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Research Article

Lump and Lump-Type Solutions of the Generalized (3+1)-Dimensional Variable-Coefficient B-Type Kadomtsev-Petviashvili Equation

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Based on the Hirota bilinear form of the generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation, the lump and lump-type solutions are generated through symbolic computation, whose analyticity can be easily achieved by taking special choices of the involved parameters. The property of solutions is investigated and exhibited vividly by three-dimensional plots and contour plots.

1. Introduction

The generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation [1] is

$$P_{BKP}(u) = a(t)\, u_{xxxy} + \rho a(t) \left(u_x u_y\right)_x + \left(u_x + u_y + u_z\right)_t + b(t)\left(u_{xx} + u_{zz}\right) = 0 \tag{1}$$

which is extended from the Kadomtsev-Petviashvili equation and can describe some interesting (3+1)-dimensional wave in fluid dynamics.

In this paper, by using Hirota bilinear forms, we begin with studying the lump and lump-type solutions of the generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation. Exact solutions, especially the rational solutions, are important to descriptions of some physical phenomena [2–4]. Lump and lump-type solutions are special kinds of rational solutions. In recent years, there has been a growing interest in lump function solutions. Particular examples of lump and lump-type solutions are found for many nonlinear partial differential equations, such as the Kadomtsev-Petviashvili equation [5], the generalized Bogoyavlensky-Konopelchenko equation [6], the (2+1)-dimensional generalized fifth-order KdV equation [7], the generalized Kadomtsev-Petviashvili-Boussinesq equation [8], and the $(3+1)$-dimensional nonlinear evolution equation [9]. The interactions of lump solutions and interactions of other types solutions have also attracted much attention [10–13].

In Section 2, searching for the quadratic function solutions, we get the lump and lump-type solutions and analyze their dynamics. In the last section, some concluding remarks will be given.

2. Lump and Lump-Type Solutions

The Hirota bilinear form of a soliton equation needs to use appropriate variable transformation adopted in Bell polynomial theories [14, 15] to search. Based on the Bell polynomial method, under the transformation between f and u:

$$u = \frac{6}{\rho}\left(\ln f\right)_x . \tag{2}$$

A Hirota bilinear equation can be proposed as follows:

$$\begin{aligned} B_{BKP}(f) &= \left[a(t) D_x^3 D_y + D_x D_t + D_y D_t + D_z D_t \right. \\ &\quad \left. + b(t)\left(D_x^2 + D_z^2\right)\right] f \cdot f = 2a(t)\left[f f_{xxxy} - f_{xxx} f_y \right. \\ &\quad \left. + 3 f_{xx} f_{xy} - 3 f_{xxy} f_x\right] + 2\left[f_{xt} f - f_x f_t\right] + 2\left[f_{yt} f \right. \\ &\quad \left. - f_y f_t\right] + 2\left[f_{zt} f - f_z f_t\right] + 2b(t)\left[f_{xx} f - f_x^2 \right. \\ &\quad \left. + f_{yy} f - f_y^2 + f_{zz} f - f_z^2\right] = 0 \end{aligned} \tag{3}$$

where f is a function of x, y, z, and t. $D_x^3 D_y$, $D_x D_t$, $D_y D_t$, D_x^2, and D_z^2 are the Hirota bilinear operators defined by

$$\begin{aligned} & D_{x_1}^{n_1} \ldots D_{x_l}^{n_l} F \cdot G \\ &\quad = \left(\partial x_1 - \partial x_1'\right)^{n_1} \ldots \left(\partial x_l - \partial x_l'\right)^{n_l} F\left(x_1, \ldots, x_l\right) \\ &\quad\quad \times G\left(x_1, \ldots, x_l\right)\Big|_{x_1' = x_1, \ldots, x_l' = x_l}, \end{aligned} \tag{4}$$

It is precise to find

$$P_{BKP}(u) = \left(\frac{B_{BKP}(f)}{f^2}\right)_x \tag{5}$$

Therefore, when f solves the bilinear equation (3), $u = (6/\rho)(\ln f)_x$ will solve the generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation (1).

2.1. Lump Solution. We apply the symbolic computation with Mathematica to show the quadratic function solutions of the (3+1)-dimensional Hirota bilinear equation (2). A direct Mathematica symbolic computation starts with

$$f = g^2 + h^2 + a_5, \tag{6}$$

and

$$\begin{aligned} g &= x + a_1 y + a_2 z + a_3 t + a_4, \\ h &= b_0 x + b_1 y + b_2 z + b_3 t + b_4 \end{aligned} \tag{7}$$

where a_i $(1 \le i \le 5)$ and b_i $(0 \le i \le 4)$ are all real parameters to be determined.

With the aid of symbolic computation, substituting (3) into (2) and eliminating the coefficients of the polynomial yield the following three sets of constraining equations on the parameters:

$$\begin{aligned} a_1 &= a_1, \\ a_2 &= a_2, \\ a_4 &= a_4, \\ a_3 &= \frac{\left[a_2\left(a_2 + b_2\right)^2 + \left(a_1 + 1\right)\left(a_2^2 - 1\right) + a_2\left(b_0^2 - 1\right) - 2b_0 b_1 + A\right] b(t)}{\left(1 + a_1 + a_2\right)^2 + \left(b_0 + b_1 + b_2\right)^2}, \\ b_0 &= b_0, \\ b_1 &= b_1, \\ b_2 &= b_2, \\ b_4 &= b_4, \\ b_3 &= -\frac{\left[b_2\left(\left(1 + a_2\right)^2 + 2a_1 a_2 + b_2^2\right) + b_1\left(1 - a_2^2 + b_2^2\right) - b_0 B\right] b(t)}{\left(1 + a_1 + a_2\right)^2 + \left(b_0 + b_1 + b_2\right)^2}, \\ a_5 &= -\frac{3\left(1 + b_0^2\right)\left(a_1 + b_0 b_1\right)\left[\left(1 + a_1 + a_2\right)^2 + \left(b_0 + b_1 + b_2\right)^2\right] a(t)}{\left[\left(a_1 + 2a_2\right) b_0 + \left(a_2 - 1\right) b_1 - \left(2 + a_1\right) b_2\right]\left[a_1\left(b_0 + b_2\right) - \left(1 + a_2\right) b_1\right] b(t)} \end{aligned} \tag{8}$$

where

$$\begin{aligned} A &= a_1\left(b_0^2 - b_2^2\right) - \left(b_0 - b_2\right)^2, \\ B &= 2a_1 + b_0 b_1 + b_0 b_2 + \left(1 + a_2^2\right) - b_2^2 + b_0^2 \end{aligned} \tag{9}$$

with the condition $[(a_1+2a_2)b_0+(a_2-1)b_1-(2+a_1)b_2][a_1(b_0+b_2)-(1+a_2)b_1] \neq 0$, and $a_1 + b_0 b_1 < 0$. to guarantee the analyticity and rational localization of the solutions.

Through the dependent variable transformation $u = (6/\rho)(\ln f)_x$, substituting (6) into (3), respectively, we present three families of lump solutions for (1). To get the

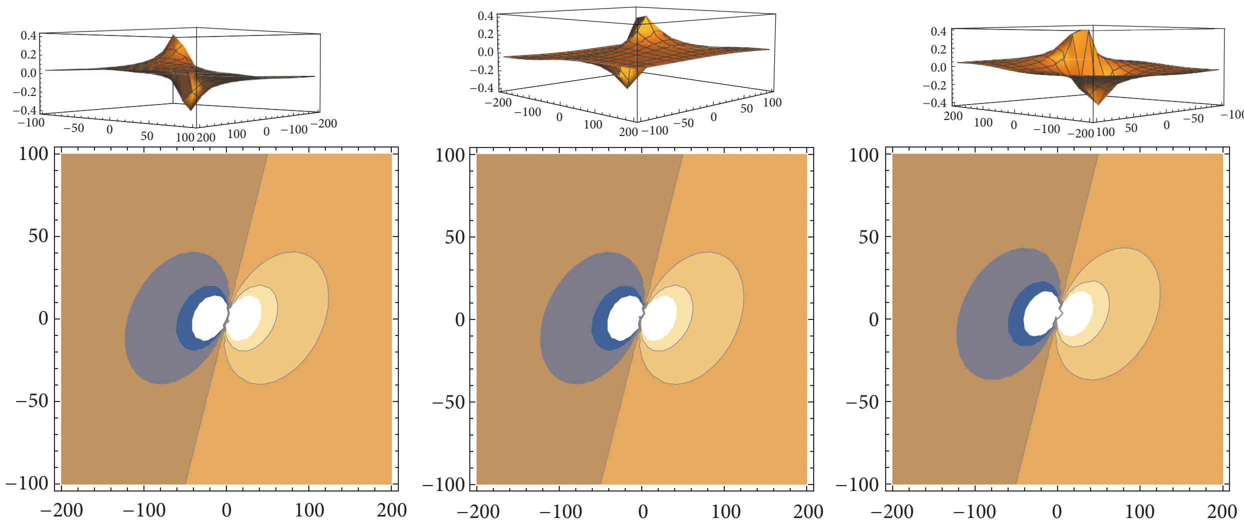

FIGURE 1: Profiles of equation (1) with $t = -0.04, 0.1, 3$: 3d plots (top) and contour plots (bottom).

special lump solution of the generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation, let us choose the following special case: $a(t)$ and $b(t)$ as constants at the same time which has no particular effect on the shape of the solution to this equation and choose the set of the parameters:

$$\begin{aligned} a_1 &= -2, \\ a_2 &= \frac{5}{2}, \\ a_4 &= 1, \\ b_0 &= 1, \\ b_1 &= 1, \\ b_2 &= 1, \\ b_4 &= -1 \end{aligned} \tag{10}$$

The corresponding special lump solution reads

$$u = \frac{12\left(2x - y + (7/2)z + (1/2)t\right)}{\left(x - 2y + (5/2)z + (3/2)t\right)^2 + \left(x + y + z - t - 1\right)^2 - 1} \tag{11}$$

The obtained lump solutions of the three nonlinear evolution equations are same in the form except that the coefficients t are different. Therefore, the plots of the lump solutions of the three equations are similar and their properties are shown by giving the three-dimensional plots and the contour plots for the lump solution of the KP equation (1) (see Figure 1).

2.2. Lump-Type Solution. In this section, we obtain the lump-type solution by setting f into bilinear equation (2) as the quadratic function. We suppose f into bilinear equation (2) is shown in the form

$$f = g^2 + h^2 + a_{11}, \tag{12}$$

and

$$\begin{aligned} g &= a_1 x + a_2 y + a_3 z + a_4 t + a_5, \\ h &= a_6 x + a_7 y + a_8 z + a_9 t + a_{10} \end{aligned} \tag{13}$$

where a_i $(1 \le i \le 11)$ are all real parameters to be determined.

The resulting quadratic function solution presents the lump-type solution u, under the transformation (2) to (1). The analyticity of the rational solution can be achieved and the solution u involves six parameters a_1, a_2, a_3, a_6, a_7, and a_8. a_4, a_5, and a_9 are arbitrary. It yields the following set of constraining equations for the parameters:

$$\begin{aligned} a_1 &= a_1, \\ a_2 &= a_2, \\ a_3 &= a_3, \\ a_4 &= \frac{\left[a_2\left(a_3^2 + a_6^2 - a_8^2\right) + a_3\left(a_3^2 + a_6^2 + a_8\left(2a_6 + a_7 + a_8\right)\right) + C\right] b(t)}{\left(a_1 + a_2 + a_3\right)^2 + \left(a_6 + a_7 + a_8\right)^2}, \end{aligned}$$

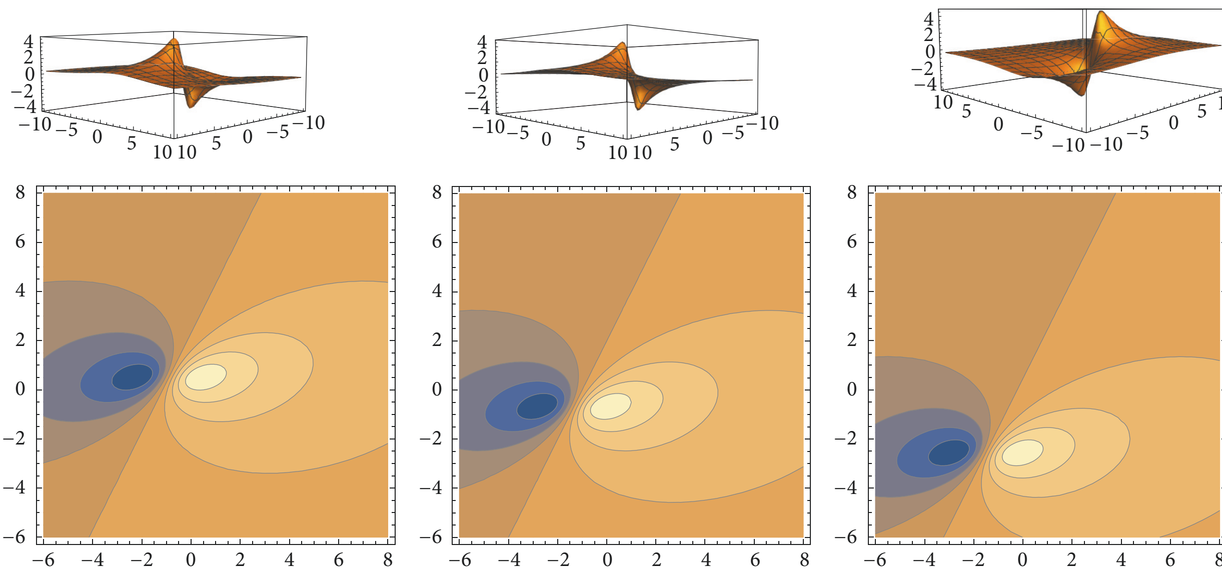

FIGURE 2: Profiles of equation (1) with $t = -1, 0, 5$: 3d plots (top) and contour plots (bottom).

$$
\begin{aligned}
a_5 &= a_5,\\
a_6 &= a_6,\\
a_7 &= a_7,\\
a_8 &= a_8,\\
a_9 &= -\frac{\left[a_3^2\left(a_6+a_7-a_8\right)-2a_2a_3a_8+\left(a_6+a_7+a_8\right)\left(a_6^2-a_8^2\right)+D\right]b(t)}{\left(a_1+a_2+a_3\right)^2+\left(a_6+a_7+a_8\right)^2},\\
a_{10} &= a_{10},\\
a_{11} &= -\frac{3\left(a_1^2+a_6^2\right)\left(a_1a_2+a_6a_7\right)\left[\left(a_1+a_2+a_3\right)^2+\left(a_6+a_7+a_8\right)^2\right]a(t)}{\left[a_2\left(a_6+a_8\right)-\left(a_1+a_3\right)a_7\right]+\left[a_3\left(2a_6+a_7\right)+a_2\left(a_6-a_8\right)-a_1\left(a_7+2a_8\right)\right]b(t)},
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
C &= a_1^3-a_1^2\left(a_2+a_3\right)\\
&\quad+a_1\left(a_3^2-a_6^2-a_8^2-\left(2a_6\left(a_7+a_8\right)\right)\right), \qquad (15)\\
D &= 2a_1\left[a_2a_6+a_3\left(a_6-a_8\right)\right]+a_1^2\left(a_6-a_7-a_8\right).
\end{aligned}
$$

If we choose the parameters guaranteeing $a11 > 0$, this satisfies

$$a_1a_7+a_2a_8 \neq 0, \tag{16}$$

$$a_3a_7+a_2a_6 \neq 0, \tag{17}$$

$$a_1a_2+a_6a_7 < 0. \tag{18}$$

Let us choose the following special set of parameters: $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_6 = 1$, $a_7 = -2$, and $a_8 = -1$. The corresponding special lump-type solution reads

$$u = \frac{12\left(2x-y-(4/13)t+2\right)}{\left(x+y+z+(8/13)t+1\right)^2+\left(x-2y-z-(12/13)t+1\right)^2+13/4} \tag{19}$$

We have found that all the three families of rational solutions exhibit the bright-dark lump wave structure. The property of the lump-type solution is shown by giving the three-dimensional plots and contour plots (see Figure 2).

3. Conclusion

In this paper, Based on the Hirota form of the generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation, the lump and lump-type solutions of the generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation are obtained by symbolic computations and the resulting classes of solutions provide supplements to the existing lump and soliton solutions. Plots of some special lump and lump-type solutions were given which assist in describing complicated nonlinear physical phenomena in fluid mechanics and enrich the dynamic changes of high-dimensional nonlinear wave fields. Therefore, we expect that the results presented in this work will also be useful to study lump solutions in a variety of other high-dimensional nonlinear equations.

Data Availability

All data generated or analysed during this study are included in this published article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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