigma_\psi(f)(x_1)+\nu^{-1}(\ell)(y_1) \\ \nu(h)(x_1)+\sigma_{\psi'}(g)(y_1) \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) \\ &= \psi\big(\sigma_\psi(f)(x_1), x_2\big)+\psi\big(\nu^{-1}(\ell)(y_1), x_2\big) \\ &\quad+\psi'\big(\nu(h)(x_1), y_2\big) \\ &\quad+\psi'\big(\sigma_{\psi'}(g)(y_1), y_2\big).\end{aligned} \tag{3.14}$$

Now $\nu : \mathrm{Hom}_D(W,V) \to \mathrm{Hom}_D(V,W)$ has the property that

$$\psi\big(h(w), v\big) = \psi'\big(w, \nu(h)(v)\big) \tag{3.15}$$

for all $h \in \mathrm{Hom}_D(W,V)$. Since ν is bijective, $h = \nu^{-1}(\ell)$ for some $\ell \in \mathrm{Hom}_D(V,W)$, and we have that

$$\psi\big(\nu^{-1}(\ell)(w), v\big) = \psi'\big(w, \ell(v)\big) \tag{3.16}$$

for all $\ell \in \mathrm{Hom}_D(V,W)$, which implies that

$$\begin{aligned}
(\psi\oplus\psi')&\left(\begin{pmatrix}\sigma_\psi(f)(x_1)+\nu^{-1}(\ell)(y_1)\\ \nu(h)(x_1)+\sigma_{\psi'}(g)(y_1)\end{pmatrix},\begin{pmatrix}x_2\\ y_2\end{pmatrix}\right)\\
&=\psi(\sigma_\psi(f)(x_1),x_2)+\psi'(y_1,\ell(x_2))\\
&\quad+\psi(x_1,h(y_2))+\psi'(\sigma_{\psi'}(g)(y_1),y_2)\\
&=\psi(x_1,f(x_2))+\psi'(y_1,\ell(x_2))\\
&\quad+\psi(x_1,h(y_2))+\psi'(y_1,g(y_2))\\
&=\psi(x_1,f(x_2)+h(y_2))+\psi'(y_1,\ell(x_2)+g(y_2))\\
&=(\psi\oplus\psi')\left(\begin{pmatrix}x_1\\ y_1\end{pmatrix},\begin{pmatrix}f & h\\ \ell & g\end{pmatrix}\begin{pmatrix}x_2\\ y_2\end{pmatrix}\right),
\end{aligned}\tag{3.17}$$

and this proves (3.12).

Observe that $\mathrm{Sym}(\mathrm{End}_D(V\oplus W),\sigma_{\psi\oplus\psi'})$ is linearly generated by elements of the two following types.

Type 1. The first type of generators is

$$\varphi_{\psi\oplus\psi'}\left(\begin{pmatrix}x\\ y\end{pmatrix}d\otimes{}^*\begin{pmatrix}x\\ y\end{pmatrix}\right)=\begin{pmatrix}\varphi_\psi(xd\otimes{}^*x) & xd\psi'(y,\cdot)\\ yd\psi(x,\cdot) & \varphi_{\psi'}(yd\otimes{}^*y)\end{pmatrix}.\tag{3.18}$$

Type 2. The second type is

$$\begin{aligned}
\varphi_{\psi\oplus\psi'}&\left(\begin{pmatrix}x_1\\ y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\ y_2\end{pmatrix}+\begin{pmatrix}x_2\\ y_2\end{pmatrix}\otimes{}^*\begin{pmatrix}x_1\\ y_1\end{pmatrix}\right)\\
&=\varphi_{\psi\oplus\psi'}\left(\begin{pmatrix}x_1\\ y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\ y_2\end{pmatrix}\right)+\sigma\left(\varphi_{\psi\oplus\psi'}\left(\begin{pmatrix}x_1\\ y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\ y_2\end{pmatrix}\right)\right).
\end{aligned}\tag{3.19}$$

For two symmetric elements f and g, we have, by definition,

$$f_Q\oplus f_{Q'}\begin{pmatrix}f & h\\ \nu(h) & g\end{pmatrix}=f_Q(f)+f_{Q'}(g).\tag{3.20}$$

So it suffices to show the following equality:

$$f_{Q\oplus Q'}\begin{pmatrix}f & h\\ \nu(h) & g\end{pmatrix}=f_Q(f)+f_{Q'}(g),\tag{3.21}$$

where

$$\begin{pmatrix} f & h \\ \nu(h) & g \end{pmatrix} \tag{3.22}$$

is a generator of Type 1 or Type 2.

We have the identification

$$\begin{gathered} (V\oplus V')\otimes_D {}^*(V\oplus V') \longmapsto \operatorname{End}_D(V\oplus V'), \\ (V\oplus V')\otimes_D {}^*(V\oplus V') = (V\otimes {}^*V)\oplus(V\otimes {}^*V')\oplus(V'\otimes {}^*V)\oplus(V'\otimes {}^*V'). \end{gathered} \tag{3.23}$$

The definition of $\varphi_{\psi\oplus\psi'}$ implies that

$$\varphi_{\psi\oplus\psi'}(x_1\otimes {}^*x_2)\begin{pmatrix} v \\ v' \end{pmatrix} = x_1(\psi\oplus\psi')(x_2,v) = x_2\psi(x_2,v) \tag{3.24}$$

for all $x_1,x_2\in V$, and it follows that

$$\varphi_{\psi\oplus\psi'}(x_1\otimes {}^*x_2) = \begin{pmatrix} \varphi_\psi(x_1\otimes {}^*x_2) & 0 \\ 0 & 0 \end{pmatrix}. \tag{3.25}$$

Now take $x\in V$, $y\in V'$, and $d\in \operatorname{Sym}(D,*)$. Then

$$\begin{aligned} &f_{\psi\oplus\psi'}\left(\varphi_{\psi\oplus\psi'}\left[\begin{pmatrix} x \\ y \end{pmatrix}\cdot d\otimes {}^*\begin{pmatrix} x \\ y \end{pmatrix}\right]\right) \\ &\quad = \operatorname{Trd}_D\left(d\cdot(Q+Q')\begin{pmatrix} x \\ y \end{pmatrix}\right) \\ &\quad = \operatorname{Trd}_D(d\cdot Q(x)) + \operatorname{Trd}_D(d\cdot Q'(y)) \end{aligned} \tag{3.26}$$

by the definition of the associated quadratic pair.

On the other hand,

$$\begin{aligned} &f_\psi\oplus f_{\psi'}\begin{pmatrix} \varphi_\psi(xd\otimes {}^*x) & xd\psi'(y,\cdot) \\ yd\psi(x,\cdot) & \varphi_{\psi'}(yd\otimes {}^*y) \end{pmatrix} \\ &\quad = f_\psi(\varphi_\psi(xd\otimes {}^*x)) + f_{\psi'}(\varphi_{\psi'}(yd\otimes {}^*y)) \\ &\quad = \operatorname{Trd}_D(dQ(x)) + \operatorname{Trd}_D(dQ'(y)), \end{aligned} \tag{3.27}$$

which implies that (3.21) holds for Type 1 generators of $\operatorname{Sym}(\operatorname{End}_D(V\oplus W),\sigma_{\psi\oplus\psi'})$. Now take $x_1,x_2\in V$ and $y_1,y_2\in V'$. Since $(\sigma_{\psi\oplus\psi'},f_{\psi\oplus\psi'})$ is a quadratic pair, we have

$$\begin{aligned}
&f_{\psi\oplus\psi'}\left(\varphi_{\psi\oplus\psi'}\left(\begin{pmatrix}x_1\\y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\y_2\end{pmatrix}+\begin{pmatrix}x_2\\y_2\end{pmatrix}\otimes{}^*\begin{pmatrix}x_1\\y_1\end{pmatrix}\right)\right)\\
&\quad=f_{\psi\oplus\psi'}\left(\varphi_{\psi\oplus\psi'}\left(\begin{pmatrix}x_1\\y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\y_2\end{pmatrix}\right)\right.\\
&\qquad\qquad\left.+\sigma\left(\varphi_{\psi\oplus\psi'}\begin{pmatrix}x_1\\y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\y_2\end{pmatrix}\right)\right)\\
&\quad=\operatorname{Trd}_{\operatorname{End}_D(V\oplus V')}\left(\varphi_{\psi\oplus\psi'}\begin{pmatrix}x_1\\y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\y_2\end{pmatrix}\right)\\
&\quad=\operatorname{Trd}_D\left(\psi\oplus\psi'\left(\begin{pmatrix}x_2\\y_2\end{pmatrix},\begin{pmatrix}x_1\\y_1\end{pmatrix}\right)\right)\\
&\quad=\operatorname{Trd}_D\left(\psi(x_2,x_1)\right)+\operatorname{Trd}_D\left(\psi'(y_2,y_1)\right)\\
&\quad=\operatorname{Trd}_D\left(\psi(x_1,x_2)\right)+\operatorname{Trd}_D\left(\psi'(y_1,y_2)\right).
\end{aligned}\tag{3.28}$$

On the other hand,

$$\begin{aligned}
&f_\psi\oplus f_{\psi'}\left(\varphi_{\psi\oplus\psi'}\left(\begin{pmatrix}x_1\\y_1\end{pmatrix}\otimes{}^*\begin{pmatrix}x_2\\y_2\end{pmatrix}+\begin{pmatrix}x_2\\y_2\end{pmatrix}\otimes{}^*\begin{pmatrix}x_1\\y_1\end{pmatrix}\right)\right)\\
&\quad=f_\psi\left(x_1\otimes{}^*x_2+x_2\otimes{}^*x_1\right)+f_{\psi'}\left(y_1\otimes{}^*y_2+y_2\otimes{}^*y_1\right)\\
&\quad=\operatorname{Trd}_D\left(\psi(x_1,x_2)\right)+\operatorname{Trd}_D\left(\psi'(y_1,y_2)\right),
\end{aligned}\tag{3.29}$$

which implies that (3.21) also holds for Type 2 generators, and this completes our proof. □

Assume that (σ,f), (σ',f'), and (σ'',f'') are quadratic pairs on A such that

$$(\sigma,f)\perp(\sigma',f')\simeq(\sigma,f)\perp(\sigma'',f'').\tag{3.30}$$

Does this imply that $(\sigma',f')\simeq(\sigma'',f'')$?

Proposition 3.4. *There is no Witt cancellation theorem for quadratic pairs in characteristic* 2*.*

Counterexample 3.5. Let k be a field of characteristic 2 and $F=k(x,y,z,t)$. We consider the quadratic forms

$$q=\langle 1,1,x,y\rangle[1,t],\qquad q'=\langle 1,1,x,z\rangle[1,t],\qquad q''=\langle 1,x,y,yz\rangle[1,t]\tag{3.31}$$

(see [1, page 5] for notation). Then $q\perp q'$ and $q\perp q''$ are isometric up to a scalar factor, but q' and q'' are not isometric up to a scalar factor since the first form is isotropic whereas the second is anisotropic. We conclude that, in general, there is no Witt cancellation theorem for quadratic pairs.

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