Hindawi
International Journal of Mathematics and Mathematical Sciences
Volume 2020, Article ID 2649514, 12 pages
https://doi.org/10.1155/2020/2649514

Research Article

Control Strategies to Curtail Transmission of COVID-19

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Received 30 March 2020; Revised 24 April 2020; Accepted 27 April 2020; Published 21 May 2020

Academic Editor: Vladimir V. Mityushev

Recently, the World Health Organization has declared the outbreak of a severe acute respiratory syndrome coronavirus 2 (SARS-Cov-2) as a pandemic and declared it as Public Health Emergency of International Concern. As on 29 March 2020, the novel corona virus 2019 (COVID-19) has affected 199 countries and territories, causing 683,536 positive cases and 32,139 deaths. This pandemic can transform into an extremely destructive form if we still do not take it seriously. In the present study, we propose a generalized SEIR model of COVID-19 to study the behaviour of its transmission under different control strategies. In the model, all possible cases of human-to-human transmission are taken care and its reproduction number is formulated to analyse the accurate transmission dynamics of the coronavirus outbreak. Optimal control theory is applied in the model to pretend the impact of various intervention strategies, including voluntary quarantine, isolation of infected individuals, improving an individual's immunity, and hospitalisation. Also, the effect of control strategies on the model is analysed graphically by simulatin`g the model numerically.

1. Introduction

In December 2019, the pandemic outbreak of a novel coronavirus disease named "COVID-19" raised intense attention not only within China but internationally[1]. Doctors and scientists tested the previously developed drugs to treat the infected people, but those failed to succeed. Then, in February 2020, the World Health Organization (WHO) declared the outbreak of this highly contagious COVID-19 as a pandemic globally [2]. To control the human-to-human virus transmission, the central government of China as well as all local governments had tightened preventive measures. However, the virus had spread rapidly across most of the regions in China and in other countries and territories around the world. One major cause of the quick spread of COVID-19 is the lack of information and awareness about the virus during its early stages of infection. As on March 29, 2020, 13:29 GMT, COVID-19 has affected 199 countries and territories around the world with 683,536 confirmed cases, of which 32,139 have passed away and 25,422 are serious or critical (worldometers.info). Still, there is a possibility that the spread of this virus could be more intense and cause high mortality. New year celebration accelarated the outbreak of COVID-19, as most of the people were on their way to hometown or else travelling to different places for relaxation.

Symptoms of COVID-19 take at least 2 to 10 days to develop, which makes it tough to isolate infected individuals during the initial stage of the infection. The major symptoms of COVID-19 include dry coughing and high fever with difficulties in breathing [3]. The virus may spread in the environment through respiratory droplets of infected individuals when they cough or sneeze. An unaffected population further becomes infected when they are exposed by touching the infected surface or while breathing in an infected environment [4]. During the initial stages of COVID-19 outbreak, such human transmissions were taking place because a wide range of public was unaware of these risk factors, and the infected individuals were also not isolated and were spreading the virus unknowingly to other individuals. Moreover, the risk factor of contamination is very high since the virus can remain viable in the environment for several days in favourable conditions [5, 6]. Several studies reveal that old-aged people, children, and those with major diseases have low immunity and tend to be seriously affected once they become infected [7, 8]. Still, we lack any proper treatment or vaccines as a cure for this

disease. Hence, to control its transmission further, isolating the infected individuals in special quarantine cells has been implemented in most of the countries. Despite these prevention strategies, we are in danger as the transmission is still ongoing and the mortality due to the virus maintains a high level.

To combat this situation, studies such as mathematical modelling play a crucial role to understand the pandemic behaviour of the infectious disease. Bi et al. worked on stability and bifurcation of a mathematics model in the Zika virus. Also, they established an optimal control problem including several popular disease intervention strategies which helped to mitigate the Zika epidemics [9]. Zhao et al. modeled a vector-born visceral leishmaniasis disease transmission system, and in addition, they have performed bifurcation analysis to support control conditions and analyse optimal control strategies to control zoonotic disease transmission [10]. Several studies have already been undertaken to analyse the COVID-19 transmission dynamics. Based on the database studies of COVID-19 outbreak from 31 December 2019 to 28 January 2020, Wu et al. introduced a SEIR model to estimate the spread of the disease nationally as well as globally [11]. Tang et al. proposed a compartmental model by dividing each group into two subpopulations, the quarantined and unquarantined. Moreover, they redesigned their previous model by using diagnosis and time-dependent contact rates and reestimated the reproduction ratio to quantify the evolution in a better way [12, 13]. Peng et al. planned a generalized SEIR model that suitably incorporates the intrinsic impact of hidden exposed and infectious cases of COVID-19 [14]. Chen et al. presented the incubation period behaviour of local outbreak of COVID-19 by constructing a dynamic system [15]. Khan and Atangana described brief details of interaction among the bats, unknown hosts, humans, and infection reservoirs by formulating the mathematical results of the mathematical fractional model [16]. Chen et al. have also developed a Bats-Hosts-Reservoir-People transmission network model for simulating the potential transmission of COVID-19 [17]. Zhao et al. divided susceptible people from Wuhan City into different age groups and developed a SEIARW model based on market-to-person and person-to-person transmission routes [18]. Zhong et al. constructed a mathematical model using epidemiological data and examined characteristics of the historical epidemic to make an early prediction of the 2019-nCoV outbreak in China [19]. Yang and Wang have studied the multiple transmission pathways in the infection dynamics and formed a mathematical model that describes the role of the environmental reservoir in the transmission and spread of COVID-19 [20].

In this work, a COVID-19 model is constructed to study human-to-human transmission of the virus in Section 2. Optimal control theory is introduced and applied to the model for development in Section 3, and in Section 4, the model is simulated numerically to observe the effect of control strategies on the model.

2. COVID-19 Model Formulation

To analyse the human-to-human transmission dynamics of COVID-2019, a compartmental model is constructed. The model consists of all possible human-to-human transmission dynamics of the virus. COVID-2019 is highly contagious in nature, and infected cases are seen in most of the countries around the world; hence, in the model, the susceptible population class is ignored and whole population is divided into five compartments: class of exposed individuals $E(t)$ (individuals surrounded by infection but not yet infected), class of infected individuals by COVID-19 $I(t)$, class of critically infected individuals by COVID-19 $C(t)$, class of hospitalised individuals $H(t)$, and class of dead individuals due to COVID-19 $D(t)$. Human-to-human transmission dynamics of COVID-19 are described graphically in Figure 1. Parameters used in the model are described in Table 1.

Using the above depiction, a dynamical system of set of nonlinear differential equations for the model is formulated as follows:

$$\begin{aligned}
\frac{dE}{dt} &= B - \beta_1 EI + \beta_7 ED + \beta_9 H + \beta_{10} EI - \mu E,\\
\frac{dI}{dt} &= \beta_1 EI - \beta_2 I - \beta_6 I - \beta_8 I - \beta_{10} EI - \mu I,\\
\frac{dC}{dt} &= \beta_2 I - \beta_5 C - \beta_3 C + \beta_4 H - \mu C,\\
\frac{dH}{dt} &= \beta_3 C - \beta_4 H + \beta_8 I - \beta_9 H - \mu H,\\
\frac{dD}{dt} &= \beta_5 C + \beta_6 I - \beta_7 DE.
\end{aligned} \tag{1}$$

Note that all the parameters used in this COVID-19 model are nonnegative. Consider the feasible region as follows:

$$\Lambda = \left\{(E, I, C, H, D) \in R_+^5 \colon E + I + C + H + D \le \frac{B}{\mu}\right\}. \tag{2}$$

The region Λ is positively invariant; all the solutions of system (1) remain in feasible region (2).

2.1. Equilibrium Points. By solving system (1), we get two equilibrium points:

(i) Disease-free equilibrium point:

$$E_0 = \left(\frac{B}{\mu}, 0, 0, 0, 0\right). \tag{3}$$

(ii) Endemic equilibrium point:

$$E^* = \left(\frac{\beta_2 + \beta_6 + \beta_8 + \mu}{\beta_1 - \beta_{10}}, \frac{i}{\mu q_1}, \frac{c}{\mu q_1}, \frac{h}{\mu q_1}, \frac{d}{\mu \beta_7 q_2}\right), \tag{4}$$

where

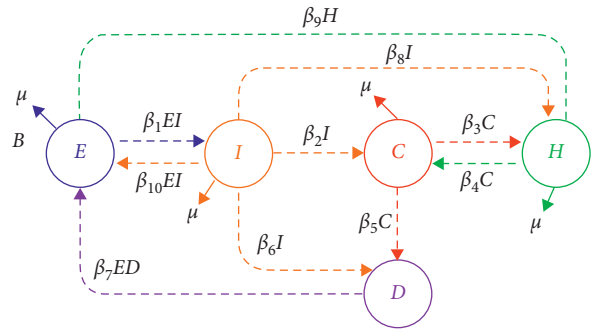


Figure 1: Transmission dynamics of COVID-19.

Table 1: Parameters used in the model.

Parameters			
B	Birth rate of class of exposed individuals	0.80	Calculated
μ	Natural death rate	0.01	Assumed
β_1	Transmission rate of individuals moving from exposed to infected class	0.55	Calculated
β_2	Rate at which infected individuals go into severe condition or in critical condition	0.40	Calculated
β_3	Rate at which critically infected individuals get hospitalised	0.60	Calculated
β_4	Rate at which hospitalised individuals are not recovered and remain in critical condition	0.80	Calculated
β_5	Mortality rate of critically infected individuals	0.34	Calculated
β_6	Mortality rate of infected individuals	0.30	Calculated
β_7	Rate at which infected dead body spreads infection	0.35	Assumed
β_8	Rate at which infected individuals get hospitalised	0.30	Calculated
β_9	Rate at which hospitalised individuals get recovered and become exposed again	0.35	Assumed
β_{10}	Rate at which infected individuals recovered themselves due to strong immunity and again become exposed	0.32	Assumed

$$
\begin{aligned}
i &= (\beta_5+\mu)\left(B\beta_4(\beta_1-\beta_{10})-\beta_4\mu(\beta_2+\beta_6)\right)-(\beta_8+\mu)(\beta_5+\mu)\beta_4\mu+(\beta_9+\mu)\left((\beta_3+\beta_5)(B\beta_1-B\beta_{10}-\beta_2\mu)\right.\\
&\quad\left.-\mu(\beta_6+\beta_8)(\beta_3+\beta_5+\mu)-B\mu(\beta_1+\beta_{10})+\mu^2(\beta_2+\beta_3+\beta_5+\mu)\right),\\
c &= (\beta_4+\beta_9+\mu)\left(B\beta_2(\beta_1-\beta_{10})\beta_2^2\mu\right)-\mu\beta_4(\beta_8+\mu)(\beta_2+\beta_8)-\beta_4\beta_8(B\beta_1-B\beta_{10}-\beta_6\mu)-\mu\beta_2(\beta_9+\mu)(1+\beta_6+\beta_8),\\
h &= \beta_8(\beta_5+\mu)(B\beta_1-B\beta_{10}-\beta_2\mu)-\beta_8\mu^2(\beta_6+\mu)-\mu\beta_8(\beta_8+\mu)(\beta_3+\beta_5)-\mu\beta_6\beta_8(\beta_3+\beta_5)-B\beta_1\beta_2(\beta_3+\beta_8)\\
&\quad-B\beta_{10}\beta_3(\beta_2+\beta_8)-\mu\beta_2\beta_3(\beta_2+\beta_6)-\mu\beta_8(\beta_2\beta_3+\mu),\\
d &= (\beta_9+\mu)\left(B\beta_6(\beta_1-\beta_{10})-\mu\beta_6^2\right)-(\beta_8+\mu)(\beta_2\beta_5\mu+\beta_6\mu(\beta_3+\beta_5+\beta_6+\mu))-\beta_2\beta_5\mu(\beta_2+\beta_6)-\beta_6\mu(\beta_2(\beta_3+\mu)+\beta_3\beta_6)\\
&\quad+B(\beta_1-\beta_{10})(\beta_3\beta_6+\beta_2\beta_5)+\beta_4\beta_6(\beta_5+\mu)(B(\beta_1-\beta_{10})-\mu(\beta_2+\beta_8))-\mu\beta_4(\beta_6+\mu)(\beta_2\beta_5+\beta_5\beta_6+\beta_6\mu)\\
&\quad-(\beta_8+\mu)\beta_4\beta_5\beta_8\mu-\beta_4\beta_5(\beta_2+\beta_6)(\mu(\beta_8+\beta_2)+B(\beta_{10}+\beta_1)),\\
q_1 &= (\beta_1-\beta_{10})(\beta_2(\beta_4+\beta_9+\mu)+(\beta_8+\beta_9+\mu)(\beta_3+\beta_5)+\beta_4(\beta_5+\beta_8+\mu)+\mu(\beta_9+\mu)+\beta_2\beta_3+\beta_8\mu),\\
q_2 &= \mu\beta_7(\beta_2+\beta_6+\beta_8)(\beta_2\beta_3+\beta_2(\beta_4+\beta_8+\mu)+\beta_4(\beta_5+\beta_8+\mu)+(\beta_8+\beta_9+\mu)(\beta_3+\beta_5+\mu)).
\end{aligned}
\tag{5}
$$

2.2. Basic Reproduction Number. The basic reproduction number (R_0) for the model can be established using the next-generation matrix method [21, 22]. The basic reproduction number (R_0) is obtained as the spectral radius of matrix (FV^{-1}) at the disease-free equilibrium point, where F and V are as follows:

$$F = \begin{bmatrix} 0 & \frac{B\beta_{10}}{\mu} & 0 & 0 & \frac{B\beta_7}{\mu} \\ 0 & \frac{B\beta_1}{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} \mu & \frac{B\beta_1}{\mu} & 0 & -\beta_9 & 0 \\ 0 & \frac{B\beta_{10}}{\mu} + \beta_8 + \beta_6 + \beta_2 + \mu & 0 & 0 & 0 \\ 0 & -\beta_2 & \beta_3 + \beta_5 + \mu & -\beta_4 & 0 \\ 0 & -\beta_8 & -\beta_3 & \beta_9 + \beta_4 + \mu & 0 \\ 0 & -\beta_6 & -\beta_5 & 0 & \frac{B\beta_7}{\mu} \end{bmatrix}. \tag{6}$$

The defined effective basic reproduction number is

$$R_0 = \frac{B\beta_1}{B\beta_{10} + \mu(\beta_2 + \beta_6 + \beta_8 + \mu)}. \tag{7}$$

3. Optimal Control Theory

Control measures have a significant role to control the epidemic of COVID-19 at a certain level. In this control theory, five control variables are used as five possible control strategies. Since the virus is highly contagious, it quickly infects any people who come in contact with an infected individual. To avoid this situation, we have taken u_1 control variable to self-quarantine exposed individuals and u_2 control variable as an isolation of infected individuals. Moreover, to minimise the mortality rate of COVID-19, u_3 control variable is taken which helps to reduce critically infected cases by taking extra medical care of infected individuals. u_4 and u_5 control variables are taken to improve hospitalisation facility for infected and critically infected individuals, respectively. The purpose of this study of control theory is to protect people from the outbreak by applying control or treatment in each stage. Objective function for the required scenario is

$$J(c_i, \Omega) = \int_0^T \left(A_1E^2 + A_2I^2 + A_3C^2 + A_4H^2 + A_5D^2 + w_1u_1^2 + w_2u_2^2 + w_3u_3^2 + w_4u_4^2 + w_5u_5^2\right)\mathrm{d}t, \tag{8}$$

where Ω denotes the feasible region for the set of compartmental variables, A_1, A_2, A_3, A_4, and A_5 denote the nonnegative weight constants for compartments E, I, C, H, and D, respectively. w_1, w_2, w_3, w_4, and w_5 are the weight constants for each control u_i, where $i = 1, 2, \ldots, 5$, respectively. The modified model is given in Figure 2.

Hence, the modified system for Figure 2 is stated as follows:

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}t} &= B - \beta_1EI + \beta_7ED + \beta_9H + \beta_{10}EI - u_1E + u_2I - \mu E, \\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \beta_1EI - \beta_2I - \beta_6I - \beta_8I - \beta_{10}EI + u_1E - u_2I - u_5I - u_3I - \mu I, \\ \frac{\mathrm{d}C}{\mathrm{d}t} &= \beta_2I - \beta_5C - \beta_3C + \beta_4H + u_3I - u_4C - \mu C, \\ \frac{\mathrm{d}H}{\mathrm{d}t} &= \beta_3C - \beta_4H + \beta_8I - \beta_9H + u_5I + u_4C - \mu H, \\ \frac{\mathrm{d}D}{\mathrm{d}t} &= \beta_5C + \beta_6I - \beta_7DE. \end{aligned} \tag{9}$$

The weight parameters w_i, $i = 1, 2, \ldots, 5$, are constants applied on the control variable, respectively, from which the optimal condition is normalized. Now, every value of control variables is calculated from $t = 0$ to $t = T$ such that

$$J(u_i(t)) = \min\{J(u_i^*, \Omega)/(u_i) \in \phi\}, \quad i = 1, 2, \ldots, 5, \tag{10}$$

where ϕ is a smooth function on the interval $[0, 1]$. The optimal effect is found by using the results of Fleming and Lions [23]. We use Pontryagin's Maximum Principle to derive the optimal amount of control [24]. The associated Lagrangian function with adjoint variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5 is given by

$$\begin{aligned} L(\Omega, A) = {} & A_1E^2 + A_2I^2 + A_3C^2 + A_4H^2 + A_5D^2 + w_1u_1^2 \\ & + w_2u_2^2 + w_3u_3^2 + w_4u_4^2 + w_5u_5^2 \\ & + \lambda_1(B - \beta_1EI + \beta_7ED + \beta_9H + \beta_{10}EI - \mu E) \\ & + \lambda_2(\beta_1EI - \beta_2I - \beta_6I - \beta_8I - \beta_{10}EI - \mu I) \\ & + \lambda_3(\beta_2I - \beta_5C - \beta_3C + \beta_4H - \mu C) \\ & + \lambda_4(\beta_3C - \beta_4H + \beta_8I - \beta_9H - \mu H) \\ & + \lambda_5(\beta_5C + \beta_6I - \beta_7DE). \end{aligned} \tag{11}$$

The partial derivatives of the Lagrangian function with respect to each variable of the compartment give the adjoint equation variables $A_i = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ corresponding to the system which are as follows:

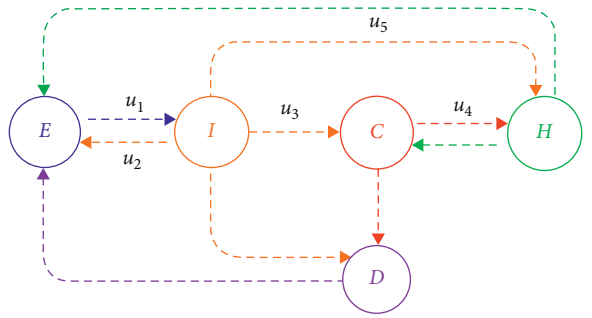


Figure 2: COVID-19 model with control variables.

$$\begin{aligned}
\dot{\lambda}_1 &= -\frac{\partial L}{\partial E} = -2A_1E + (\lambda_1 - \lambda_2)\beta_1 I - (\lambda_1 - \lambda_2)\beta_{10} I \\
&\quad - \lambda_1\beta_7 \mathrm{D} - \lambda_1\mu + \lambda_5\beta_7 \mathrm{D} + (\lambda_1 - \lambda_2)u_1, \\
\dot{\lambda}_2 &= -\frac{\partial L}{\partial I} = -2A_2I + (\lambda_1 - \lambda_2)(\beta_1 E - \beta_{10}E - u_2) \\
&\quad + (\lambda_2 - \lambda_3)(\beta_2 + u_3) + (\lambda_2 - \lambda_4)(\beta_8 + u_5) + (\lambda_2 - \lambda_5)\beta_6 + \lambda_2\mu, \\
\dot{\lambda}_3 &= -\frac{\partial L}{\partial C} = -2A_3C + (\lambda_3 - \lambda_4)(\beta_3 + u_4) + (\lambda_3 - \lambda_5)\beta_5 + \lambda_3\mu, \\
\dot{\lambda}_4 &= -\frac{\partial L}{\partial H} = -2A_4H - (\lambda_1 - \lambda_4)\beta_9 - (\lambda_3 - \lambda_4)\beta_4 + \lambda_4\mu, \\
\dot{\lambda}_5 &= -\frac{\partial L}{\partial D} = -2A_5D - (\lambda_1 - \lambda_5)\beta_7 E.
\end{aligned} \tag{12}$$

Hence, the optimal controls are given by

$$\begin{aligned}
u_1 &= \frac{(\lambda_1 - \lambda_2)E}{2w_1}, \\
u_2 &= \frac{(\lambda_2 - \lambda_1)I}{2w_2}, \\
u_3 &= \frac{(\lambda_2 - \lambda_3)I}{2w_3}, \\
u_4 &= \frac{(\lambda_3 - \lambda_4)C}{2w_4}, \\
u_5 &= \frac{(\lambda_2 - \lambda_4)I}{2w_5}.
\end{aligned} \tag{13}$$

Also, optimal conditions are given as follows:

$$\begin{aligned}
u_1^* &= \max\left(a_1, \min\left(b_1, \frac{(\lambda_1 - \lambda_2)E}{2w_1}\right)\right), \\
u_2^* &= \max\left(a_2, \min\left(b_2, \frac{(\lambda_2 - \lambda_1)I}{2w_2}\right)\right), \\
u_3^* &= \max\left(a_3, \min\left(b_3, \frac{(\lambda_2 - \lambda_3)I}{2w_3}\right)\right), \\
u_4^* &= \max\left(a_4, \min\left(b_4, \frac{(\lambda_3 - \lambda_4)C}{2w_4}\right)\right), \\
u_5^* &= \max\left(a_5, \min\left(b_5, \frac{(\lambda_2 - \lambda_4)I}{2w_5}\right)\right).
\end{aligned} \tag{14}$$

This calculation gives analytical behaviour of optimal control on the system. Numerical interpretation of optimal control theory is simulated in the next section.

4. Numerical Simulation

In this section, the COVID-19 model is simulated numerically, wherein the parametric values for simulation are taken from the recent pandemic outbreak of coronavirus (https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports). The initial condition taken to plot variations in all the compartments is $y_0(E, I, C, H, D) = (8, 4, 0.12, 1, 5)$.

Figure 3 represents the variations in all the compartments of the COVID-19 model with respect to time. Also, the pandemic behaviour of the COVID-19 outbreak can be clearly seen here. We can say that a large population of exposed individuals becomes infected before a week. Moreover, the critically infected cases and hospitalisation cases also increase with time. Furthermore, it clearly shows that after one week, the mortality rate is also increased.

Figure 4(a) shows variation in each compartment with and without control variables. It is observed that COVID-19

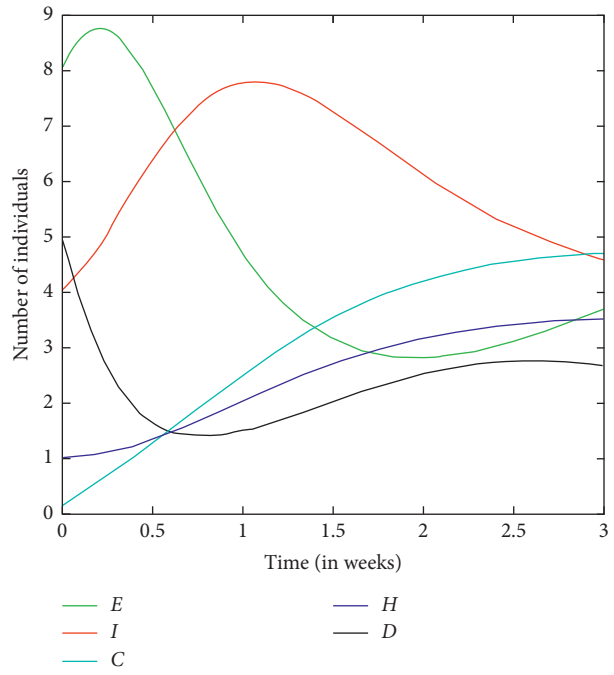


Figure 3: Variations in all compartments.

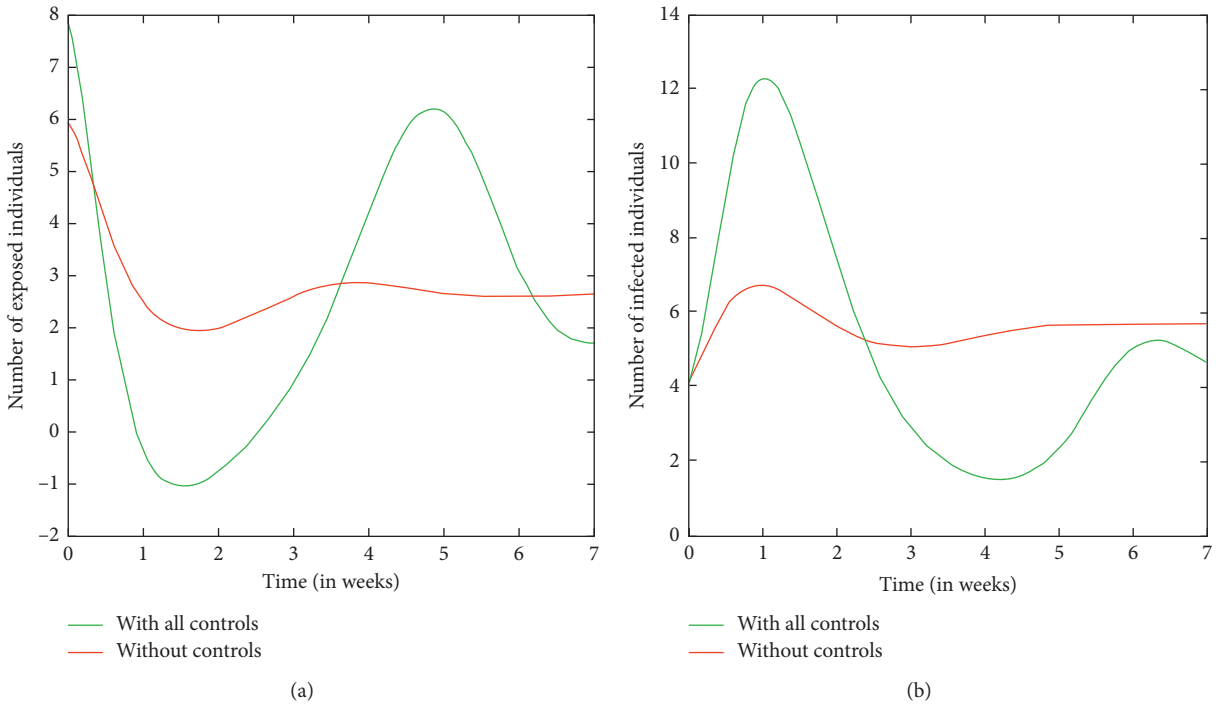


Figure 4: Continued.

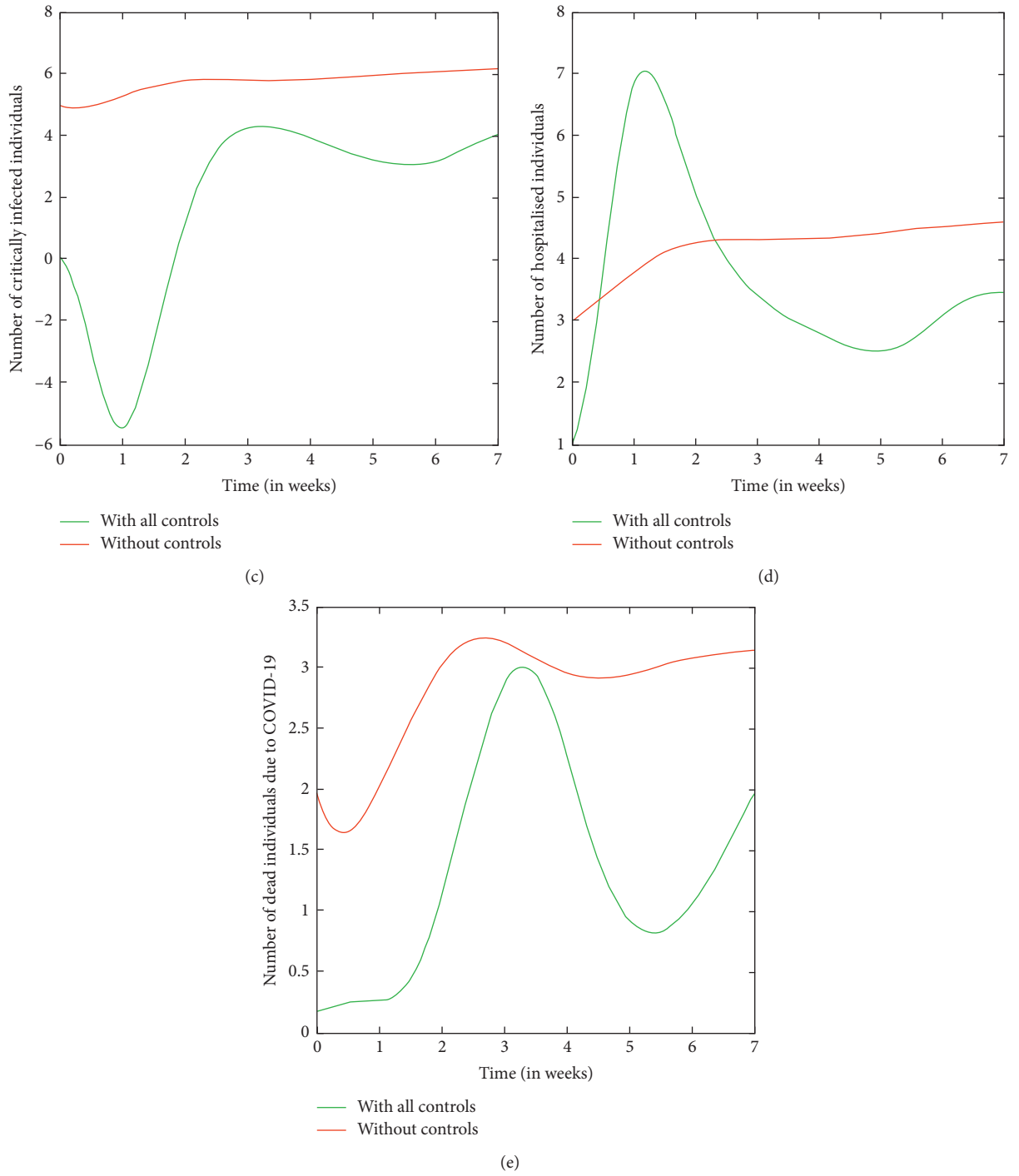


Figure 4: Variation in compartments with and without optimal controls.

outbreak can be controlled up to a certain level in seven weeks after applying control strategies. Meanwhile, Figures 4(b) and 4(c) represent the number of infected and critically infected individuals decreasing; which is further leading to a decrease in hospitalised class as shown in Figure 4(d). Thus, reduction in the mortality rate under the effect of control strategies can be clearly observed in Figure 4(e).

Variation in the intensity of control strategies with time is shown in Figure 5. Moreover, the figure also demonstrates which control can be applied at how much intensity to control COVID-19 outbreak in seven weeks. The range of all the four controls u_1, u_2, u_3, and u_4 is $[-0.7481, 0.4776]$, $[-1.733, 0.8913]$, $[-7.409, 1.04]$, and $[0.0158, 0.6338]$, respectively. The high fluctuation in u_3 control variable at an initial stage suggests that it is very important to control

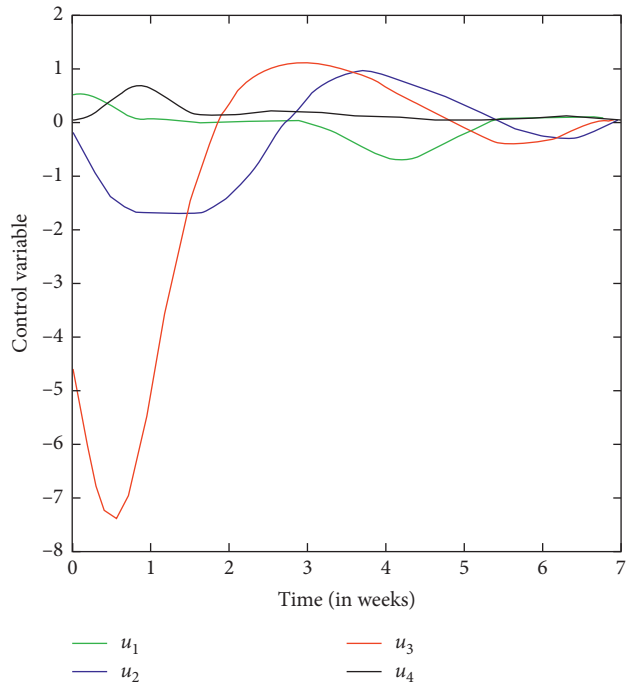
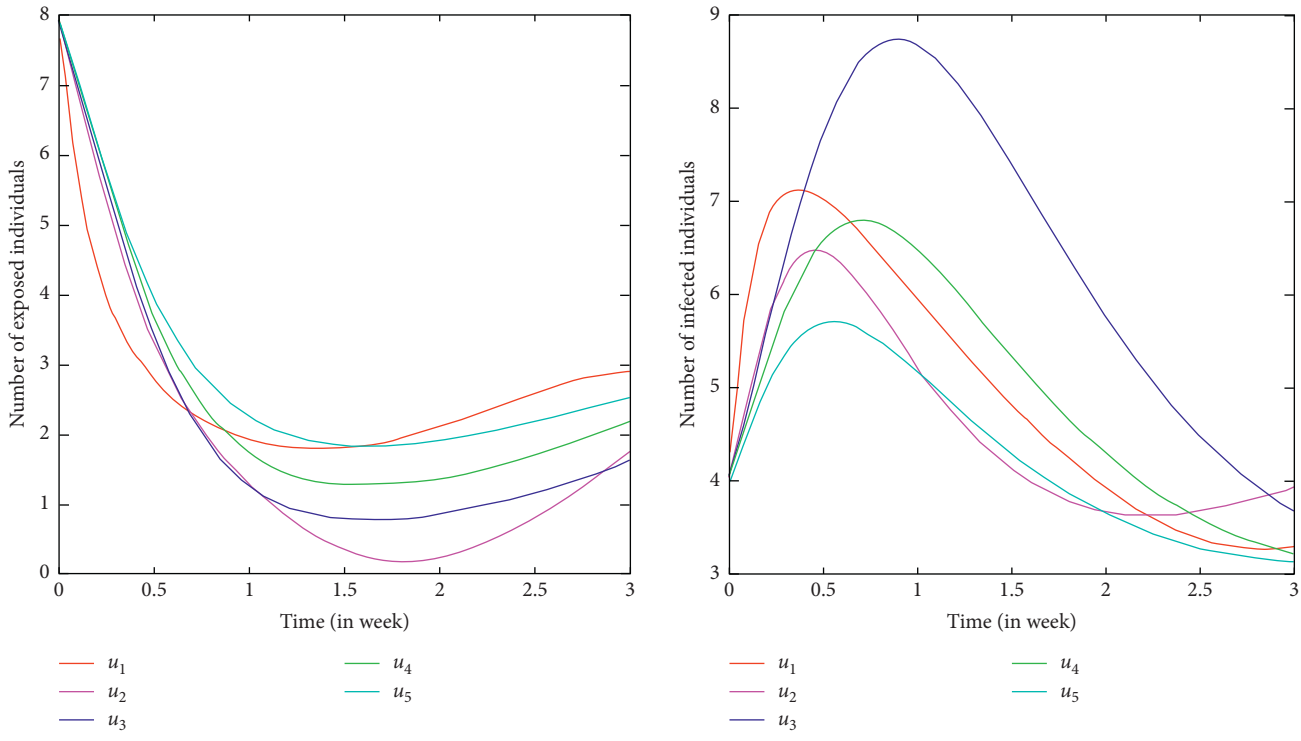


Figure 5: Variation in control variable with time.



(a) (b)

Figure 6: Continued.

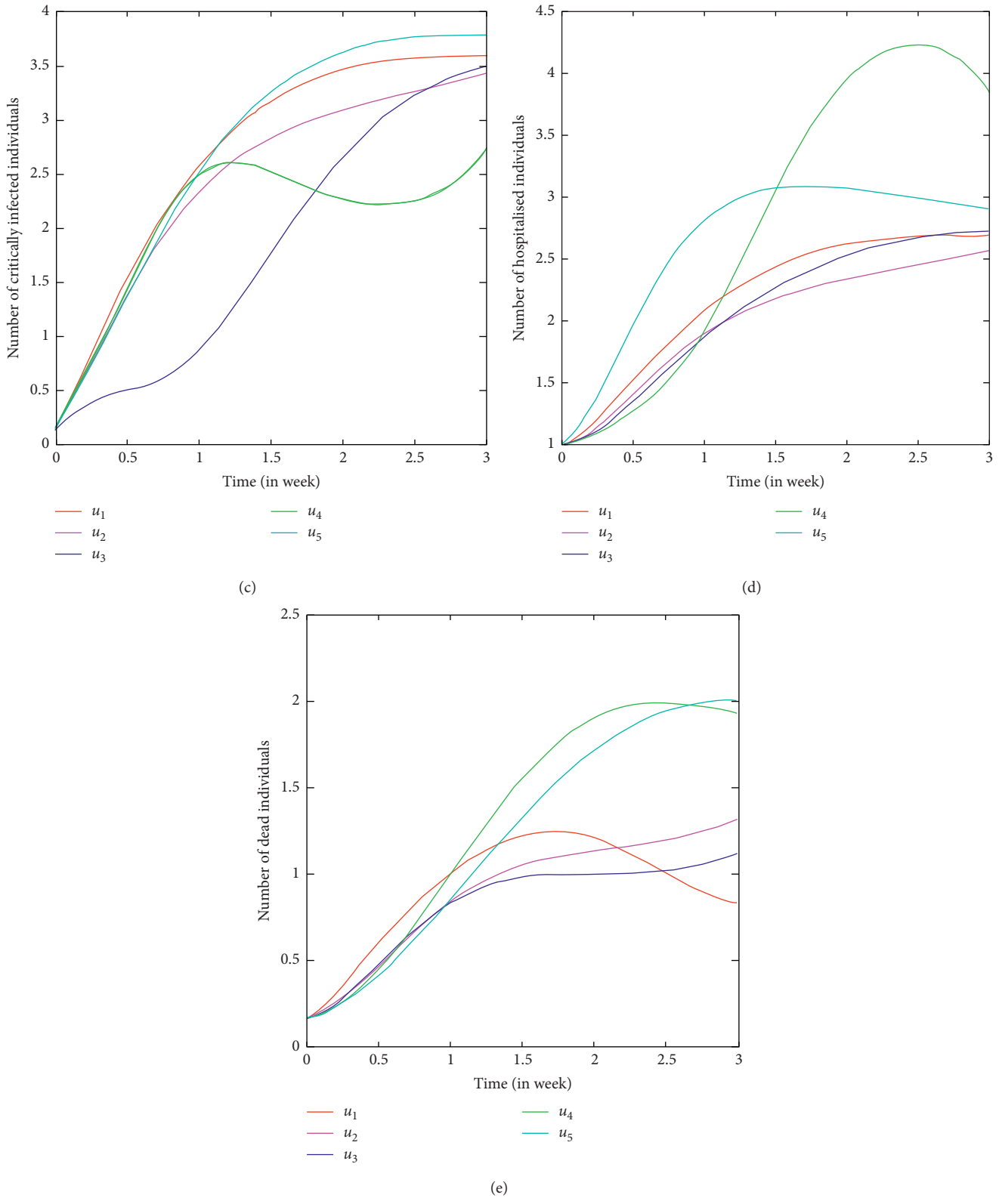


Figure 6: Variation in each compartment under individual effect of control variables.

infected individuals moving to the critical stage to reduce mortality due to COVID-19. Also, this can be achieved easily if an infected individual gets proper vaccination for this disease. Since effective vaccination is not available for coronavirus, one should take proper care of infected individuals to improve their immunity so that their body

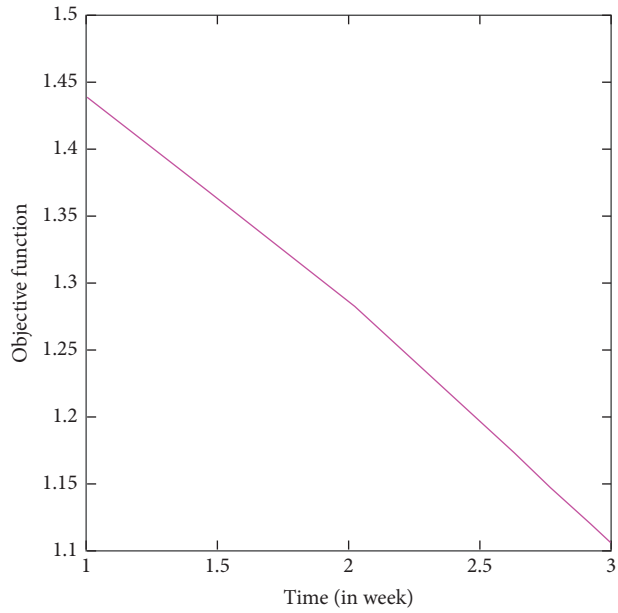


Figure 7: Change in objective function with time.

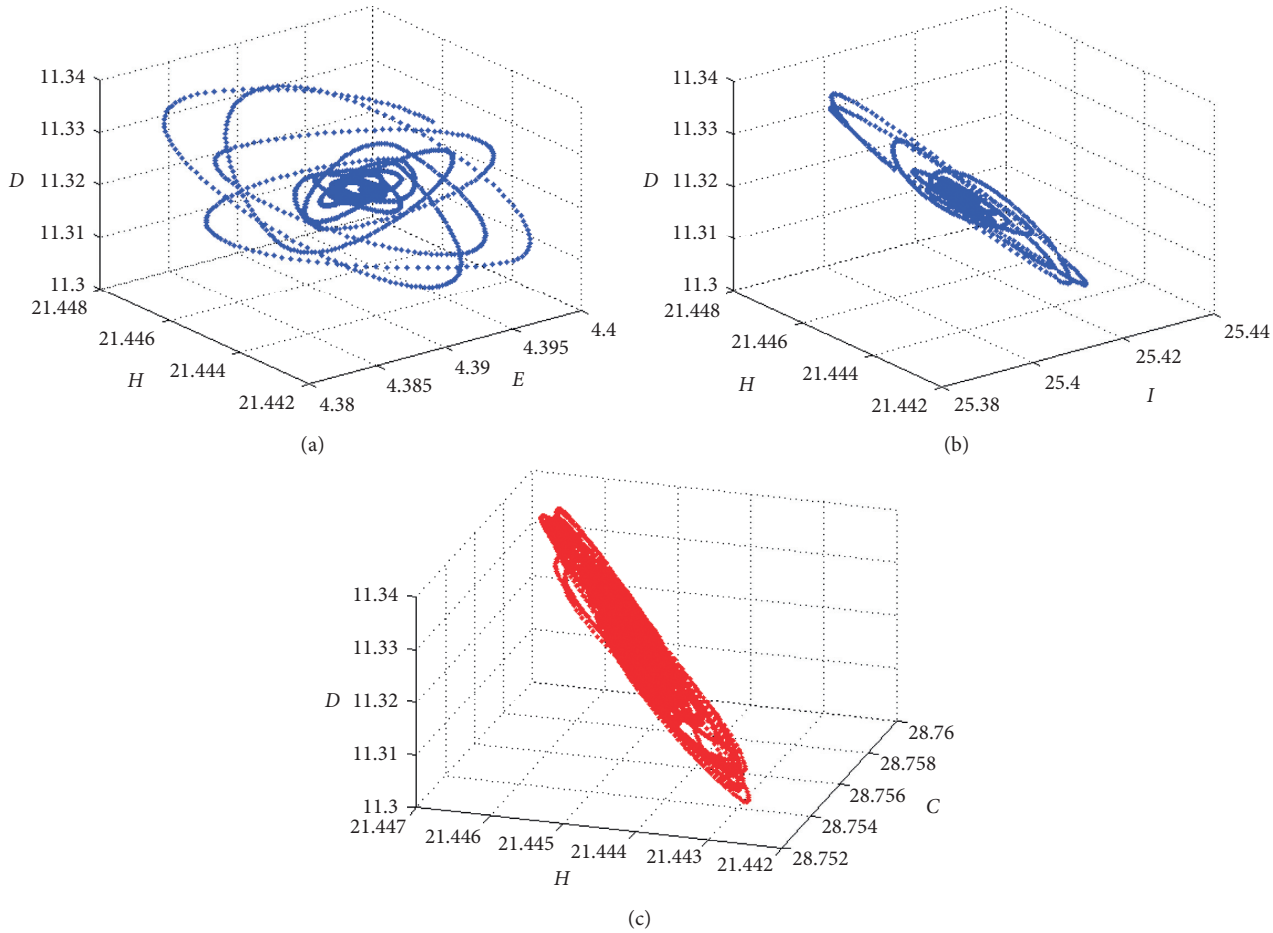


Figure 8: Scatter diagram for COVID-19 outbreak.

becomes capable to fight against the virus and not reach a critical stage. Moreover, fluctuation in u_2 control variable suggests that it is very important to isolate or quarantine infected individuals to control this pandemic outbreak.

Separate effect of control variables on each compartment can be observed in Figure 6. From this figure, we can interpret that u_2 control variable is highly effective to stabilise this epidemic situation. Figure 6(b) depicts that the population class of infected individuals is lowest under the influence of u_5 control variable which suggests that rapid hospitalisation of infected individuals is an effective step to reduce infected cases of COVID-19. Figure 6(c) suggests that to reduce critical cases of COVID-19, first, we should provide control on infected individuals from becoming critically infected and we should improve hospitalisation and medical facility for critically infected individuals to save their lives. Figure 6(e) shows that mortality rate due to COVID-19 can be reduced effectively within three weeks of outbreak by applying u_1, u_2, and u_3 control strategies. This means that self-quarantine for an exposed individual, isolation of an infected individual, and reducing critical cases by taking extra care of infected individuals are effective strategies to control further transmission of COVID-19.

Figure 7 represents change in the objective function under influence of all the controls. Combined effect of all the controls gives fruitful effect on the model.

Figure 8 demonstrates the scatter diagram representing chaotic situation created during COVID-19 outbreak. Figures 8(a)–8(c) show periodic mortality from classes of exposed individuals, infected individuals, and critically infected individuals with respect to time (in week), respectively, when under hospitalisation. By comparing Figures 8(a) and 8(b), it can be interpreted that mortality ratio in the class of infected individuals is higher and much quicker than in the class of exposed individuals. The chaotic Figure 8(c) represents a very high mortality rate of critically infected individuals. Hence, in the absence of vaccination for COVID-19, it becomes a challenging situation to cure critically infected individuals.

5. Conclusion

In this study, a compartmental model is constructed to examine transmission of COVID-19 in the human population class. Moreover, the basic reproduction number is formulated to calculate threshold value of the disease. In order to develop strategies to prevent the epidemic of COVID-19, optimal control theory is applied to the model. Further to advance control theory, five control variables are introduced in the model in the form of control strategies. These strategies include self-quarantine of exposed individuals, isolation of infected individuals, taking extra care of infected individuals to reduce critical case of COVID-19, and increased hospitalisation facility for infected and critically infected individuals. Distinctive and combined effects of these control variables on all the compartments are observed and examined graphically by simulating the COVID-19 model. Numerical simulation of the model reflects that quarantine and better medical treatment of infected individuals reduce the critically infected cases, which will further reduce the transmission risk and demises.

Data Availability

The data used to support the findings of this study are included within this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors are thankful to DST-FIST file # MSI-097 for technical support to the Department of Mathematics, Gujarat University. AHS is funded by a Junior Research Fellowship from the Council of Scientific & Industrial Research (file no. 09/070(0061)/2019-EMR-I). ENJ is funded by UGC granted National Fellowship for Other Backward Classes (NFO-2018-19-OBC-GUJ-71790).

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