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Research Article

On Certain Subclasses of Analytic Functions Defined by Differential Subordination

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We introduce and study certain subclasses of analytic functions which are defined by differential subordination. Coefficient inequalities, some properties of neighborhoods, distortion and covering theorems, radius of starlikeness, and convexity for these subclasses are given.

1. Introduction

Let $\mathcal{T}(j)$ be the class of analytic functions f of the form

$$f(z) = z - \sum_{k=j+1}^{\infty} a_k z^k, \quad \left(a_k \geq 0,\ j \in \mathbb{N} = \{1, 2, \ldots\}\right), \tag{1.1}$$

defined in the open unit disc $\mathcal{U} = \{z \in \mathbb{C} : |z| < 1\}$.

Let Ω be the class of functions ω analytic in $\mathcal{U}$ such that $\omega(0) = 0$, $|\omega(z)| < 1$.

For any two functions f and g in $\mathcal{T}(j)$, f is said to be subordinate to g that is denoted $f \prec g$, if there exists an analytic function $\omega \in \Omega$ such that $f(z) = g(\omega(z))$ [1].

Definition 1.1 (see [2]). For $n \in \mathbb{N}$ and $\lambda \geq 0$, the Al-Oboudi operator $D_\lambda^n : \mathcal{T}(j) \to \mathcal{T}(j)$ is defined as $D_\lambda^0 f(z) = f(z)$, $D_\lambda^1 f(z) = (1 - \lambda) f(z) + \lambda z f'(z) = D_\lambda f(z)$, and $D_\lambda^n f(z) = D_\lambda(D_\lambda^{n-1} f(z))$.

For $\lambda = 1$, we get Sălăgean differential operator [3].

Further, if $f(z) = z - \sum_{k=j+1}^{\infty} a_k z^k$, then

$$D_\lambda^n f(z) = z - \sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n a_k z^k \quad (a_k \geq 0). \tag{1.2}$$

For any function $f \in \mathcal{T}(j)$ and $\delta \geq 0$, the (j,δ)-neighborhood of f is defined as

$$\mathcal{N}_{j,\delta}(f) = \left\{ g(z) = z - \sum_{k=j+1}^{\infty} b_k z^k \in \mathcal{T}(j) : \sum_{k=j+1}^{\infty} k|a_k - b_k| \leq \delta \right\}. \tag{1.3}$$

In particular, for the identity function $e(z) = z$, we see that

$$\mathcal{N}_{j,\delta}(e) = \left\{ g(z) = z - \sum_{k=j+1}^{\infty} b_k z^k \in \mathcal{T}(j) : \sum_{k=j+1}^{\infty} k|b_k| \leq \delta \right\}. \tag{1.4}$$

The concept of neighborhoods was first introduced by Goodman [4] and then generalized by Ruscheweyh [5].

Definition 1.2. A function $f \in \mathcal{T}(j)$ is said to be in the class $\mathcal{T}_j(n,m,A,B,\lambda)$ if

$$\frac{D_\lambda^{n+m} f(z)}{D_\lambda^n f(z)} \prec \frac{1+Az}{1+Bz}, \quad z \in \mathcal{U}, \tag{1.5}$$

where $n \in \mathbb{N}_0$, $m \in \mathbb{N}$, $\lambda \geq 1$, and $-1 \leq B < A \leq 1$.

We observe that $\mathcal{T}_j(n,m,1-2\alpha,-1,1) \equiv \mathcal{T}_j(n,m,\alpha)$ [6], $\mathcal{T}_j(0,1,1-2\alpha,-1,1) \equiv \mathcal{S}_j^\star(\alpha)$ [7], the class of starlike functions of order α and $\mathcal{T}_j(1,1,1-2\alpha,-1,1) \equiv \mathcal{C}_j(\alpha)$ [7], the class of convex functions of order α.

2. Neighborhoods for the Class $\mathcal{T}_j(n,m,A,B,\lambda)$

Theorem 2.1. *A function $f \in \mathcal{T}(j)$ belongs to the class $\mathcal{T}_j(n,m,A,B,\lambda)$ if and only if*

$$\sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{(1-B)[1+(k-1)\lambda]^m - (1-A)\} a_k \leq A - B \tag{2.1}$$

for $n \in \mathbb{N}_0$, $m \in \mathbb{N}$, $\lambda \geq 1$, and $-1 \leq B < A \leq 1$.

Proof. Let $f \in \mathcal{T}_j(n,m,A,B,\lambda)$. Then,

$$\frac{D_\lambda^{n+m} f(z)}{D_\lambda^n f(z)} \prec \frac{1+Az}{1+Bz}, \quad z \in \mathcal{U}. \tag{2.2}$$

Therefore,

$$\omega(z) = \frac{D_\lambda^n f(z) - D_\lambda^{n+m} f(z)}{B D_\lambda^{n+m} f(z) - A D_\lambda^n f(z)}. \tag{2.3}$$

Hence,

$$\begin{aligned} |\omega(z)| &= \left| \frac{D_\lambda^n f(z) - D_\lambda^{n+m} f(z)}{B D_\lambda^{n+m} f(z) - A D_\lambda^n f(z)} \right| \\ &= \left| \frac{\sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{[1+(k-1)\lambda]^m - 1\} a_k z^k}{(A-B)z + \sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{B[1+(k-1)\lambda]^m - A\} a_k z^k} \right| < 1. \end{aligned} \tag{2.4}$$

Thus,

$$\Re\left\{ \frac{\sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{[1+(k-1)\lambda]^m - 1\} a_k z^k}{(A-B)z + \sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{B[1+(k-1)\lambda]^m - A\} a_k z^k} \right\} < 1. \tag{2.5}$$

Taking $|z| = r$, for sufficiently small r with $0 < r < 1$, the denominator of (2.5) is positive and so it is positive for all r with $0 < r < 1$, since $\omega(z)$ is analytic for $|z| < 1$. Then, inequality (2.5) yields

$$\begin{aligned} &\sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{[1+(k-1)\lambda]^m - 1\} a_k r^k \\ &\quad < (A-B)r + B \sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^{n+m} a_k r^k - A \sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n a_k r^k. \end{aligned} \tag{2.6}$$

Equivalently,

$$\sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{(1-B)[1+(k-1)\lambda]^m - (1-A)\} a_k r^k \le (A-B)r, \tag{2.7}$$

and (2.1) follows upon letting $r \to 1$.

Conversely, for $|z| = r$, $0 < r < 1$, we have $r^k < r$. That is,

$$\begin{aligned} &\sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{(1-B)[1+(k-1)\lambda]^m - (1-A)\} a_k r^k \\ &\quad \le \sum_{k=j+1}^{\infty} [1+(k-1)\lambda]^n \{(1-B)[1+(k-1)\lambda]^m - (1-A)\} a_k r \le (A-B)r. \end{aligned} \tag{2.8}$$

From (2.1), we have

$$\begin{aligned}
&\left|\sum_{k=j+1}^{\infty}[1+(k-1)\lambda]^{n}\left\{[1+(k-1)\lambda]^{m}-1\right\}a_k z^k\right| \\
&\quad\leq \sum_{k=j+1}^{\infty}[1+(k-1)\lambda]^{n}\left\{[1+(k-1)\lambda]^{m}-1\right\}a_k r^k \\
&\quad< (A-B)r+\sum_{k=j+1}^{\infty}\left\{B[1+(k-1)\lambda]^{m}-A\right\}[1+(k-1)\lambda]^{n}a_k r^k \\
&\quad< \left|(A-B)z+\sum_{k=j+1}^{\infty}\left\{B[1+(k-1)\lambda]^{m}-A\right\}[1+(k-1)\lambda]^{n}a_k z^k\right|.
\end{aligned} \tag{2.9}$$

This proves that

$$\frac{D_{\lambda}^{n+m}f(z)}{D_{\lambda}^{n}f(z)} \prec \frac{1+Az}{1+Bz}, \quad z\in\mathcal{U}, \tag{2.10}$$

and hence $f\in\mathcal{T}_j(n,m,A,B,\lambda)$. □

Theorem 2.2. *If*

$$\delta=\frac{(A-B)}{(1+\lambda j)^{n-1}\left[(1-B)(1+\lambda j)^{m}-(1-A)\right]}, \tag{2.11}$$

then $\mathcal{T}_j(n,m,A,B,\lambda)\subset N_{j,\delta}(e)$.

Proof. It follows from (2.1) that if $f\in\mathcal{T}_j(n,m,A,B,\lambda)$, then

$$(1+\lambda j)^{n-1}\left[(1-B)(1+\lambda j)^{m}-(1-A)\right]\sum_{k=j+1}^{\infty}ka_k\leq(A-B), \tag{2.12}$$

which implies

$$\sum_{k=j+1}^{\infty}ka_k\leq\frac{(A-B)}{(1+\lambda j)^{n-1}\left[(1-B)(1+\lambda j)^{m}-(1-A)\right]}=\delta. \tag{2.13}$$

Using (1.4), we get the result. □

3. Neighborhoods for the Classes $\mathcal{R}_j(n,A,B,\lambda)$ and $\mathcal{P}_j(n,A,B,\lambda)$

Definition 3.1. A function $f \in \mathcal{T}(j)$ is said to be in the class $\mathcal{R}_j(n,A,B,\lambda)$ if it satisfies

$$\left(D_\lambda^n f(z)\right)' \prec \frac{1+Az}{1+Bz} \quad (z \in \mathcal{U}), \tag{3.1}$$

where $-1 \le B < A \le 1$, $\lambda \ge 1$ and $n \in \mathcal{N}_0$.

Definition 3.2. A function $f \in \mathcal{T}(j)$ is said to be in the class $\mathcal{P}_j(n,A,B,\lambda)$ if it satisfies

$$\frac{D_\lambda^n f(z)}{z} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathcal{U}), \tag{3.2}$$

where $-1 \le B < A \le 1$, $\lambda \ge 1$ and $n \in \mathcal{N}_0$.

Lemma 3.3. *A function $f \in \mathcal{T}(j)$ belongs to the class $\mathcal{R}_j(n,A,B,\lambda)$ if and only if*

$$\sum_{k=j+1}^{\infty} (1-B)[1+(k-1)\lambda]^{n+1} a_k \le A-B. \tag{3.3}$$

Lemma 3.4. *A function $f \in \mathcal{T}(j)$ belongs to the class $\mathcal{P}_j(n,A,B,\lambda)$ if and only if*

$$\sum_{k=j+1}^{\infty} (1-B)[1+(k-1)\lambda]^{n} a_k \le A-B. \tag{3.4}$$

Theorem 3.5. $\mathcal{R}_j(n,A,B,\lambda) \subset \mathcal{N}_{j,\delta}(e)$, *where*

$$\delta = \frac{(A-B)}{[1+\lambda j]^n (1-B)}. \tag{3.5}$$

Proof. If $f \in \mathcal{R}_j(n,A,B,\lambda)$, we have

$$[1+\lambda j]^n \sum_{k=j+1}^{\infty} (1-B) k a_k \le A-B, \tag{3.6}$$

which implies

$$\sum_{k=j+1}^{\infty} k a_k \le \frac{(A-B)}{[1+\lambda j]^n (1-B)} = \delta. \tag{3.7}$$

□

Theorem 3.6. $\mathcal{P}_j(n,A,B,\lambda) \subset \mathcal{N}_{j,\delta}(e)$, *where*

$$\delta = \frac{(A-B)}{[1+\lambda j]^{n-1} (1-B)}. \tag{3.8}$$

Proof. If $f \in \mathcal{D}_j(n, A, B, \lambda)$, we have

$$[1+\lambda j]^{n-1} \sum_{k=j+1}^{\infty} (1-B) k a_k \leq A - B, \tag{3.9}$$

which implies

$$\sum_{k=j+1}^{\infty} k a_k \leq \frac{(A-B)}{[1+\lambda j]^{n-1}(1-B)} = \delta. \tag{3.10}$$

Thus, in view of condition (1.4), we get the required result of Theorem 3.6. □

4. Neighborhood of the Class $\mathcal{K}_j^{\lambda}(n, m, A, B, C, D)$

Definition 4.1. A function $f \in \mathcal{T}(j)$ is said to be in the class $\mathcal{K}_j^{\lambda}(n, m, A, B, C, D)$ if it satisfies

$$\left| \frac{f(z)}{g(z)} - 1 \right| < \frac{A-B}{1-B}, \quad z \in \mathcal{U}, \tag{4.1}$$

for $-1 \leq B < A \leq 1$, $-1 \leq D < C \leq 1$, $\lambda \geq 1$ and $g \in \mathcal{T}_j(n, m, C, D, \lambda)$.

Theorem 4.2. *For $g \in \mathcal{T}_j(n, m, C, D, \lambda)$, one has $\mathcal{N}_{j,\delta}(g) \subset \mathcal{K}_j^{\lambda}(n, m, A, B, C, D)$ and*

$$\frac{1-A}{1-B} = 1 - \frac{[1+\lambda j]^{n-1}[(1-D)[1+\lambda j]^m - (1-C)]\delta}{[1+\lambda j]^n[(1-D)[1+\lambda j]^m - (1-C)] - (C-D)}, \tag{4.2}$$

where

$$\delta \leq (1-D)(1+\lambda j) - (C-D)[1+\lambda j]^{1-n}\{(1-D)[1+\lambda j]^m - (1-C)\}^{-1}. \tag{4.3}$$

Proof. Let $f \in \mathcal{N}_{j,\delta}(g)$ for $g \in \mathcal{T}_j(n, m, C, D, \lambda)$. Then,

$$\sum_{k=j+1}^{\infty} k|a_k - b_k| \leq \delta, \qquad \sum_{k=j+1}^{\infty} b_k \leq \frac{C-D}{[1+\lambda j]^n[(1-D)[1+\lambda j]^m - (1-C)]}. \tag{4.4}$$

Consider

$$\begin{aligned}\left|\frac{f(z)}{g(z)}-1\right| &\leq \frac{\sum_{k=j+1}^{\infty}|a_k-b_k|}{1-\sum_{k=j+1}^{\infty}b_k}\\ &\leq \frac{\delta}{(1+\lambda j)}\ \frac{[1+\lambda j]^n\{(1-D)[1+\lambda j]^m-(1-C)\}}{[1+\lambda j]^n\{(1-D)[1+\lambda j]^m-(1-C)\}-(C-D)}\\ &= \frac{[1+\lambda j]^{n-1}\{(1-D)[1+\lambda j]^m-(1-C)\}\delta}{[1+\lambda j]^n\{(1-D)[1+\lambda j]^m-(1-C)\}-(C-D)}\\ &= \frac{A-B}{1-B}.\end{aligned} \tag{4.5}$$

This implies that $f \in \mathcal{K}_j^{\lambda}(n,m,A,B,C,D)$. □

5. Distortion and Covering Theorems

Theorem 5.1. *If $f \in \mathcal{T}_j(n,m,A,B,\lambda)$, then*

$$\begin{aligned}&r-\frac{A-B}{(1+j\lambda)^n\{(1-B)(1+j\lambda)^m-(1-A)\}}r^{j+1}\\ &\quad\leq |f(z)| \leq r+\frac{A-B}{(1+j\lambda)^n\{(1-B)(1+j\lambda)^m-(1-A)\}}r^{j+1} \quad (0<|z|=r<1),\end{aligned} \tag{5.1}$$

with equality for

$$f(z)=z-\frac{A-B}{(1+j\lambda)^n\{(1-B)(1+j\lambda)^m-(1-A)\}}r^{j+1} \quad (z=\pm r). \tag{5.2}$$

Proof. In view of Theorem 2.1, we have

$$\begin{aligned}&(1+j\lambda)^n\{(1-B)(1+j\lambda)^m-(1-A)\}\sum_{k=j+1}^{\infty}a_k\\ &\quad\leq \sum_{k=j+1}^{\infty}[1+(k-1)\lambda]^n\{(1-B)[1+(k-1)\lambda]^m-(1-A)\}a_k \leq A-B.\end{aligned} \tag{5.3}$$

Hence,

$$\begin{aligned}|f(z)| &\leq r+\sum_{k=j+1}^{\infty}a_kr^k \leq r+r^{j+1}\sum_{k=j+1}^{\infty}a_k \leq r+\frac{A-B}{(1+j\lambda)^n\{(1-B)(1+j\lambda)^m-(1-A)\}}r^{j+1},\\ |f(z)| &\geq r-\sum_{k=j+1}^{\infty}a_kr^k \geq r-r^{j+1}\sum_{k=j+1}^{\infty}a_k \geq r-\frac{A-B}{(1+j\lambda)^n\{(1-B)(1+j\lambda)^m-(1-A)\}}r^{j+1}.\end{aligned} \tag{5.4}$$

This completes the proof. □

Theorem 5.2. *Any function $f \in \mathcal{T}_j(n, m, A, B, \lambda)$ maps the disk $|z| < 1$ onto a domain that contains the disk*

$$|w| < 1 - \frac{A - B}{(1 + j\lambda)^n \{(1 - B)(1 + j\lambda)^m - (1 - A)\}}. \tag{5.5}$$

Proof. The proof follows upon letting $r \to 1$ in Theorem 5.1. □

Theorem 5.3. *If $f \in \mathcal{T}_j(n, m, A, B, \lambda)$, then*

$$\begin{aligned} 1 &- \frac{(A - B)}{(1 + j\lambda)^{n-1} \{(1 - B)(1 + j\lambda)^m - (1 - A)\}} r^j \\ &\leq |f'(z)| \leq 1 + \frac{A - B}{(1 + j\lambda)^{n-1} \{(1 - B)(1 + j\lambda)^m - (1 - A)\}} r^j \quad (0 < |z| = r < 1), \end{aligned} \tag{5.6}$$

with equality for

$$f(z) = z - \frac{A - B}{(1 + j\lambda)^{n-1} \{(1 - B)(1 + j\lambda)^m - (1 - A)\}} z^{j+1} \quad (z = \pm r). \tag{5.7}$$

Proof. We have

$$|f'(z)| \leq 1 + \sum_{k=j+1}^{\infty} k a_k |z|^{k-1} \leq 1 + r^j \sum_{k=j+1}^{\infty} k a_k. \tag{5.8}$$

In view of Theorem 2.1,

$$\sum_{k=j+1}^{\infty} k a_k \leq \frac{A - B}{(1 + j\lambda)^{n-1} \{(1 - B)(1 + j\lambda)^m - (1 - A)\}}. \tag{5.9}$$

Thus,

$$|f'(z)| \leq 1 + \frac{A - B}{(1 + j\lambda)^{n-1} \{(1 - B)(1 + j\lambda)^m - (1 - A)\}} r^j. \tag{5.10}$$

On the other hand,

$$\begin{aligned} |f'(z)| &\geq 1 - \sum_{k=j+1}^{\infty} k a_k |z|^{k-1} \geq 1 - r^j \sum_{k=j+1}^{\infty} k a_k \\ &\geq 1 - \frac{A - B}{(1 + j\lambda)^{n-1} \{(1 - B)(1 + j\lambda)^m - (1 - A)\}} r^j. \end{aligned} \tag{5.11}$$

This completes the proof. □

6. Radii of Starlikeness and Convexity

In this section, we find the radius of starlikeness of order ρ and the radius of convexity of order ρ for functions in the class $\mathcal{T}_j(n,m,A,B,\lambda)$.

Theorem 6.1. *If $f \in \mathcal{T}_j(n,m,A,B,\lambda)$, then f is starlike of order ρ, $(0 \le \rho < 1)$ in $|z| < r_1(n,m,A,B,\lambda,\rho)$, where*

$$r_1(n,m,A,B,\lambda,\rho) = \inf_k \left[\frac{[1+(k-1)\lambda]^n\{(1-B)[1+(k-1)\lambda]^m-(1-A)\}(1-\rho)}{(k-\rho)(A-B)}\right]^{1/(k-1)}. \tag{6.1}$$

Proof. It is sufficient to show that $|z(f'(z)/f(z))-1| \le 1-\rho$ $(0 \le \rho < 1)$ for $|z| < r_1(n,m,A,B,\lambda,\rho)$.

We have

$$\left|z\frac{f'(z)}{f(z)}-1\right| \le \frac{\sum_{k=j+1}^{\infty}(k-1)a_k|z|^{k-1}}{1-\sum_{k=j+1}^{\infty}a_k|z|^{k-1}}. \tag{6.2}$$

Thus, $|z(f'(z)/f(z))-1| \le 1-\rho$ if

$$\sum_{k=j+1}^{\infty}\frac{(k-\rho)a_k|z|^{k-1}}{(1-\rho)} \le 1. \tag{6.3}$$

Hence, by Theorem 2.1, (6.3) will be true if

$$\frac{(k-\rho)|z|^{k-1}}{(1-\rho)} \le \frac{[1+(k-1)\lambda]^n\{(1-B)[1+(k-1)\lambda]^m-(1-A)\}}{(A-B)} \tag{6.4}$$

or if

$$|z| \le \left[\frac{[1+(k-1)\lambda]^n\{(1-B)[1+(k-1)\lambda]^m-(1-A)\}(1-\rho)}{(k-\rho)(A-B)}\right]^{1/(k-1)} \quad (k \ge j+1). \tag{6.5}$$

This completes the proof. □

Theorem 6.2. *If $f \in \mathcal{T}_j(n,m,A,B,\lambda)$, then f is convex of order ρ, $(0 \le \rho < 1)$ in $|z| < r_2(n,m,A,B,\lambda,\rho)$, where*

$$r_2(n,m,A,B,\lambda,\rho) = \inf_k \left[\frac{[1+(k-1)\lambda]^n\{(1-B)[1+(k-1)\lambda]^m-(1-A)\}(1-\rho)}{(k-\rho)(A-B)}\right]^{1/(k-1)}. \tag{6.6}$$

Proof. It is sufficient to show that $|z(f''(z)/f'(z))| \leq 1-\rho$ $(0 \leq \rho < 1)$ for $|z| < r_1(n,m,A,B,\lambda,\rho)$.

We have

$$\left|z\frac{f''(z)}{f'(z)}\right| \leq \frac{\sum_{k=j+1}^{\infty} k(k-1)a_k|z|^{k-1}}{1-\sum_{k=j+1}^{\infty} ka_k|z|^{k-1}}. \tag{6.7}$$

Thus, $|z(f''(z)/f'(z))| \leq 1-\rho$ if

$$\sum_{k=j+1}^{\infty} \frac{k(k-\rho)a_k|z|^{k-1}}{(1-\rho)} \leq 1. \tag{6.8}$$

Hence, by Theorem 2.1, (6.8) will be true if

$$\frac{k(k-\rho)|z|^{k-1}}{(1-\rho)} \leq \frac{[1+(k-1)\lambda]^n\{(1-B)[1+(k-1)\lambda]^m-(1-A)\}}{(A-B)} \tag{6.9}$$

or if

$$|z| \leq \left[\frac{[1+(k-1)\lambda]^n\{(1-B)[1+(k-1)\lambda]^m-(1-A)\}(1-\rho)}{k(k-\rho)(A-B)}\right]^{1/(k-1)} \quad (k \geq j+1). \tag{6.10}$$

This completes the proof. □

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