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Research Article

Analyzing the Solar Energy Data Using a New Anderson-Darling Test under Indeterminacy

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The generalization of the Anderson-Darling (AD) test under neutrosophic statistics is presented in this paper. We present the designing and operating procedure of neutrosophic Anderson-Darling when the quality of interest follows the neutrosophic normal distribution. The application of the proposed test is given using data from the renewable energy field. From the analysis of the data, it is concluded that the proposed test is effective and information to be applied when the data is recorded from the complex system in the renewable energy field.

1. Introduction

The use of an appropriate statistical technique depends on the nature of the data. The nonparametric statistical tests are applied for the analysis of the data if statistical tests indicate nonnormality in the data. The statistical techniques based on normality are applied when the data follows the normal distribution. Therefore, the diagnostic of the normality of the data is a basic and important step for the deep analysis of the data. Several statistical tests have been applied to test the normality of the data. Among the tests, Anderson-Darling (AD) has been widely applied to test the normality of the data. The AD test is applied to test the null hypothesis that the data follows the normal distribution versus the alternative hypothesis that the normal distribution is not a good choice for the data. Reference [1] applied the AD test for generalized Pareto distribution. Reference [2] worked on evaluating the performance of the AD test. [3] worked on the performance of the various statistical tests. Reference [4] worked on the computation aspects of the AD test. Reference [5] applied the AD test in risk internal models. Reference [6] worked on the ranking of statistical tests. More applications of the AD test can be seen in [7–10].

The accurate prediction and estimation of renewable energy depend on the correct statistical analysis of the data. The statistical test guides renewable energy experts to give an accurate estimation of the production and consumption of energy. Therefore, effective planning about the use and saving of energy depends on statistical tests. [11] worked on the prediction of solar energy. References [12, 13] modelled the wind data using the Weibull distribution. References [14, 15] provided the statistical analysis for the energy data. Some more applications of statistical methods can be seen in [16–21].

When the data has uncertain or fuzzy observations such as measuring the water level in a river and a lifetime of a product and predicting the solar energy and melting point of a component, the existing AD test cannot be applied for the analysis of the data. In this situation, the statistical test designed using the fuzzy logic is applied for testing the normality of the data. References [22, 23] worked on the tests using the fuzzy logic. [24] worked on the Kolmogorov-Smirnov test based on fuzzy logic. Reference [25] discussed the application of fuzzy logic in decision making. For more details, the reader may see [26–31].

The neutrosophic logic introduced by [32] reduces to fuzzy logic if the measure of indeterminacy is not found. Neutrosophic logic, which is more flexible and informative than fuzzy logic, has many applications in the real world. Reference [33] showed the efficiency of neutrosophic logic over the fuzzy and interval-based approaches. More discussion about the neutrosophic logic can be read in [34–45]. The neutrosophic statistics which is worked on the neutrosophic logic is introduced by [46]. This is the branch of statistics which deals with the analysis of the data having the neutrosophic numbers. References [47, 48] introduced the analysis based on neutrosophic numbers. Classical statistics is a special case of neutrosophic statistics when no uncertainty is found in the data. The neutrosophic statistics gives information about the measure of indeterminacy that classical statistics do not provide. Reference [49] introduced quality control under neutrosophic statistics. References [50, 51] introduced statistical tests of normality under neutrosophic statistics. For more applications, the reader may read [52, 53].

A rich literature of the AD test under classical statistics and fuzzy approach is available in the literature. The existing AD tests are unable to provide the measure of indeterminacy under an uncertain environment. Reference [54] developed the goodness of fit test under neutrosophic statistics for non-normal data. By exploring the literature and best of our knowledge, no AD test is found for testing the normality of normal data under neutrosophic statistics. In this paper, we will introduce the neutrosophic Anderson-Darling (NAD) test. We will introduce the test statistics of the proposed test under neutrosophic statistics. The necessary steps are given to apply the proposed test under an uncertain environment. We will discuss the efficiency of the proposed NAD test using data from renewable energy. From the comparison, it is expected that the proposed NAD test will perform better than the existing AD test under classical statistics in terms of the measure of indeterminacy. Further, it is expected that the proposed test will be more informative, effective, and adequate than the existing test under classical statistics.

2. Preliminaries

The neutrosophic logic consists of three measures known as the measure of truth, say T; the measure of false, say F; and the measure of indeterminacy, say I_N. The neutrosophic logic is a generalization of fuzzy logic. Let a_{iN} and $b_{\mathrm{iN}}I_N\,; I_N\epsilon[I_L, I_U]$ be the determined part and indeterminate part of the neutrosophic variable $X_{\mathrm{iN}} = a_{\mathrm{iN}} + b_{\mathrm{iN}}I_N\,; I_N\epsilon[I_L, I_U]$, where $I_N\epsilon[I_L, I_U]$ denotes the indeterminate interval. Suppose that $n_N\epsilon[n_L, n_U]$ be a neutrosophic sample size and $\bar{a}_N = 1/n_N \sum_{i=1}^{n_N} a_i$ and $\bar{b}_N = 1/n_N\sum_{i=1}^{n_N} b_i$ are means of determinate and indeterminate parts, respectively. The neutrosophic standard deviation (NSD) by following [47, 48] is given as

$$s_N = \sqrt{\frac{1}{n_N}\sum_{i=1}^{n_N}\left(X_{\mathrm{iN}} - \bar{X}_N\right)^2}, \tag{1}$$

where

$$\sum_{i=1}^{n_N}\left(X_{\mathrm{iN}} - \bar{X}_N\right)^2 = \sum_{i=1}^{n_N}\left[\begin{matrix} \min\begin{pmatrix} \left(a_i + b_jI_L\right)\left(\bar{a} + \bar{b}I_L\right), \left(a_j + b_jI_L\right)\left(\bar{a} + \bar{b}I_U\right) \\ \left(a_j + b_jI_U\right)\left(\bar{a} + \bar{b}I_L\right), \left(a_j + b_jI_U\right)\left(\bar{a} + \bar{b}I_U\right) \end{pmatrix} \\ \max\begin{pmatrix} \left(a_j + b_jI_L\right)\left(\bar{a} + \bar{b}I_L\right), \left(a_j + b_jI_L\right)\left(\bar{a} + \bar{b}I_U\right) \\ \left(a_j + b_jI_U\right)\left(\bar{a} + \bar{b}I_L\right), \left(a_j + b_jI_U\right)\left(\bar{a} + \bar{b}I_U\right) \end{pmatrix}\end{matrix}\right], \quad I_N\epsilon[I_L, I_U], \tag{2}$$

where $\bar{X}_N = \bar{a}_N + \bar{b}_N I_N, I_N\epsilon[I_L, I_U]$.

3. Proposed Test

The Anderson-Darling (AD) test under classical statistics is applied to test the normality of the data having determined values. We propose neutrosophic Anderson-Darling (NAD) for testing the normality of the imprecise and indeterminate data. The null hypothesis H_{0N} is that the given neutrosophic data follows the neutrosophic normal distribution versus the alternative hypothesis H_{1N} that the neutrosophic normal distribution is not suitable. The normality test will lead the energy expert either to use statistical analysis based on the normal distribution or not. The operational process of the proposed NAD is stated as follows.

Step 1. Select a random sample $n_N\epsilon[n_L, n_U]$. Compute the neutrosophic averages of determined and indeterminate parts of the data as $\bar{a}_N = 1/n_N\sum_{i=1}^{n_N} a_i$ and $\bar{b}_N = 1/n_N\sum_{i=1}^{n_N} b_i$.

Step 2. Compute the neutrosophic average of a neutrosophic random variable as $\bar{X}_N = \bar{a}_N + \bar{b}_N I_N; I_N\epsilon[I_L, I_U]$.

Step 3. Compute the neutrosophic standard deviation is as follows: $s_N = \sqrt{1/n_N\sum_{i=1}^{n_N}(X_{\mathrm{iN}} - \bar{X}_N)^2}$, where $\sum_{i=1}^{n_N}(X_{\mathrm{iN}} - \bar{X}_N)^2$ is given as

$$\sum_{i=1}^{n_N}\left(X_{\mathrm{iN}} - \bar{X}_N\right)^2 = \left[\begin{matrix} \min\left((a_i - \bar{a}_N)^2, \left((a_i - \bar{a})\left((a_i - \bar{a}_N) + 1 \times \left(b_i - \bar{b}_N\right)\right), (a_i - \bar{a}_N) + 1 \times \left(b_i - \bar{b}_N\right)^2\right)\right) \\ \max\left((a_i - \bar{a}_N)^2, \left((a_i - \bar{a}_N)\left((a_i - \bar{a}_N) + 1 \times \left(b_i - \bar{b}_N\right)\right), (a_i - \bar{a}_N) + 1 \times \left(b_i - \bar{b}_N\right)^2\right)\right)\end{matrix}\right]. \tag{3}$$

Table 1: The solar data.

M	T	WD	WS	DHI	DNI	GHI	PWS	RH	P	ND-GHI
1	[14.2, 15.9]	[9, 111]	[2.5, 3.5]	[2030, 2311]	[1127, 3648]	[2863, 3972]	[6.9, 9.9]	[41.5, 62.1]	[941, 945.1]	[2863, 4251]
2	[17, 18.3]	[96, 144]	[2.3, 3.4]	[1423, 2274]	[3676, 6170]	[4519, 5051]	[6.9, 10.4]	[48.7, 54.6]	[934, 940.1]	[4746, 5051]
3	[21.1, 24.1]	[53, 124]	[2.1, 3.6]	[1447, 3350]	[1680, 7773]	[4573, 6365]	[7.2, 9.6]	[25.8, 35.5]	[938.6, 940.4]	[3873, 5802]
4	[20.1, 24.8]	[118, 354]	[3.7, 4.1]	[3643, 3697]	[1330, 4028]	[4741, 6859]	[9.9, 15.7]	[22.2, 45.7]	[931.2, 935.3]	[4741, 7099]
5	[30.9, 33.3]	[28, 198]	[2.6, 4.2]	[4079, 4302]	[2334, 3380]	[6122, 6791]	[9.1, 16]	[20.2, 29.7]	[932.8, 935]	[4653, 6703]
6	[34.5, 34.9]	[337, 341]	[4.5, 5.6]	[3552, 4842]	[2894, 5389]	[7098, 8121]	[14.7, 16]	[7.9, 9.5]	[933.6, 934.4]	[7098, 7864]
7	[36.7, 38]	[314, 339]	[2.8, 4.1]	[1955, 2845]	[6228, 8217]	[7894, 8151]	[11.7, 17.3]	[8.1, 10.3]	[927.7, 928.8]	[6921, 7894]
8	[35.6, 36.4]	[315, 316]	[2.6, 3.9]	[2137, 3046]	[5581, 7726]	[7523, 7961]	[10.1, 13.1]	[9.1, 9.3]	[928.2, 928.8]	[7104, 7798]
9	[38.6, 39.2]	[86, 285]	[2.1, 2.5]	[2416, 3337]	[3657, 5107]	[6175, 6362]	[7.7, 9.9]	[9.4, 14.9]	[931.5, 933.2]	[6175, 7010]
10	[31.5, 32.5]	[62, 107]	[2, 3.1]	[1409, 1793]	[6540, 7888]	[6105, 6586]	[7.2, 9.3]	[13.1, 18]	[934.5, 937.2]	[6105, 6416]
11	[26.7, 28.8]	[123, 187]	[2.6, 3.6]	[1176, 1705]	[5548, 7005]	[5019, 5286]	[8.3, 10.4]	[25.6, 28.3]	[937.5, 939.3]	[3596, 5114]
12	[19.5, 21.9]	[240, 332]	[2, 2.3]	[776, 941]	[7040, 7768]	[4641, 4747]	[6.4, 8.8]	[35, 49.9]	[941.1, 945.7]	[4641, 4747]

Step 4. Compute the cumulative probabilities using the following transformation:

$$F_0(Z_N) = \Phi_N\left(\frac{X_N - \bar{X}_N}{s_N}\right), \quad I_N \epsilon [I_L, I_U], \tag{4}$$

where $\Phi_N(x_N)$ denotes the neutrosophic cumulative distribution function.

Step 5. Compute NAD using the following functional form:

$$\text{NAD} = \sum_{i=1}^{n_N} \frac{1-2i}{n_N}\left\{\ln\left(F_0\left[Z_{N(i)}\right]\right) + \ln\left(1 - F_0\left[Z_{N(n_N+1-i)}\right]\right)\right\} - n_N, \quad n_N \epsilon [n_L, n_U]. \tag{5}$$

Step 6. Compute the critical value (CV) as follows:

$$\text{CV} = \frac{0.752}{\left(1 + 0.75/n_N + 2.25/n_N^2\right)}, \quad n_N \epsilon [n_L, n_U]. \tag{6}$$

Step 7. The null hypothesis H_{0N} will be accepted if NAD < CV.

4. Application of NAD Test

The application of the proposed NAD test is given with the help of solar data recorded from Riyadh satiation, Saudi Arabia. The data is taken from [11]. According to [11], "in order to predict solar radiation, the system will use historical observed data: the data of ten variables including temperature (T), average wind direction at 3 m (degree from the north), average wind speed at 3 m (m/s), Diffuse Horizontal Irradiance (DHI) (Wh/m^2), Direct Normal Irradiance (DNI) (Wh/m^2), Global Horizontal Irradiance (GHI) of the current day (Wh/m^2), peak wind speed at 3 m (m/s), relative humidity (percent), station pressure (mB (hPa equivalent)), and next-day GHI (Wh/m^2) (model output)." The data is reported in Table 1. From Table 1, it can be seen that the solar data has neutrosophy. Therefore, the analysis of the data using the AD test under classical statistics may mislead the experimenters. In this situation, the use of the proposed NAD test will be quite effective and informative. The proposed NAD test on this data for variable T is implemented as follows.

Step 1. Select a random sample $n_N \epsilon [12, 12]$. Compute the neutrosophic averages of determined and indeterminate parts of the data as $\bar{a}_N = 1/n_N \sum_{i=1}^{n_N} a_i = (14.2 + \cdots + 19.5)/12 = 27.2$ and $\bar{b}_N = 1/n_N \sum_{i=1}^{n_N} b_i = (15.9 + \cdots + 21.9)/12 = 29.008$.

Step 2. Compute the neutrosophic average of a neutrosophic random variable as $\bar{X}_N = 27.2 + 29.008 I_N$, $I_N \epsilon [0, 1]$.

Step 3. Compute the neutrosophic standard deviation as follows:

$$s_N = \sqrt{\frac{1}{n_N}\sum_{i=1}^{n_N}\left(X_{\text{iN}} - \bar{X}_N\right)^2} = s_N \epsilon [5.7606, 11.0750], \tag{7}$$

where $\sum_{i=1}^{n_N}\left(X_{\text{iN}} - \bar{X}_N\right)^2$ is given as

$$\sum_{i=1}^{n_N}\left(X_{\text{iN}} - \bar{X}_N\right)^2 = \sum_{i=1}^{n_N}\begin{bmatrix} \min\left(\left(a_i - \bar{a}_N\right)^2, \left(\left(a_i - \bar{a}\right)\left(\left(a_i - \bar{a}_N\right) + 1 \times \left(b_i - \bar{b}_N\right)\right), \left(a_i - \bar{a}_N\right) + 1 \times \left(b_i - \bar{b}_N\right)^2\right)\right) \\ \max\left(\left(a_i - \bar{a}_N\right)^2, \left(\left(a_i - \bar{a}_N\right)\left(\left(a_i - \bar{a}_N\right) + 1 \times \left(b_i - \bar{b}_N\right)\right), \left(a_i - \bar{a}_N\right) + 1 \times \left(b_i - \bar{b}_N\right)^2\right)\right) \end{bmatrix}. \tag{8}$$

Step 4. Compute the cumulative probabilities using the following transformation:

$$F_0(Z_N) = \Phi_N\left(\frac{X_N - \bar{X}_N}{s_N}\right) = \Phi_N\left(\frac{[14.2,15.9] - [27.2,56.20]}{[5.7606,11.0750]}\right), \cdots, \Phi_N\left(\frac{[19.5,21.9] - [27.2,56.20]}{[5.7606,11.0750]}\right), \quad I_N \epsilon [0, 1]. \tag{9}$$

Step 5. Compute NAD using the following functional form:

$$\text{NAD} = \sum_{i=1}^{n_N} \frac{1-2i}{n_N}\left\{\ln\left(F_0\left[Z_{N(i)}\right]\right) + \ln\left(1 - F_0\left[Z_{N(n_N+1-i)}\right]\right)\right\} - n_N = \text{NAD}\epsilon[1.79, 49.26]. \tag{10}$$

Step 6. Compute the critical value (CV) as follows:

$$\text{CV} = \frac{0.752}{\left(1 + 0.75/n_N + 2.25/n_N^2\right)} = 0.6975. \tag{11}$$

Step 7. The null hypothesis H_{0N} will be rejected as NAD > CV . From the proposed NAD test, it is concluded that the variable temperature does not follow the normal distribution.

5. Comparative Study

As mentioned earlier, the proposed NAD test under neutrosophic statistics is the generalization of the AD test under classical statistics. The proposed NAD test reduces to an AD test under classical statistics if uncertainty does not exist. The indeterminate value of the NAD statistic is $\text{NAD}\epsilon[1.79, 49.26]$. The neutrosophic form of NAD can be written as $\text{NAD} = \text{AD} + 49.26 I_N, I_N \epsilon [0,0.9636]$, where $\text{AD} = 1.79$ shows the values of the AD test under classical statistics. The part $49.26 I_N$ shows the indeterminate part of the neutrosophic test. The proposed NAD test becomes the same as the AD test when $I_L = 0$. From the study, it can be noted that the proposed test has the values in indeterminacy interval. It means, under uncertainty, that the NAD test can take the values between 1.79 and 49.26. On the other hand, the existing AD test under classical statistics provides the determined value of the statistics. Therefore, the proposed test is more flexible than the existing test under uncertainty. In a neutrosophic analysis, the total probability can be more than one due to uncertainty which is called paraconsistent probability (see [46]). In addition, the proposed test provides the probability of indeterminacy that is 0.9636. The proposed NAD test can be interpreted as follows: under an indeterminate environment, the null hypothesis that the solar data follows the normal distribution will be accepted with the probability 0.95 and rejected with the probability 0.05, and the probability of indeterminacy is 0.9636. By comparing both tests, it can be seen that for the proposed, the sum of the probabilities is larger than 1 while in the existing test, the sum of probabilities is always equal to one. In addition, the proposed test provides information about the measure of indeterminacy while the existing test does not provide such information. The proposed test results in indeterminate intervals; therefore, the theory of the proposed test is the same as in [48]. From this comparison, it is concluded that the proposed test is quite informative, effective, and flexible to be applied for the renewable energy data as compared to the existing test under classical statistics.

6. Concluding Remarks

The existing AD test cannot be applied for testing the normality of the data in intervals, having neutrosophy and uncertainty. The generalization of the Anderson-Darling (AD) test under neutrosophic statistics that can be used to test the normality of such data was presented in this paper. We presented the designing and operational procedure of neutrosophic Anderson-Darling when the quality of interest followed the neutrosophic normal distribution. The application of the proposed test was given using data from the renewable energy field. From the analysis of the data, it was concluded that the proposed test is effective and information to be applied when the data is recorded from the complex system in the renewable energy field. The proposed test provides the results in indeterminate intervals that are required in dealing with the problem under uncertainty. We recommend that the renewable energy experts should apply the proposed test under an indeterminate environment. The proposed test for nonnormal distribution can be considered future research. Developing software to run the proposed test is also a fruitful area of future research. The application of the proposed test for big data can be considered future research.

Data Availability

The data is given in the paper.

Conflicts of Interest

The authors declare no conflict of interest regarding this paper.

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References

[1] M. Arshad, M. T. Rasool, and M. I. Ahmad, “Anderson Darling and modified Anderson Darling tests for generalized Pareto distribution,” *Journal of Applied Sciences*, vol. 3, no. 2, pp. 85–88, 2003.

[2] G. Marsaglia and J. Marsaglia, “Evaluating the Anderson-darling distribution,” *Journal of Statistical Software*, vol. 9, no. 2, pp. 1–5, 2004.

[3] N. M. Razali and Y. B. Wah, "Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests," *Journal of statistical modeling and analytics*, vol. 2, no. 1, pp. 21–33, 2011.

[4] L. Jäntschi and S. D. Bolboacă, "Computation of probability associated with Anderson–Darling statistic," *Mathematics*, vol. 6, no. 6, p. 88, 2018.

[5] M. Formenti, L. Spadafora, M. Terraneo, and F. Ramponi, "The efficiency of the Anderson–Darling test with a limited sample size: an application to backtesting counterparty credit risk internal models," *Journal of Risk*, vol. 21, no. 6, pp. 69–100, 2019.

[6] T. U. Islam, "Ranking of normality tests: an appraisal through skewed alternative space," *Symmetry*, vol. 11, no. 7, p. 872, 2019.

[7] M. Rahman, L. M. Pearson, and H. C. Heien, "A modified Anderson-Darling test for uniformity," *Bulletin of the Malaysian Mathematical Sciences Society*, vol. 29, pp. 11–16, 2006.

[8] T. W. Anderson, "Anderson-Darling tests of goodness-of-fit," *International Encyclopedia of Statistical Science*, vol. 1, pp. 52–54, 2011.

[9] Y. Li, Y. Wei, B. Li, and G. Alterovitz, "Modified Anderson-Darling test-based target detector in non-homogenous environments," *Sensors*, vol. 14, no. 9, pp. 16046–16061, 2014.

[10] D. K. Wijekularathna, A. B. Manage, and S. M. Scariano, "Power analysis of several normality tests: a Monte Carlo simulation study," *Communications in Statistics-Simulation and Computation*, pp. 1–17, 2019.

[11] M. Almaraashi, "Short-term prediction of solar energy in Saudi Arabia using automated-design fuzzy logic systems," *PloS one*, vol. 12, no. 8, p. e0182429, 2017.

[12] A. Chauhan and R. Saini, "Statistical analysis of wind speed data using Weibull distribution parameters," in *2014 1st International Conference on Non Conventional Energy (ICONCE 2014)*, pp. 160–163, Kalyani, India, 2014.

[13] K. Azad, M. Rasul, P. Halder, and J. Sutariya, "Assessment of wind energy prospect by Weibull distribution for prospective wind sites in Australia," *Energy Procedia*, vol. 160, pp. 348–355, 2019.

[14] D. Mohammed, A. S. M. Abdelaziz, E. Mohammed, and E. Elmostapha, "Analysis of Wind speed data and wind energy potential using Weibull distribution in Zagora, Morocco," *International Journal of Renewable Energy Development*, vol. 8, no. 3, pp. 267–273, 2019.

[15] H. Bidaoui, I. El Abbassi, A. El Bouardi, and A. Darcherif, "Wind speed data analysis using Weibull and Rayleigh distribution functions, case study: five cities northern Morocco," *Procedia Manufacturing*, vol. 32, pp. 786–793, 2019.

[16] F. H. Mahmood, A. K. Resen, and A. B. Khamees, "Wind characteristic analysis based on Weibull distribution of Al-Salman site, Iraq," *Energy Reports*, vol. 6, Supplement 3, pp. 79–87, 2020.

[17] Y. Min, Y. Chen, and H. Yang, "A statistical modeling approach on the performance prediction of indirect evaporative cooling energy recovery systems," *Applied Energy*, vol. 255, article 113832, 2019.

[18] E. K. Akpinar and S. Akpinar, "A statistical analysis of wind speed data used in installation of wind energy conversion systems," *Energy Conversion and Management*, vol. 46, no. 4, pp. 515–532, 2005.

[19] Y. Zhao, P. Liu, Z. Wang, L. Zhang, and J. Hong, "Fault and defect diagnosis of battery for electric vehicles based on big data analysis methods," *Applied Energy*, vol. 207, pp. 354–362, 2017.

[20] V. Katinas, G. Gecevicius, and M. Marciukaitis, "An investigation of wind power density distribution at location with low and high wind speeds using statistical model," *Applied Energy*, vol. 218, pp. 442–451, 2018.

[21] M. Vogt, F. Marten, and M. Braun, "A survey and statistical analysis of smart grid co-simulations," *Applied Energy*, vol. 222, pp. 67–78, 2018.

[22] P. Grzegorzewski and H. Szymanowski, "Goodness-of-fit tests for fuzzy data," *Information Sciences*, vol. 288, pp. 374–386, 2014.

[23] H. A. Noughabi and M. Akbari, "Testing normality based on fuzzy data," *International Journal of Intelligent Technologies & Applied Statistics*, vol. 9, no. 1, pp. 37–52, 2016.

[24] F. Momeni, B. S. Gildeh, and G. Hesamian, "Kolmogorov-Smirnov two-sample test in fuzzy environment," *Journal of Hyperstructures*, vol. 6, no. 2, 2018.

[25] L. Wang and N. Li, "Pythagorean fuzzy interaction power Bonferroni mean aggregation operators in multiple attribute decision making," *International Journal of Intelligent Systems*, vol. 35, pp. 150–183, 2019.

[26] B. Van Cutsem and I. Gath, "Detection of outliers and robust estimation using fuzzy clustering," *Computational Statistics & Data Analysis*, vol. 15, no. 1, pp. 47–61, 1993.

[27] M. Montenegro, "Two-sample hypothesis tests of means of a fuzzy random variable," *Information Sciences*, vol. 133, no. 1-2, pp. 89–100, 2001.

[28] V. Mohanty and P. AnnanNaidu, "Fraud detection using outlier analysis: a survey," *International Journal of Engineering Sciences and Research Technology*, vol. 2, no. 6, 2013.

[29] Y. M. Moradnezhadi, "Determination of a some simple methods for outlier detection in maximum daily rainfall (case study: Baliglichay Watershed Basin–Ardebil Province–Iran)," *Bulletin of Environment, Pharmacology and Life Sciences*, vol. 3, no. 3, pp. 110–117, 2014.

[30] C. Moewes, R. Mikut, and R. Kruse, "Fuzzy Control," in *Springer Handbook of Computational Intelligence*, pp. 269–283, Springer, 2015.

[31] Y. Choi, H. Lee, and Z. Irani, "Big data-driven fuzzy cognitive map for prioritising IT service procurement in the public sector," *Annals of Operations Research*, vol. 270, no. 1-2, pp. 75–104, 2018.

[32] F. Smarandache, *Neutrosophy. neutrosophic probability, set, and logic, ProQuest information & learning*, vol. 105, 1998Ann Arbor, Michigan, USA, 1998.

[33] F. Smarandache and H. E. Khalid, *Neutrosophic Precalculus and Neutrosophic Calculus: Infinite Study*, Europanova Asbl, Bruxelles, Belgium, 2015.

[34] I. Hanafy, A. Salama, and M. Mahfouz, "Correlation Coefficients of Neutrosophic Sets by Centroid Method: Infinite Study," *International Journal of Probability and Statistics*, vol. 2, no. 1, pp. 9–12, 2013.

[35] S. Broumi and F. Smarandache, "Correlation coefficient of interval neutrosophic set," *Applied Mechanics and Materials*, vol. 436, pp. 511–517, 2013.

[36] Y. Guo and A. Sengur, "NCM: neutrosophic c-means clustering algorithm," *Pattern Recognition*, vol. 48, no. 8, pp. 2710–2724, 2015.

[37] Y. Guo and A. Sengur, “NECM: neutrosophic evidential c-means clustering algorithm,” *Neural Computing and Applications*, vol. 26, no. 3, pp. 561–571, 2015.

[38] Y. Guo, A. Şengür, and J.-W. Tian, “A novel breast ultrasound image segmentation algorithm based on neutrosophic similarity score and level set,” *Computer Methods and Programs in Biomedicine*, vol. 123, pp. 43–53, 2016.

[39] S. Patro and F. Smarandache, “The Neutrosophic Statistical Distribution,” in *More Problems, More Solutions*, Infinite Study, 2016.

[40] S. Broumi, M. Talea, A. Bakali, F. Smarandache, and K. Ullah, “Bipolar Neutrosophic Minimum Spanning Tree,” *SSRN Electronic Journal*, 2018.

[41] X. Peng and J. Dai, “Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function,” *Neural Computing and Applications*, vol. 29, no. 10, pp. 939–954, 2018.

[42] M. Abdel-Baset, V. Chang, and A. Gamal, “Evaluation of the green supply chain management practices: a novel neutrosophic approach,” *Computers in Industry*, vol. 108, pp. 210–220, 2019.

[43] M. Abdel-Basset, M. Mohamed, M. Elhoseny, F. Chiclana, and A. E.-N. H. Zaied, “Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases,” *Artificial Intelligence in Medicine*, vol. 101, article 101735, 2019.

[44] M. Abdel-Basset, N. A. Nabeeh, H. A. El-Ghareeb, and A. Aboelfetouh, “Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises,” *Enterprise Information Systems*, vol. 14, no. 9-10, pp. 1304–1324, 2020.

[45] N. A. Nabeeh, F. Smarandache, M. Abdel-Basset, H. A. El-Ghareeb, and A. Aboelfetouh, “An integrated neutrosophic-topsis approach and its application to personnel selection: a new trend in brain processing and analysis,” *IEEE Access*, vol. 7, pp. 29734–29744, 2019.

[46] F. Smarandache, “Introduction to Neutrosophic Statistics: Infinite Study,” 2014.

[47] J. Chen, J. Ye, and S. Du, “Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics,” *Symmetry*, vol. 9, no. 10, 2017.

[48] J. Chen, J. Ye, S. Du, and R. Yong, “Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers,” *Symmetry*, vol. 9, no. 7, 2017.

[49] M. Aslam, “A new sampling plan using neutrosophic process loss consideration,” *Symmetry*, vol. 10, no. 5, 2018.

[50] M. Aslam, “Introducing Kolmogorov–Smirnov Tests under Uncertainty: An Application to Radioactive Data,” *ACS Omega*, vol. 5, no. 1, pp. 914–917, 2019.

[51] M. Aslam, “Design of the Bartlett and Hartley tests for homogeneity of variances under indeterminacy environment,” *Journal of Taibah University for Science*, vol. 14, no. 1, pp. 6–10, 2020.

[52] M. Aslam and M. Albassam, “Application of neutrosophic logic to evaluate correlation between prostate cancer mortality and dietary fat assumption,” *Symmetry*, vol. 11, no. 3, p. 330, 2019.

[53] M. Aslam, “Neutrosophic analysis of variance: application to university students,” *Complex & Intelligent Systems*, vol. 5, no. 4, pp. 403–407, 2019.

[54] M. Aslam, “A new goodness of fit test in the presence of uncertain parameters,” *Complex & Intelligent Systems*, no. article 214, pp. 1–7, 2020.