

## Research Article

# Solutions of Two-Dimensional Nonlinear Sine-Gordon Equation via Triple Laplace Transform Coupled with Iterative Method

Alemayehu Tamirie Deresse , Yesuf Obsie Mussa , and Ademe Kebede Gizaw 

Department of Mathematics, College of Natural Sciences, Jimma University, Ethiopia

Correspondence should be addressed to Ademe Kebede Gizaw; kebedeademe2020@gmail.com

Received 29 May 2021; Revised 27 July 2021; Accepted 26 August 2021; Published 4 October 2021

Academic Editor: Kai Diethelm

Copyright © 2021 Alemayehu Tamirie Deresse et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This article presents triple Laplace transform coupled with iterative method to obtain the exact solution of two-dimensional nonlinear sine-Gordon equation (NLSGE) subject to the appropriate initial and boundary conditions. The noise term in this equation is vanished by successive iterative method. The proposed technique has the advantage of producing exact solution, and it is easily applied to the given problems analytically. Four test problems from mathematical physics are taken to show the accuracy, convergence, and the efficiency of the proposed method. Furthermore, the results indicate that the introduced method is promising for solving other type systems of NLPDEs.

## 1. Introduction

The sine-Gordon (SG) equation is a nonlinear hyperbolic PDE, which was originally considered in the nineteenth century in the course of study of surfaces of constant negative curvature often used to describe and simulate the physical phenomena in a variety of fields of engineering and science, such as nonlinear waves, propagation of fluxons, and dislocation of metals [1–4]. Because sine-Gordon equation leads to different types of soliton solutions, it has been receiving an enormous amount of attention. Soliton solution travels without experiencing any deformation through the medium, even when it collides with another soliton. The solitons, identified in many wave and particle systems, are of importance in the theory of nonlinear differential equations. As one of the crucial equations in nonlinear science, the sine-Gordon equation has been constantly investigated and solved numerically and analytically in recent years [5–8]. For the one-dimensional sine-Gordon equation, Maitama and Hamza [5] introduced analytical method called the Natural Decomposition Method (NDM) for solving nonlinear Sine-Gordon equation. Fayadh and Faraj [9] also applied combined Laplace transform method and VIM to get the approximate solution of the one-dimensional sine-Gordon equation.

For the two-dimensional sine-Gordon equation, Su [6] obtained numerical solution of two-dimensional nonlinear sine-Gordon equation using localized method of approximate particular solutions. In [10], the author developed and analyzed an energy-conserving local discontinuous Galerkin method for the two-dimensional SGE on Cartesian grids. Duan et al. [11] proposed a numerical model based on lattice Boltzmann method to obtain the numerical solutions of two-dimensional generalized sine-Gordon equation, and the method was extended to solve the other two-dimensional wave equations, such as nonlinear hyperbolic telegraph equation as indicated in [12].

In 2020, [13] developed a local Kriging meshless solution to the nonlinear  $(2 + 1)$ -dimensional sine-Gordon equation. The meshless shape function is constructed by Kriging interpolation method to have Kronecker delta function property for the two-dimensional field function, which leads to convenient implementation of imposing essential boundary conditions.

In paper [14], the authors constructed kink wave solutions and traveling wave solutions of the  $(2 + 1)$ -dimensional sine-Gordon equation from the well-known AKNS system. For more related research about solving the sine-Gordon equations, readers may refer to [15–20].

The Laplace transform method (LTM) is one of the integral transform methods that have been intensively used to solve linear and nonlinear equations [21]. The Laplace transform method is used frequently in engineering and physics; the output of a linear time invariant system can be calculated by convolving its unit impulse response with the input signal. Performing this calculation in Laplace space turns the convolution into a multiplication; the latter is easier to solve because of its algebraic form. The Laplace transform can also be used to solve differential equations and is used extensively in electrical engineering [22–25]. The Laplace transform reduces a linear differential equation to an algebraic equation, which can then be solved by the formal rules of algebra. The original differential equation can then be solved by applying the inverse Laplace transform.

Recently, the concept of single Laplace transform is extended to double Laplace transform to solve some kind of differential equations and fractional differential equations such as linear/nonlinear space-time fractional telegraph equations, functional, integral, and partial differential equations [26–28]. Dhunde and Waghmare [29] applied the double Laplace transform method for solving a one-dimensional boundary value problems. Through this method, the boundary value problem is solved without converting it into ordinary differential equation; therefore, there is no need to find complete solution of ordinary differential equation. This is the biggest advantage of the proposed method.

Furthermore, different scholars extended the double Laplace transform method to triple Laplace transform (TLT) to solve two-dimensional nonlinear partial differential equations arising in various natural phenomena. In [30], the authors used triple Laplace transform method to the solution of fractional-order partial differential equations by using Caputo fractional derivative. Through [31–34], the triple Laplace transform method was applied to obtain the solution of fractional-order telegraph equation in two dimensions, linear Volterra integro-differential equations in three dimensions, third-order Mboctara equations, and the proof of some of its properties like linearity property, change of scale property, first shifting property, and convolution theorem property, and differential property and triple integral property are given.

Similar to the Laplace transform method, an iteration method (IM) is a fascinating task in an applied scientific branches to find the solution of nonlinear differential equation. The iterative procedure of the proposed method leads to a series, which can be summed up to find an analytical formula, or it can form a suitable approximation [35]. The error of the approximation can be controlled by properly truncating the series [36]. More surprisingly, an IM has showed effective and more rapid convergent series solution (see [37]).

The purpose of this paper, is to apply triple Laplace transform (TLT) and iterative method (IM) developed in [38] to find the exact solution of two-dimensional nonlinear sine-Gordon equation (NLSGE) subject to appropriate initial and boundary conditions. Dhunde and Waghmare in [37, 39] applied double Laplace transform iteration method (TLTIM) to solve nonlinear Klein-Gordon and telegraph

equations. By this method, the noise terms disappear in the iteration process, and single iteration gives the exact solution.

In the present study, we are interested in the following two-dimensional sine-Gordon equation [40, 41].

$$u_{tt}(x, y, t) + \beta u_t(x, y, t) = \alpha(u_{xx}(x, y, t) + u_{yy}(x, y, t)) - \phi(x, y) \sin(u(x, y, t)) + h(x, y, t), \quad (1)$$

subject to the initial conditions

$$u(x, y, 0) = \phi_1(x, y), u_t(x, y, 0) = \phi_2(x, y), x, y \in \Omega, \quad (2)$$

and boundary conditions (Cauchy type BCs)

$$u(0, y, t) = g_1(y, t), u_x(0, y, t) = g_2(y, t), u(x, 0, t) = g_3(x, t), u_y(x, 0, t) = g_4(x, t), \quad (3)$$

within the problem domain of  $\Omega = \{(x, y): a \leq x \leq b, c \leq y \leq d\}$  for  $t > 0$ . In Equation (1), the parameter  $\beta \geq 0$  is the damping factor for the dissipative term. When  $\beta = 0$ , this equation will be reduced to an undamped sine-Gordon equation and when  $\beta > 0$  to the damped one. The function  $\phi(x, y)$  represents Josephson current density, and the functions  $\phi_1(x, y)$  and  $\phi_2(x, y)$  are specified wave modes and velocity.

The remaining parts of this paper is structured as follows. In Section 2, we begin with some basic definitions, properties, and theorems of triple Laplace transform method. Section 3 illustrates the details of the new iterative method and its convergence. Section 4 presents the description of the model; how the approximate analytical solutions of the given SGE equations is obtained using triple Laplace transform method coupled with iterative method. In Section 5, we apply the proposed method to four illustrative examples in order to show its liability, convergence, and efficiency. Finally, concluding remarks are given in Section 6.

## 2. Definitions and Properties of Triple Laplace Transform Method

In this section, we give some essential definitions, properties, and theorems of triple Laplace transform of partial differential equation, which should be used in the present study.

*Definition 1* (see [31]). Let  $f(x, y, t)$  be a function of three variables  $x, y$ , and  $t$  defined in the positive  $xyt$ -plane. The triple Laplace transform of the function  $f(x, y, t)$  is defined by

$$L_{xyt}[f(x, y, t)] = F(k, p, s) = \int_0^{\infty} e^{-kx} \int_0^{\infty} e^{-py} \int_0^{\infty} e^{-st} f(x, y, t) dt dy dx, \quad (4)$$

whenever the integral exist. Here,  $L_{xyt}[f(x, y, t)]$  denotes  $L_x L_y L_t[f(x, y, t)]$ , and  $k, p$ , and  $s$  are complex

numbers. From this definition, we deduce that:

$$L_{xyt}\{f(x)g(y)h(t)\} = F(k)G(p)H(s) = L_x\{f(x)\}L_y\{g(y)\}L_t\{h(t)\}. \tag{5}$$

**Definition 2** (see [34]). The inverse triple Laplace transform of  $F(k, p, s)$ ,  $L_{xyt}^{-1}[F(k, p, s)] = f(x, y, t)$  is given by the complex triple integral formula

$$L_{xyt}^{-1}[F(k, p, s)] = f(x, y, t) = \frac{1}{2\pi i} \int_{q-i\infty}^{q+i\infty} e^{-kx} \cdot \left( \frac{1}{2\pi i} \int_{r-i\infty}^{r+i\infty} e^{-py} \left[ \frac{1}{2\pi i} \int_{z-i\infty}^{z+i\infty} e^{-st} F(k, p, s) ds \right] dp \right) dk, \tag{6}$$

where  $L_{xyt}^{-1}[F(k, p, s)]$  denotes  $L_x^{-1}L_y^{-1}L_t^{-1}[F(k, p, s)]$ , and  $F(k, p, s)$  must be an analytic function for all  $k, p$ , and  $s$  in the region defined by the inequality  $\text{Re}(k) \geq q$ ,  $\text{Re}(p) \geq r$ , and  $\text{Re}(s) \geq z$ .

**Property 3** (see [34]). The triple Laplace transform of second-order partial derivatives are given by

$$\begin{aligned} L_{xyt} \left\{ \frac{\partial^2}{\partial x^2} f(x, y, t) \right\} &= k^2 F(k, p, s) - kF(0, p, s) - F_x(0, p, s), \\ L_{xyt} \left\{ \frac{\partial^2}{\partial y^2} f(x, y, t) \right\} &= p^2 F(k, p, s) - pF(k, 0, s) - F_y(k, 0, s), \\ L_{xyt} \left\{ \frac{\partial^2}{\partial t^2} f(x, y, t) \right\} &= s^2 F(k, p, s) - sF(k, p, 0) - F_t(k, p, 0), \\ L_{xyt} \left\{ \frac{\partial^2}{\partial x \partial y} f(x, y, t) \right\} &= kpF(k, p, s) - F(0, p, s) - F(k, 0, s). \end{aligned} \tag{7}$$

Furthermore, the triple Laplace transform of first-order partial derivatives are given by

$$\begin{aligned} L_{xyt} \left\{ \frac{\partial}{\partial x} f(x, y, t) \right\} &= sF(k, p, s) - F(0, p, s), \\ L_{xyt} \left\{ \frac{\partial}{\partial y} f(x, y, t) \right\} &= sF(k, p, s) - F(k, 0, s), \\ L_{xyt} \left\{ \frac{\partial}{\partial t} f(x, y, t) \right\} &= sF(k, p, s) - F(k, p, 0). \end{aligned} \tag{8}$$

**2.1. Existence and Uniqueness of the Triple Laplace Transform**

**Theorem 4 (Existence).** Let  $f(x, y, t)$  be a continuous function on the interval  $[0, \infty)$  which is of exponential order, that is, for some  $a, b, c \in \mathbb{R}$ . Consider

$$\sup_{x,y,t>0} \left| \frac{f(x, y, t)}{\exp[ax + by + ct]} \right| < 0. \tag{9}$$

Under this condition, the triple transform,  $F(k, p, s) = \int_0^\infty \int_0^\infty \int_0^\infty e^{-kx} e^{-py} e^{-st} f(x, y, t) dx dy dt$ , exists for all  $p > a, s > b$  and  $k > c$  and is in actuality infinitely differentiable with respect to  $p > a, s > b$  and  $k > c$ .

**Theorem 5 (Uniqueness).** Let  $f(x, y, t)$  and  $g(x, y, t)$  be continuous functions defined for  $x, y, t \geq 0$  and having Laplace transforms,  $F(k, p, s)$  and  $G(k, p, s)$ , respectively. If  $F(k, p, s) = G(k, p, s)$ , then  $f(x, y, t) = g(x, y, t)$ .

For the proof, see [34].

**2.2. Some Properties of Triple Laplace Transform**

**Property 6 (Linearity Property of TLT).** If  $f(x, y, t)$  and  $g(x, y, t)$  are two functions of  $x, y$ , and  $t$  such that  $L_{xyt}\{f(x, y, t)\} = F(k, p, s)$  and  $L_{xyt}\{g(x, y, t)\} = G(k, p, s)$ , then

$$L_{xyt}\{\alpha f(x, y, t) + \beta g(x, y, t)\} = \alpha L_{xyt}f(x, y, t) + \beta L_{xyt}g(x, y, t) = \alpha F(k, p, s) + \beta G(k, p, s), \tag{10}$$

where  $\alpha$  and  $\beta$  are constants.

For the proof, see [31–33].

**Property 7 (Change of Scale Property).** Let  $L_{xyt}\{f(x, y, t)\} = F(k, p, s)$ , then

$$L_{xyt}\{f(ax, by, ct)\} = \frac{1}{abc} F\left(\frac{k}{a}, \frac{p}{b}, \frac{s}{c}\right), \tag{11}$$

where  $a, b$ , and  $c$  are nonzero constants.

For the proof, see [31, 33, 34].

**Property 8 (First Shifting Property).** If  $L_{xyt}\{f(x, y, t)\} = F(k, p, s)$ , then

$$L_{xyt}\{e^{ax+by+ct} f(x, y, t)\} = F(k - a, p - b, s - c). \tag{12}$$

For the proof, see [31, 33, 34].

**Property 9 (Second Shifting Property).** If  $L_{xyt}\{f(x, y, t)\} = F(k, p, s)$ , then

$$L_{xyt}\{f(x - a, y - b, t - c)H(x - a, y - b, t - c)\} = e^{-ak-bp-cs} F(k, p, s), \tag{13}$$

where  $H(x, y, t)$  is the Heaviside unit step function defined by

$$H(x, y, t) = \begin{cases} H(x - a, y - b, t - c): x > a, y > b, t > c, \\ H(x - a, y - b, t - c): x < a, y < b, t < c. \end{cases} \quad (14)$$

For the proof, see [31, 33].

*Property 10.* If  $L_{xyt}\{f(x, y, t)\} = F(k, p, s)$ , then  $L_{xyt}\{e^{ax+by+ct}\} = 1/((k - a)(p - b)(s - c))$ , for all  $x, y$ , and  $t$ .

For the proof, see [31].

### 3. The New Iterative Method

Consider the following functional equation [38]:

$$u(\bar{x}) = N(u(\bar{x})) + f(\bar{x}), \quad (15)$$

where  $N$  is a nonlinear operator in a Banach space such that  $N : B \rightarrow B$ , and  $f$  is a known function.

$\bar{x} = (x_1, x_2, x_3, \dots, x_n)$ , and  $u$  is assumed to be the solution of Equation (15) having the series form:

$$u(\bar{x}) = \sum_{i=0}^{\infty} u_i(\bar{x}). \quad (16)$$

The nonlinear operation  $N$  can then be decomposed as

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{r=0}^i u_r\right) - N\left(\sum_{r=0}^{i-1} u_r\right) \right\}. \quad (17)$$

Using Equations (16) and (17), Equation (15) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{r=0}^i u_r\right) - N\left(\sum_{r=0}^{i-1} u_r\right) \right\}. \quad (18)$$

From Equation (18), we define the following recurrence relation:

$$\begin{aligned} u_0 &= f, u_1 = N(u_0), \\ u_2 &= N(u_0 + u_1) - N(u_0), \\ u_{(n+1)} &= N(u_0 + \dots + u_n) - N(u_0 + \dots + u_{(n-1)}), \quad n = 1, 2, \dots \end{aligned} \quad (19)$$

Thus,

$$(u_1 + \dots + u_{(n+1)}) = N(u_0 + \dots + u_{(n)}), \quad n = 1, 2, \dots, \quad (20)$$

and hence,

$$\sum_{i=0}^{\infty} u_i = f + N\left(\sum_{i=0}^{\infty} u_i\right). \quad (21)$$

Therefore, the  $n^{\text{th}}$  term approximate solution of Equation (16) is given by

$$u = u_0 + \sum_{i=1}^{n-1} u_i = u_0 + u_1 + u_2 + \dots + u_{n-1}, \quad n > 1. \quad (22)$$

**Theorem 11.** If  $N$  is a continuously differentiable functional in a neighborhood of  $u_0$  and  $\|N^{(n)}(u_0)\| \leq M \leq e^{-1}$  for all  $n$ , then the series  $\sum_{n=0}^{\infty} u_{n+1}$  is absolutely convergent [35].

### 4. Description of the Method

Steps to be followed to apply triple Laplace Transform properties as in Table 1 coupled with new iterative method are as follows:

*Step 1.* Applying triple Laplace transform on both sides of Equation (1) and using Property 3, we get

$$\begin{aligned} s^2 \bar{u}(k, p, s) - s \bar{u}(k, p, 0) - \bar{u}_t(k, p, 0) + \beta(s \bar{u}(k, p, s) - \bar{u}(k, p, 0)) \\ + \alpha(-p^2 \bar{u}(k, p, s) + p \bar{u}(k, 0, s) + \bar{u}_y(k, 0, s)) \\ - k^2 \bar{u}(k, p, s) + k \bar{u}(0, p, s) + \bar{u}_x(0, p, s) \\ + L_{xyt}(\phi(x, y) \sin(u(x, y, t))) = L_{xyt}(h(x, y, t)), \end{aligned} \quad (23)$$

where  $h(x, y, t)$  is the source functioning Equation (1).

*Step 2.* Now, employing double Laplace transform to Equations (2) and (3), we have

$$\begin{aligned} \bar{\varphi}_1(k, p) = \bar{u}(k, p, 0), \bar{\varphi}_2(k, p) = \bar{u}_t(k, p, 0), \text{ and } \bar{g}_1(p, s) \\ = \bar{u}(0, p, s), \bar{g}_2(p, s) = \bar{u}_x(0, p, s), \end{aligned} \quad (24)$$

$$\bar{g}_3(k, s) = \bar{u}(k, 0, s), \bar{g}_4(k, s) = \bar{u}_y(k, 0, s), \quad (25)$$

respectively.

By substituting Equations (24) and (25) into Equation (23) and simplifying, we obtain

$$\begin{aligned} \bar{u}(k, p, s) = \frac{1}{s^2 - \alpha p^2 - \alpha k^2 + \beta s} \{ (s + \beta) \bar{u}(k, p, 0) + \bar{u}_t(k, p, 0) \\ - \alpha(p \bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) + k \bar{u}(0, p, s) + \bar{u}_x(0, p, s)) \\ + L_{xyt}\{h(x, y, t) - \phi(x, y) \sin(u(x, y, t))\} \}. \end{aligned} \quad (26)$$

*Step 3.* By implementing the triple inverse Laplace transformation of Equation (26), we obtain

TABLE 1: Triple Laplace transform for some functions of three variables [33, 34].

Functions $f(x, y, t)$	Triple Laplace transform $F(k, p, s)$
$abc$	$\frac{abc}{kps}$
$xyt$	$\frac{1}{k^2 p^2 s^2}$
$x^m y^n t^r$	$\frac{m!n!r!}{k^{m+1} p^{n+1} s^{r+1}}$
$e^{-ax-by-ct}$	$\frac{1}{(a+k)(b+p)(c+s)}$
$\cos(x) \cos(y) \cos(t)$	$\frac{kps}{(1+k^2)(1+p^2)(1+s^2)}$
$\sin(x) \sin(y) \sin(t)$	$\frac{1}{(1+k^2)(1+p^2)(1+s^2)}$
$\sin(x+y+t)$	$\frac{-1+ps+k(p+s)}{(1+k^2)(1+p^2)(1+s^2)}$
$\cos(x+y+t)$	$-\frac{k+p+s-kps}{(1+k^2)(1+p^2)(1+s^2)}$
$\cosh(ax+by+ct)$	$\frac{1}{2} \left[ \frac{1}{(k-a)(p-b)(s-c)} + \frac{1}{(k+a)(p+b)(s+c)} \right]$
$\sinh(ax+by+ct)$	$\frac{1}{2} \left[ \frac{1}{(k-a)(p-b)(s-c)} - \frac{1}{(k+a)(p+b)(s+c)} \right]$

$$\begin{aligned} \bar{u}(k, p, s) = L_{xyt}^{-1} & \left[ \frac{1}{s^2 - \alpha p^2 - \alpha k^2 + \beta s} \{ (s + \beta) \bar{u}(k, p, 0) \right. \\ & + \bar{u}_t(k, p, 0) - \alpha(p\bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) \\ & + k\bar{u}(0, p, s) + \bar{u}_x(0, p, s)) + L_{xyt} \{ (h(x, y, t) \\ & \left. - \phi(x, y) \sin(u(x, y, t))) \} \right]. \end{aligned} \tag{27}$$

Step 4. Assume that

$$u(x, y, t) = \sum_{i=0}^{\infty} u_i(x, y, t), \tag{28}$$

is the solution of Equation (1).

Then, substituting Equation (28) into Equation (27), we obtain

$$\begin{aligned} \sum_{i=0}^{\infty} u_i(x, y, t) = L_{xyt}^{-1} & \left[ \frac{1}{s^2 - \alpha p^2 - \alpha k^2 + \beta s} \{ (s + \beta) \bar{u}(k, p, 0) \right. \\ & + \bar{u}_t(k, p, 0) - \alpha(p\bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) \\ & + k\bar{u}(0, p, s) + \bar{u}_x(0, p, s)) + L_{xyt} [h(x, y, t) \\ & \left. - \phi(x, y) \sin \left( \sum_{i=0}^{\infty} u_i(x, y, t) \right) \right] \right]. \end{aligned} \tag{29}$$

Step 5. By implementing the new iterative method, the non-linear term  $\sin(u(x, y, t))$  in Equation (29) is decomposed as

$$\begin{aligned} \sin \left( \sum_{i=0}^{\infty} u_i(x, y, t) \right) & = \sin(u_0(x, y, t)) \\ & + \sum_{i=1}^{\infty} \left\{ \sin \left( \sum_{r=0}^i u_r(x, y, t) \right) - \sin \left( \sum_{r=0}^{i-1} u_r(x, y, t) \right) \right\}. \end{aligned} \tag{30}$$

Using Equation (30), Equation (29) is equivalent to

$$\begin{aligned} \sum_{i=0}^{\infty} u_i(x, y, t) = L_{xyt}^{-1} & \left[ \frac{1}{s^2 - \alpha p^2 - \alpha k^2 + \beta s} \{ (s + \beta) \bar{u}(k, p, 0) \right. \\ & + \bar{u}_t(k, p, 0) - \alpha(p\bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) \\ & + k\bar{u}(0, p, s) + \bar{u}_x(0, p, s)) + L_{xyt} \{ h(x, y, t) \\ & - \phi(x, y) [ \sin(u_0(x, y, t)) \\ & + \sum_{i=1}^{\infty} \left( \sin \left( \sum_{r=0}^i u_r(x, y, t) \right) \right. \\ & \left. \left. - \sin \left( \sum_{r=0}^{i-1} u_r(x, y, t) \right) \right) \right] \right] \right]. \end{aligned} \tag{31}$$

Step 6. Using triple Laplace transform coupled with new iterative method, we introduce the recursive relations and get

$$u_0(x, y, t) = L_{xyt}^{-1} \left[ \frac{1}{s^2 - \alpha p^2 - \alpha k^2 + \beta s} \{ (s + \beta) \bar{u}(k, p, 0) + \bar{u}_t(k, p, 0) - \alpha (p \bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) + k \bar{u}(0, p, s) + \bar{u}_x(0, p, s)) \} \right],$$

$$u_1(x, y, t) = L_{xyt}^{-1} \left[ \frac{1}{s^2 - \alpha p^2 - \alpha k^2 + \beta s} L_{xyt} \{ (h(x, y, t) - \phi(x, y) \sin(u_0(x, y, t))) \} \right],$$

$$u_{n+1}(x, y, t) = -L_{xyt}^{-1} \left[ \frac{1}{s^2 - \alpha p^2 - \alpha k^2 + \beta s} L_{xyt} \left\{ \phi(x, y) \sum_{i=1}^{\infty} \left( \sin \left( \sum_{r=0}^i u_r(x, y, t) \right) - \sin \left( \sum_{r=0}^{i-1} u_r(x, y, t) \right) \right) \right\} \right], \quad n \geq 1. \tag{32}$$

Step 7. The solution of Equations (1)–(3) in series form is given by

$$u(x, y, t) = u_0(x, y, t) + u_1(x, y, t) + u_2(x, y, t) + \dots = u_0(x, y, t). \tag{33}$$

### 5. Illustrative Examples

In order to show the validity and effectiveness of the method under consideration, some examples are presented here.

Example 1. Consider the two-dimensional NLSGE (1) with  $\beta = 0, \alpha = 1, \phi(x, y) = 1, h(x, y, t) = \sin(e^{x+y-2t}) + 2e^{x+y-2t}$  on the domain  $\Omega = [0, 2]^2, t \geq 0$ . That is,

$$u_{tt} = u_{xx} + u_{yy} - \sin(u(x, y, t)) + \sin(e^{x+y-2t}) + 2e^{x+y-2t}, \tag{34}$$

with initial conditions

$$u(x, y, 0) = e^{x+y}, u_t(x, y, 0) = -2e^{x+y}, \tag{35}$$

and boundary conditions

$$u(0, y, t) = e^{y-2t}, u_x(0, y, t) = e^{y-2t}, u(x, 0, t) = e^{x-2t}, u_y(x, 0, t) = e^{x-2t}. \tag{36}$$

Solution: Applying properties of triple Laplace transform on both sides of Equation (34), we get

$$\begin{aligned} & \{ s^2 \bar{u}(k, p, s) - s \bar{u}(k, p, 0) - \bar{u}_t(k, p, 0) - p^2 \bar{u}(k, p, s) \\ & + p \bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) - k^2 \bar{u}(k, p, s) + k \bar{u}(0, p, s) \\ & + \bar{u}_x(0, p, s) + L_{xyt}(\sin(u(x, y, t))) \} = L_{xyt}[\sin(e^{x+y-2t})] \\ & + \frac{2}{(k-1)(p-1)(s+2)}. \end{aligned} \tag{37}$$

Applying double Laplace transform to Equations (35) and (36), we obtain

$$\begin{aligned} \bar{u}(k, p, 0) &= \frac{1}{(k-1)(p-1)}, \bar{u}_t(k, p, 0) = -\frac{2}{(k-1)(p-1)}, \\ \bar{u}(0, p, s) &= \frac{1}{(p-1)(s+2)}, \bar{u}_x(0, p, s) = \frac{1}{(p-1)(s+2)}, \end{aligned} \tag{38}$$

$$\bar{u}(k, 0, s) = \frac{1}{(k-1)(s+2)}, \bar{u}_y(k, 0, s) = \frac{1}{(k-1)(s+2)}, \tag{39}$$

respectively.

By substituting Equations (38) and (39) into Equation (37), we get

$$(s^2 - p^2 - k^2) \bar{u}(k, p, s) = \left( \frac{2}{(k-1)(p-1)(s+2)} + \frac{s-2}{(k-1)(p-1)} - \frac{p+1}{(k-1)(s+2)} - \frac{k+1}{(p-1)(s+2)} \right) + L_{xyt}[\sin(e^{x+y-2t}) - \sin(u(x, y, t))]. \tag{40}$$

Simplifying (40) gives us

$$\bar{u}(k, p, s) = \frac{1}{(k-1)(p-1)(s+2)} + \frac{1}{(s^2 - p^2 - k^2)} \cdot L_{xyt}[\sin(e^{x+y-2t}) - \sin(u(x, y, t))]. \tag{41}$$

Applying inverse triple Laplace transform to Equation (41), we get

$$u(x, y, t) = e^{x+y-2t} + L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} [\sin(e^{x+y-2t}) - \sin(u(x, y, t))] \right\}. \tag{42}$$

Now, applying the new iterative method to Equation

(42), we obtain the components of the solution as follows:

$$u_0(x, y, t) = e^{x+y-2t}, \tag{43}$$

$$u_1(x, y, t) = L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} [\sin (e^{x+y-2t}) - \sin (u(x, y, t))] \right\} = 0. \tag{44}$$

$$u_{n+1}(x, y, t) = -L_{xyt}^{-1} \left[ \frac{1}{s^2 - p^2 - k^2} L_{xyt} \left\{ \sin \left( \sum_{r=0}^n u_r(x, y, t) \right) - \sin \left( \sum_{r=0}^{n-1} u_r(x, y, t) \right) \right\} \right], \quad n \geq 1. \tag{45}$$

Now, we define the recurrence relation from Equation (45) for  $n \geq 1$  as follows:

$$u_2(x, y, t) = L_{xyt}^{-1} \left[ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \{ \sin (u_0(x, y, t)) + u_1(x, y, t) - \sin (u_0(x, y, t)) \} \right] = 0,$$

$$u_3(x, y, t) = L_{xyt}^{-1} \left[ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \{ \sin (u_0(x, y, t)) + u_1(x, y, t) + u_2(x, y, t) - \sin (u_0(x, y, t)) + u_1(x, y, t) \} \right] = 0. \tag{46}$$

In the same way, we obtain  $u_4(x, y, t) = u_5(x, y, t) = 0$  and so on.

Therefore, the solution of Example 1 by using Equation (33) is

$$u(x, y, t) = e^{x+y-2t}. \tag{47}$$

Let us now test the convergence of the obtained series solution. From Equation (42), we have

$$u_0(x, y, t) = e^{x+y-2t},$$

$$N(u(x, y, t)) = L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} [\sin (e^{x+y-2t}) - \sin (u(x, y, t))] \right\}. \tag{48}$$

Thus, for all  $x, y, t \geq 0$ , we have

$$\begin{aligned} N(u_0(x, y, t)) &= L_{ytx}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} [\sin (e^{x+y-2t}) - \sin (u_0(x, y, t))] \right\} \\ &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} [\sin (e^{x+y-2t}) - \sin (e^{x+y-2t})] \right\} = 0, \end{aligned} \tag{49}$$

Therefore,  $\|N(u_0(x, y, t))\| = \|0\| = 0 < (1/e)$ .

$$\begin{aligned} N'(u(x, y, t)) &= L_{ytx}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_x L_y L_t \cdot [(e^{x+y-2t})' \cos (e^{x+y-2t}) - \cos (u(x, y, t)) u'(x, y, t)] \right\}, \end{aligned} \tag{50}$$

where  $N'(u(x, y, t))$  represents the partial derivatives  $(\partial/\partial x)u(x, y, t)$  or  $(\partial/\partial y)u(x, y, t)$  or  $(\partial/\partial t)u(x, y, t)$ .

$$N\left(\frac{\partial}{\partial t} u(x, y, t)\right) = L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \left[ \begin{array}{l} -2e^{x+y-2t} \cos (e^{x+y-2t}) \\ -\cos (u(x, y, t)) \frac{\partial}{\partial t} u(x, y, t) \end{array} \right] \right\}. \tag{51}$$

Then,

$$\begin{aligned} N\left(\frac{\partial}{\partial t} u_0(x, y, t)\right) &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \left[ \begin{array}{l} -2e^{x+y-2t} \cos (e^{x+y-2t}) \\ +2e^{x+y-2t} \cos (e^{x+y-2t}) \end{array} \right] \right\} = 0, \\ N\left(\frac{\partial}{\partial x} u_0(x, y, t)\right) &= N\left(\frac{\partial}{\partial y} u_0(x, y, t)\right) = L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \left[ \begin{array}{l} e^{x+y-2t} \cos (e^{x+y-2t}) \\ -e^{x+y-2t} \cos (e^{x+y-2t}) \end{array} \right] \right\} = 0. \end{aligned} \tag{52}$$

Therefore,  $\|N(u_0'(x, y, t))\| = \|0\| = 0 < (1/e)$ .

$$N\left(\frac{\partial^2}{\partial t^2} u_0(x, y, t)\right) = L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \left[ \begin{array}{l} 4e^{x+y-2t} \cos(e^{x+y-2t}) - 4e^{2x+2y-4t} \sin(e^{x+y-2t}) \\ -\cos(u_0(x, y, t)) \frac{\partial^2}{\partial t^2} u_0(x, y, t) \\ +\sin(u(x, y, t)) \left(\frac{\partial}{\partial t} u_0(x, y, t)\right)^2 \end{array} \right] \right\}$$

$$= L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \left[ \begin{array}{l} 4e^{x+y-2t} \cos(e^{x+y-2t}) - 4e^{2x+2y-4t} \sin(e^{x+y-2t}) \\ -4e^{x+y-2t} \cos(e^{x+y-2t}) + 4e^{2x+2y-4t} \sin(e^{x+y-2t}) \end{array} \right] \right\} = 0,$$

$$N\left(\frac{\partial^2}{\partial x^2} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial y^2} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial x \partial y} u_0(x, y, t)\right)$$

$$= L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \left[ \begin{array}{l} e^{x+y-2t} \cos(e^{x+y-2t}) - e^{2x+2y-4t} \sin(e^{x+y-2t}) \\ -e^{x+y-2t} \cos(e^{x+y-2t}) + e^{2x+2y-4t} \sin(e^{x+y-2t}) \end{array} \right] \right\} = 0,$$

$$N\left(\frac{\partial^2}{\partial x \partial t} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial y \partial t} u_0(x, y, t)\right) = L_{xyt}^{-1} \left\{ \frac{1}{(s^2 - p^2 - k^2)} L_{xyt} \left[ \begin{array}{l} -2e^{x+y-2t} \cos(e^{x+y-2t}) + 2e^{2x+2y-4t} \sin(e^{x+y-2t}) \\ +2e^{x+y-2t} \cos(e^{x+y-2t}) - 2e^{2x+2y-4t} \sin(e^{x+y-2t}) \end{array} \right] \right\} = 0. \tag{53}$$

Therefore,  $\|N''(u_0(x, y, t))\| = \|0\| = 0 < (1/e)$ ,

where  $N''(u_0(x, y, t))$  represents all the second-order partial derivatives of  $u_0(x, y, t)$ .

Similarly, by principle of Mathematical induction, we have  $\|N^{(3)}(u_0(x, y, t))\| = \|N^{(4)}(u_0(x, y, t))\| = \dots = \|N^{(k)}(u_0(x, y, t))\| = 0 < (1/e)$ , for all  $k \geq 0$ .

As the condition of Theorem 11 are satisfied, the series solution  $u(x, y, t) = \sum_{i=0}^{\infty} u_i(x, y, t)$  is absolutely convergent, and hence, the solution obtained by the new iterative method is convergent on the domain of interest.

*Example 2.* Consider Equation (1) with  $\beta = 0, \alpha = 1, \phi(x, y) = 1, h(x, y, t) = \sin(\sin(x, y, t)) + \sin(x, y, t)$  on the domain  $\Omega = [0, 2\pi]^2, t \geq 0$ . That is,

$$u_{tt}(x, y, t) = u_{xx}(x, y, t) + u_{yy}(x, y, t) - \sin(u(x, y, t)) + \sin(\sin(x + y + t)) + \sin(x + y + t), \tag{54}$$

with initial conditions

$$u(x, y, 0) = \sin(x + y), u_t(x, y, 0) = \cos(x + y), \tag{55}$$

and boundary conditions

$$u(0, y, t) = \sin(y + t), u_x(0, y, t) = \cos(y + t), \tag{56}$$

$$u(x, 0, t) = \sin(x + t), u_y(x, 0, t) = \cos(x + t).$$

**Solution:** Applying properties of triple Laplace transform on both sides of Equation (54), we obtain

$$(s^2 \bar{u}(k, p, s) - s\bar{u}(k, p, 0) - \bar{u}_t(k, p, 0) - p^2 \bar{u}(k, p, s) + p\bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) - k^2 \bar{u}(k, p, s) + k\bar{u}(0, p, s) + \bar{u}_x(0, p, s) + L_{xyt}(\sin(u(x, y, t))))$$

$$= L_{xyt}[\sin\{\sin(x + y + t)\}] + \frac{-1 + ps + k(p + s)}{(1 + k^2)(1 + p^2)(1 + s^2)}. \tag{57}$$

Applying double Laplace transform to Equations (55) and (56), we obtain

$$\bar{u}(k, p, 0) = \frac{k + p}{(1 + p^2)(1 + k^2)}, \bar{u}_t(k, p, 0) = \frac{kp - 1}{(1 + p^2)(1 + k^2)}, \tag{58}$$



$$\begin{aligned} \bar{u}(0, p, s) &= \frac{k+s}{(1+p^2)(1+s^2)}, \bar{u}_x(0, p, s) = \frac{ps-1}{(1+p^2)(1+s^2)}, \\ \bar{u}(k, 0, s) &= \frac{k+s}{(1+k^2)(1+s^2)}, \bar{u}_y(k, 0, s) = \frac{ks-1}{(1+k^2)(1+s^2)}, \end{aligned} \tag{59}$$

respectively.

By substituting Equations (58) and (59) in to Equation (57) and simplifying, we get

$$\begin{aligned} \bar{u}(k, p, s) &= \frac{-1+ps+k(p+s)}{(1+k^2)(1+p^2)(1+s^2)} + \frac{1}{s^2-p^2-k^2} \\ &\cdot L_{xyt}[\sin\{\sin(x+y+t)\} - \sin(u(x, y, t))]. \end{aligned} \tag{60}$$

Applying inverse triple Laplace transform to Equation (60), we get

$$\begin{aligned} u(x, y, t) &= \sin(x+y+t) + L_{xyt}^{-1} \left\{ \frac{1}{s^2-p^2-k^2} L_{xyt}[\sin\{\sin(x+y+t)\} \right. \\ &\left. \cdot \{\sin(x+y+t)\} - \sin(u(x, y, t))\} \right\}. \end{aligned} \tag{61}$$

Now, applying the new iterative method to Equation (61), we obtain the components of the solution as follows:

Substituting Equation (16) into Equation (61), we obtain the components of the solution as follows:

$$\begin{aligned} u_0(x, y, t) &= \sin(x+y+t), \\ u_1(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{1}{s^2-p^2-k^2} L_{xyt}[\sin\{\sin(x+y+t)\} \right. \\ &\left. - \sin\{\sin(x+y+t)\}] \right\} = 0, \\ u_2(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{1}{s^2-p^2-k^2} L_{xyt}[\sin\{u_0(x, y, t)\} \right. \\ &\left. + u_1(x, y, t) - \sin(u_0(x, y, t))] \right\} = 0, \\ u_3(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{1}{s^2-p^2-k^2} L_{xyt}[\sin\{u_0(x, y, t)\} \right. \\ &\left. + u_1(x, y, t) + u_2(x, y, t) \right. \\ &\left. - \sin(u_0(x, y, t) + u_1(x, y, t))] \right\} = 0, \end{aligned} \tag{62}$$

and so on.

Therefore, the solution of Example 2 by using equation (33) is

$$u(x, y, t) = \sin(x+y+t), \tag{63}$$

as indicated in Baccouch [10] and Deresse et al. [41].

Next, we test the convergence of the obtained series solution. From Equation (61) we have,

$$\begin{aligned} u_0(x, y, t) &= \sin(x+y+t) \text{ and } N(u(x, y, t)) \\ &= L_{xyt}^{-1} \left\{ \frac{1}{s^2-p^2-k^2} L_{xyt}[\sin\{\sin(x+y+t)\} \right. \\ &\left. - \sin(u(x, y, t))] \right\}. \end{aligned} \tag{64}$$

Thus, for all  $x, y, t \geq 0$ , we have

$$\begin{aligned} N(u_0(x, y, t)) &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2-p^2-k^2)} L_{xyt}[\sin\{\sin(x+y+t)\} \right. \\ &\left. - \sin(u_0(x, y, t))] \right\} \\ &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2-p^2-k^2)} L_{xyt}[\sin\{\sin(x+y+t)\} \right. \\ &\left. - \sin\{\sin(x+y+t)\}] \right\} = 0. \end{aligned} \tag{65}$$

Therefore,  $\|N(u_0(x, y, t))\| = \|0\| = 0 < (1/e)$ .

$$\begin{aligned} N\left(\frac{\partial}{\partial t} u(x, y, t)\right) &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2-p^2-k^2)} L_{xyt}[\cos \right. \\ &\left. \cdot \{\sin(x+y+t)\} \cos(x+y+t) \right. \\ &\left. - \cos(u(x, y, t)) \frac{\partial}{\partial t} u(x, y, t) \right\}, \\ N\left(\frac{\partial}{\partial t} u_0(x, y, t)\right) &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2-p^2-k^2)} L_{xyt}[\cos \right. \\ &\left. \cdot \{\sin(x+y+t)\} \cos(x+y+t) \right. \\ &\left. - \cos\{\sin(x+y+t)\} \cos(x+y+t) \right\} = 0. \end{aligned} \tag{66}$$

Similarly,  $N((\partial/\partial x)u_0(x, y, t)) = N((\partial/\partial y)u_0(x, y, t)) = 0$ . Therefore,  $\|N(u_0'(x, y, t))\| = \|0\| = 0 < (1/e)$ .

---


$$\begin{aligned} N\left(\frac{\partial^2}{\partial t^2} u_0(x, y, t)\right) &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2-p^2-k^2)} L_{xyt}[-\sin\{\sin(x+y+t)\} \cos^2(x+y+t) + \cos\{\sin(x+y+t)\} \sin(x+y+t) \right. \\ &\left. - \left( \cos(u_0(x, y, t)) \frac{\partial^2}{\partial t^2} u_0(x, y, t) - \sin(u(x, y, t)) \left(\frac{\partial}{\partial t} u_0(x, y, t)\right)^2 \right) \right\} \\ &= L_{xyt}^{-1} \left\{ \frac{1}{(s^2-p^2-k^2)} L_{xyt} \left[ -\sin\{\sin(x+y+t)\} \cos^2(x+y+t) - \cos\{\sin(x+y+t)\} \sin(x+y+t) \right] \right\} = 0. \end{aligned} \tag{67}$$

Similarly,

$$\begin{aligned} N\left(\frac{\partial^2}{\partial x^2} u_0(x, y, t)\right) &= N\left(\frac{\partial^2}{\partial y^2} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial x \partial y} u_0(x, y, t)\right) \\ &= N\left(\frac{\partial^2}{\partial x \partial t} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial y \partial t} u_0(x, y, t)\right) = 0. \end{aligned} \tag{68}$$

Therefore,  $\|N''(u_0(x, y, t))\| = \|0\| = 0 < (1/e)$ , where  $N''(u_0(x, y, t))$  represents all the second-order partial derivatives of  $u_0(x, y, t)$ .

Similarly, by principle of Mathematical induction, we have  $\|N^{(3)}(u_0(x, y, t))\| = \|N^{(4)}(u_0(x, y, t))\| = \dots = \|N^{(k)}(u_0(x, y, t))\| = 0 < (1/e)$ , for all  $k \geq 0$ .

As the condition of Theorem 11 are satisfied, the series solution  $u(x, y, t) = \sum_{i=0}^{\infty} u_i(x, y, t)$  is absolutely convergent, and hence, the solution obtained by the new iterative method is convergent on the domain of interest.

*Example 3.* Consider Equation (1) on the domain  $\Omega = [-1/2, 1/2]^2$  with  $\beta = 0, \alpha = 1/2\pi^2, \phi(x, y) = 1$ , and  $h(x, y, t) = \sin(\cos(\pi x) \cos(\pi y) \cos(t)) + \sin(x, y, t)$ , That is,

$$\begin{aligned} u_{tt}(x, y, t) &= \frac{1}{2\pi^2} [u_{xx}(x, y, t) + u_{yy}(x, y, t)] \\ &\quad - \sin(u(x, y, t)) + \sin(\cos(\pi x) \cos(\pi y) \cos(t)), \end{aligned} \tag{69}$$

with initial conditions

$$u(x, y, 0) = \cos(\pi x) \cos(\pi y), u_t(x, y, 0) = 0, \tag{70}$$

and boundary conditions

$$\begin{aligned} u(0, y, t) &= \cos(\pi y) \cos(t), u_x(0, y, t) \\ &= 0, u(x, 0, t) = \cos(\pi x) \cos(t), u_y(x, 0, t) = 0. \end{aligned} \tag{71}$$

Solution: Applying properties of triple Laplace transform on both sides of Equation (69), we get

$$\begin{aligned} \left[ s^2 \bar{u}(k, p, s) - s \bar{u}(k, p, 0) - \bar{u}_t(k, p, 0) + \frac{1}{2\pi^2} (-p^2 \bar{u}(k, p, s) \right. \\ \left. + p \bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) - k^2 \bar{u}(k, p, s) + k \bar{u}(0, p, s) \right. \\ \left. + \bar{u}_x(0, p, s) \right] + L_x L_y L_t (\sin(u(x, y, t))) \\ = L_x L_y L_t [\sin\{\cos(\pi x) \cos(\pi y) \cos(t)\}]. \end{aligned} \tag{72}$$

Applying double Laplace transform to Equations (70) and (71), we obtain

$$\bar{u}(k, p, 0) = \frac{kp}{(\pi^2 + p^2)(\pi^2 + k^2)}, \bar{u}_t(k, p, 0) = 0, \tag{73}$$

$$\bar{u}(k, 0, s) = \frac{ks}{(\pi^2 + k^2)(1 + s^2)}, \bar{u}_y(k, 0, s) = 0, \tag{74}$$

$$\bar{u}(0, p, s) = \frac{ps}{(\pi^2 + p^2)(1 + s^2)}, \bar{u}_x(0, p, s) = 0$$

respectively.

By substituting Equations (73) and (74) into Equation (72) and simplifying, we get

$$\begin{aligned} \bar{u}(k, p, s) &= \frac{kps}{(\pi^2 + p^2)(\pi^2 + k^2)(1 + s^2)} + \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} \\ &\quad \cdot L_{xyt} [\sin\{\cos(\pi x) \cos(\pi y) \cos(t)\} - \sin(u(x, y, t))]. \end{aligned} \tag{75}$$

Applying inverse triple Laplace transform to Equation (75), we obtain

$$\begin{aligned} u(x, y, t) &= \left( \cos(\pi x) \cos(\pi y) \cos(t) + L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} \right. \right. \\ &\quad \left. \left. \cdot L_{xyt} [\sin\{\cos(\pi x) \cos(\pi y) \cos(t)\} - \sin(u(x, y, t))] \right\} \right). \end{aligned} \tag{76}$$

Now, applying the new iterative method to Equation (76), we obtain the components of the solution as follows:

$$u_0(x, y, t) = \cos(\pi x) \cos(\pi y) \cos(t),$$

$$\begin{aligned} u_1(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [\sin\{\cos(\pi x) \right. \\ &\quad \cdot \cos(\pi y) \cos(t)\} - \sin(u_0(x, y, t))] \right\}, \\ &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [\sin\{\cos(\pi x) \right. \\ &\quad \cdot \cos(\pi y) \cos(t)\} - \sin\{\cos(\pi x) \\ &\quad \cdot \cos(\pi y) \cos(t)\}] \right\} = 0, \end{aligned} \tag{77}$$

$$\begin{aligned} u_2(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [\sin(u_0(x, y, t) \right. \\ &\quad \left. + u_1(x, y, t)) - \sin(u_0(x, y, t))] \right\} = 0, \end{aligned}$$

$$\begin{aligned} u_3(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [\sin(u_0(x, y, t) \right. \\ &\quad \left. + u_1(x, y, t) + u_2(x, y, t)) - \sin(u_0(x, y, t) \right. \\ &\quad \left. + u_1(x, y, t))] \right\} = 0, \end{aligned} \tag{78}$$

and so on.

Therefore, the solution of Example 3 by using Equation (33) is

$$u(x, y, t) = \cos(\pi x) \cos(\pi y) \cos(t). \tag{79}$$

This is exactly the same as the result obtained by Kang et al. [16] and Deresse et al. [41].

We now test the convergence of the obtained series solution of Example 3.

From Equation (61), we have  $u_0(x, y, t) = \cos(\pi x) \cos(\pi y) \cos(t)$  and

$$\begin{aligned} N(u(x, y, t)) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [\sin\{\cos(\pi x) \right. \\ &\quad \cdot \cos(\pi y) \cos(t)\} - \sin(u(x, y, t))] \left. \right\}. \end{aligned} \tag{80}$$

Thus, for all  $x, y, t \geq 0$ , we have

$$\begin{aligned} N(u_0(x, y, t)) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [\sin\{\cos(\pi x) \right. \\ &\quad \cdot \cos(\pi y) \cos(t)\} - \sin(u_0(x, y, t))] \left. \right\} \\ &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [\sin\{\cos(\pi x) \right. \\ &\quad \cdot \cos(\pi y) \cos(t)\} - \sin\{\cos(\pi x) \\ &\quad \cdot \cos(\pi y) \cos(t)\}] \left. \right\} = 0. \end{aligned} \tag{81}$$

Therefore,  $\|N(u_0(x, y, t))\| = \|0\| = 0 < (1/e)$ .

$$\begin{aligned} N\left(\frac{\partial}{\partial t} u(x, y, t)\right) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} \right. \\ &\quad \cdot [-\cos\{\cos(\pi x) \cos(\pi y) \cos(t)\} \\ &\quad \cdot \cos(\pi x) \cos(\pi y) \sin(t) \\ &\quad \left. - \cos(u(x, y, t)) \frac{\partial}{\partial t} u(x, y, t) \right\}, \end{aligned}$$

$$\begin{aligned} N\left(\frac{\partial}{\partial t} u_0(x, y, t)\right) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} \right. \\ &\quad \cdot [-\cos\{\cos(\pi x) \cos(\pi y) \cos(t)\} \\ &\quad \cdot \cos(\pi x) \cos(\pi y) \sin(t) \\ &\quad + \cos\{\cos(\pi x) \cos(\pi y) \cos(t)\} \\ &\quad \left. \cdot \cos(\pi x) \cos(\pi y) \sin(t) \right\} = 0. \end{aligned} \tag{82}$$

Similarly,  $N((\partial/\partial x)u_0(x, y, t)) = N((\partial/\partial y)u_0(x, y, t)) = 0$ .

Therefore,  $\|N(u_0'(x, y, t))\| = \|0\| = 0 < (1/e)$ .

$$\begin{aligned} N\left(\frac{\partial^2}{\partial t^2} u_0(x, y, t)\right) &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} \right. \\ &\quad \cdot [-\sin\{\cos(\pi x) \cos(\pi y) \cos(t)\} \cos^2(\pi x) \cos^2(\pi y) \\ &\quad \cdot \sin^2(t) - \cos\{\cos(\pi x) \cos(\pi y) \cos(t)\} \cos(\pi x) \\ &\quad \cdot \cos(\pi y) \cos(t) - \left( \cos(u_0(x, y, t)) \frac{\partial^2}{\partial t^2} u_0(x, y, t) \right. \\ &\quad \left. - \sin(u(x, y, t)) \left( \frac{\partial}{\partial t} u_0(x, y, t) \right)^2 \right] \left. \right\} \\ &= L_{xyt}^{-1} \left\{ \frac{2\pi^2}{2\pi^2 s^2 - p^2 - k^2} L_{xyt} [-\sin\{\cos(\pi x) \cos(\pi y) \right. \\ &\quad \cdot \cos(t)\} \cos^2(\pi x) \cos^2(\pi y) \sin^2(t) - \cos\{\cos(\pi x) \\ &\quad \cdot \cos(\pi y) \cos(t)\} \cos(\pi x) \cos(\pi y) \cos(t) \\ &\quad + \sin\{\cos(\pi x) \cos(\pi y) \cos(t)\} \cos^2(\pi x) \cos^2(\pi y) \\ &\quad \cdot \sin^2(t) + \cos\{\cos(\pi x) \cos(\pi y) \cos(t)\} \\ &\quad \left. \cdot \cos(\pi x) \cos(\pi y) \cos(t) \right\} = 0. \end{aligned} \tag{83}$$

Similarly,

$$\begin{aligned} N\left(\frac{\partial^2}{\partial x^2} u_0(x, y, t)\right) &= N\left(\frac{\partial^2}{\partial y^2} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial x \partial y} u_0(x, y, t)\right) \\ &= N\left(\frac{\partial^2}{\partial x \partial t} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial y \partial t} u_0(x, y, t)\right) = 0, \end{aligned} \tag{84}$$

Therefore,  $\|N(u_0''(x, y, t))\| = \|0\| = 0 < (1/e)$ ,

where  $N''(u_0(x, y, t))$  represents all the second order partial derivatives of  $u_0(x, y, t)$ .

Similarly, by principle of Mathematical induction, we have  $\|N^{(3)}(u_0(x, y, t))\| = \|N^{(4)}(u_0(x, y, t))\| = \dots = \|N^{(k)}(u_0(x, y, t))\| = 0 < (1/e)$ , for all  $k \geq 0$ .

As the condition of Theorem 11 are satisfied, the solution series  $u(x, y, t) = \sum_{i=0}^{\infty} u_i(x, y, t)$  is absolutely convergent, and hence, the solution obtained by the new iterative method is convergent on the domain of interest.

*Example 4.* Consider Equation (1) with  $\beta = 1, \alpha = 1, \phi(x, y) = 2, h(x, y, t) = 2 \sin[e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] - \pi^2 e^{-t}[\cos(\pi x) + \cos(\pi y) - 2 \cos(\pi x) \cos(\pi y)]$  on the domain  $\Omega = [0, 2]^2, t \geq 0$ . That is,

$$\begin{aligned} u_{tt}(x, y, t) + u_t(x, y, t) &= (u_{xx}(x, y, t) + u_{yy}(x, y, t) - 2 \sin \\ &\quad \cdot (u(x, y, t)) + 2 \sin[e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \\ &\quad - \pi^2 e^{-t}[\cos(\pi x) + \cos(\pi y) - 2 \cos(\pi x) \cos(\pi y)]), \end{aligned} \tag{85}$$

with initial conditions

$$u(x, y, 0) = (1 - \cos(\pi x))(1 - \cos(\pi y)), u_t(x, y, 0) = -(1 - \cos(\pi x))(1 - \cos(\pi y)), \tag{86}$$

and boundary conditions

$$u(0, y, t) = 0, u_x(0, y, t) = 0, u(x, 0, t) = 0, u_y(x, 0, t) = 0. \tag{87}$$

Solution: Applying properties of triple Laplace transform on both sides of Equation (85), we obtain

$$\begin{aligned} & (s^2 \bar{u}(k, p, s) - s \bar{u}(k, p, 0) - \bar{u}_t(k, p, 0) + s \bar{u}(k, p, s) - \bar{u}(k, p, 0) \\ & - p^2 \bar{u}(k, p, s) + p \bar{u}(k, 0, s) + \bar{u}_y(k, 0, s) - k^2 \bar{u}(k, p, s) \\ & + k \bar{u}(0, p, s) + \bar{u}_x(0, p, s) + 2L_{xyt}(\sin(u(x, y, t))) \\ & = (2L_{xyt}\{\sin[e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))]\}) \\ & - \frac{\pi^2}{s+1} \left[ \frac{k}{k^2 + \pi^2} + \frac{p}{p^2 + \pi^2} - \frac{2kp}{(k^2 + \pi^2)(p^2 + \pi^2)} \right] \end{aligned} \tag{88}$$

Applying double Laplace transform to Equations (86) and (87), we obtain

$$\begin{aligned} \bar{u}(k, p, 0) &= \frac{1}{s} - \frac{k}{k^2 + \pi^2} - \frac{p}{p^2 + \pi^2} + \frac{kp}{(k^2 + \pi^2)(p^2 + \pi^2)}, \\ \bar{u}_t(k, p, 0) &= -\left( \frac{1}{s} - \frac{k}{k^2 + \pi^2} - \frac{p}{p^2 + \pi^2} + \frac{kp}{(k^2 + \pi^2)(p^2 + \pi^2)} \right), \end{aligned} \tag{89}$$

and

$$\bar{u}(k, 0, s) = 0, \bar{u}_y(k, 0, s) = 0, \bar{u}(0, p, s) = 0, \bar{u}_x(0, p, s) = 0, \tag{90}$$

respectively.

By substituting Equations (89) and (90) into Equation (88) and simplifying, we get

$$\begin{aligned} \bar{u}(k, p, s) &= \left( \frac{1}{(s+1)} \left[ \frac{1}{s} - \frac{k}{k^2 + \pi^2} - \frac{p}{p^2 + \pi^2} + \frac{kp}{(k^2 + \pi^2)(p^2 + \pi^2)} \right] \right. \\ &+ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt}[\sin[e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \\ &\left. - \sin(u(x, y, t))] \right). \end{aligned} \tag{91}$$

Applying inverse triple Laplace transform to Equation (91), we obtain

$$\begin{aligned} u(x, y, t) &= (e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)) + L_{xyt}^{-1} \\ &\cdot \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt}[\sin[e^{-t}(1 - \cos(\pi x)) \right. \\ &\cdot (1 - \cos(\pi y))] - \sin(u(x, y, t))] \left. \right\}). \end{aligned} \tag{92}$$

Now, we apply the iteration process.

Substituting Equation (16) into Equation (92), we obtain the components of the solution as follows:

$$\begin{aligned} u_0(x, y, t) &= e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)), \\ u_1(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt}[\sin[e^{-t}(1 - \cos(\pi x)) \right. \\ &\cdot (1 - \cos(\pi y))] - \sin(u_0(x, y, t))] \left. \right\}, \\ &= L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt}[\sin[e^{-t}(1 - \cos(\pi x)) \right. \\ &\cdot (1 - \cos(\pi y))] - \sin[e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \left. \right\} = 0, \end{aligned}$$

$$\begin{aligned} u_2(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt}[\sin(u_0(x, y, t)) \right. \\ &\left. + u_1(x, y, t) - \sin(u_0(x, y, t))] \right\} = 0, \end{aligned}$$

$$\begin{aligned} u_3(x, y, t) &= L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt}[\sin(u_0(x, y, t)) \right. \\ &\left. + u_1(x, y, t) + u_2(x, y, t) - \sin(u_0(x, y, t)) \right. \\ &\left. + u_1(x, y, t)] \right\} = 0, \end{aligned} \tag{93}$$

and so on.

Therefore, the solution of Example 4 by using Equation (33) is

$$u(x, y, t) = e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)). \tag{94}$$

This result shows an excellent agreement with the one obtained in [12, 40, 41].

Now, we test the convergence of the obtained series solution. From Equation (61), we have

$$u_0(x, y, t) = e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)),$$

$$N(u(x, y, t)) = L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt} \cdot [\sin [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] - \sin(u(x, y, t))] \right\}. \tag{95}$$

Thus, for all  $x, y, t \geq 0$ , we have

$$N(u_0(x, y, t)) = L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt} \cdot [\sin [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] - \sin(u_0(x, y, t))] \right\} \\ = L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt} \cdot [\sin [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] - \sin [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] ] \right\} = 0. \tag{96}$$

Therefore,  $\|N(u_0(x, y, t))\| = \|0\| = 0 < (1/e)$ .

$$N\left(\frac{\partial}{\partial t} u(x, y, t)\right) = L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt} \cdot [-\cos [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \cdot e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)) - \cos(u(x, y, t)) \cdot \frac{\partial}{\partial t} u(x, y, t)] \right\},$$

$$N\left(\frac{\partial}{\partial t} u_0(x, y, t)\right) = L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt} \cdot [-\cos [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \cdot e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)) + \cos [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \cdot e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \right\} = 0. \tag{97}$$

Similarly,  $N((\partial/\partial x)u_0(x, y, t)) = N((\partial/\partial y)u_0(x, y, t)) = 0$

Therefore,  $\|N(u_0'(x, y, t))\| = \|0\| = 0 < (1/e)$ .

$$N\left(\frac{\partial^2}{\partial t^2} u_0(x, y, t)\right) = L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt} \cdot [\sin [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \cdot e^{-2t}[(1 - \cos(\pi x))^2(1 - \cos(\pi y))^2 - \cos [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)) - (\cos(u_0(x, y, t)) \cdot \frac{\partial^2}{\partial t^2} u_0(x, y, t) - \sin(u(x, y, t)) \left(\frac{\partial}{\partial t} u_0(x, y, t)\right)^2] ] \right\}$$

$$= L_{xyt}^{-1} \left\{ \frac{2}{(s^2 + s - p^2 - k^2)} L_{xyt} [[e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \cdot e^{-2t}[(1 - \cos(\pi x))^2(1 - \cos(\pi y))^2 - \cos [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \cdot (1 - \cos(\pi y))e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y)) - [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] e^{-2t}[(1 - \cos(\pi x))^2] \cdot [(1 - \cos(\pi y))^2 + \cos [e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] \cdot e^{-t}(1 - \cos(\pi x))(1 - \cos(\pi y))] ] \right\} = 0. \tag{98}$$

Similarly,

$$N\left(\frac{\partial^2}{\partial x^2} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial y^2} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial x \partial y} u_0(x, y, t)\right) \\ = N\left(\frac{\partial^2}{\partial x \partial t} u_0(x, y, t)\right) = N\left(\frac{\partial^2}{\partial y \partial t} u_0(x, y, t)\right) = 0. \tag{99}$$

Therefore,  $\|N(u_0''(x, y, t))\| = \|0\| = 0 < (1/e)$ , where  $N''(u_0(x, y, t))$  represents all the second order partial derivatives of  $u_0(x, y, t)$ .

Similarly, by principle of Mathematical induction, we have  $\|N^{(3)}(u_0(x, y, t))\| = \|N^{(4)}(u_0(x, y, t))\| = \dots = \|N^{(k)}(u_0(x, y, t))\| = 0 < (1/e)$ , for all  $k \geq 0$ .

As the condition of Theorem 11 are satisfied, the series solution  $u(x, y, t) = \sum_{i=0}^{\infty} u_i(x, y, t)$  is absolutely convergent, and hence, the solution obtained by the new iterative method is convergent on the domain of interest.

### 6. Conclusion

In this paper, triple Laplace transform coupled with iterative method is applied to obtain exact solution of two-dimensional nonlinear Sine-Gordon equation subject to initial and boundary conditions. Four illustrative examples are presented to show the validity of the method under consideration. The solutions of Examples 1, 2, 3, and 4 obtained by the proposed method are in an excellent agreement with the same problem that has been considered in [6, 10–12, 16, 40, 41], and further, nontrivial problems that are solved using earlier methods become trivial in the sense that the decomposition  $u(x, y, t) = \sum_{n=0}^{\infty} u_n(x, y, t) = u_0(x, y, t) + u_1(x, y, t) + u_2(x, y, t) + \dots + u_m(x, y, t) + \dots$  consists of only one term, i.e.,  $u(x, y, t) = u_0(x, y, t)$ . From this study, we concluded that triple Laplace transform coupled with iterative method finds quite practical analytical results with less computational work.

### Data Availability

There are no data used in this research study.

## Conflicts of Interest

The authors declare that there is no conflict of interest.

## Authors' Contributions

The authors equally contributed and approved the final manuscript for submission.

## Acknowledgments

The authors are grateful to thank Jimma University, College of Natural Sciences, and Department of Mathematics, for providing the necessary resources during conducting this research.

## References

- [1] R. K. Dodd, J. C. Eilbeck, J. D. Gibbon, and H. C. Morris, *Solitons and Nonlinear Wave Equations*, Academic, London, UK, 1982.
- [2] A. Barone, F. Esposito, C. J. Magee, and A. C. Scott, "Theory and applications of the sine-Gordon equation," *La Rivista del Nuovo Cimento*, vol. 1, no. 2, pp. 227–267, 1971.
- [3] A. M. Wazwaz, *Partial Differential Equations and Solitary Waves Theory*, Springer, Berlin, Germany, 2009.
- [4] W. G. Belayeh, Y. O. Mussa, and A. K. Gizaw, "Approximate analytic solutions of two-dimensional nonlinear Klein-Gordon equation by using the reduced differential transform method," *Mathematical Problems in Engineering*, vol. 2020, Article ID 5753974, 12 pages, 2020.
- [5] S. Maitama and Y. F. Hamza, "An analytical method for solving nonlinear sine-Gordon equation," *Sohag Journal of Mathematics*, vol. 7, no. 1, pp. 5–10, 2020.
- [6] L. Su, "Numerical solution of two-dimensional nonlinear sine-Gordon equation using localized method of approximate particular solutions," *Engineering Analysis with boundary Elements*, vol. 108, pp. 95–107, 2019.
- [7] M. Dehghan, M. Abbaszadeh, and A. Mohebbi, "An implicit RBF meshless approach for solving the time fractional nonlinear sine-Gordon and Klein-Gordon equations," *Engineering Analysis with Boundary Elements*, vol. 50, pp. 412–434, 2015.
- [8] M. Dehghan and A. Ghesmati, "Numerical simulation of two-dimensional sine-Gordon solitons via a local weak meshless technique based on the radial point interpolation method (RPIM)," *Computer Physics Communications*, vol. 181, no. 4, pp. 772–786, 2010.
- [9] A. A. Fayadh and D. S. Faraj, "Combined Laplace transform – variational iteration method for sine-Gordon equation," *Department of Mathematics and Computer Applications*, vol. 31, no. 1, pp. 61–64, 2019.
- [10] M. Baccouch, "Optimal error estimates of the local discontinuous Galerkin method the two dimensional sine-Gordon equation on Cartesian grids," *International Journal of Numerical Analysis and Modeling*, vol. 16, no. 3, pp. 436–462, 2019.
- [11] Yali Duan, Linghua Kong, Xianjin Chen, and Min Guo, "Lattice Boltzmann model for two-dimensional generalized sine-Gordon equation," *Journal of Applied Analysis and Computation*, vol. 8, no. 6, pp. 1645–1663, 2018.
- [12] D. Li, H. Lai, and B. Shi, "Mesoscopic simulation of the (2 + 1)-dimensional wave equation with nonlinear damping and source terms using the lattice Boltzmann BGK model," *Entropy*, vol. 21, no. 4, p. 390, 2019.
- [13] P. Guo, A. Boldbaatar, D. Yi, and P. Dai, "Numerical solution of sine-Gordon equation with the local kriging meshless method," *Mathematical Problems in Engineering*, vol. 2020, Article ID 9057387, 11 pages, 2020.
- [14] W. Wang, K. Yang, H. Gu, F. Guan, and A. H. Kara, "A (2+1)-dimensional sine-Gordon and sinh-Gordon equations with symmetries and kink wave solutions," *Nuclear Physics B*, vol. 953, article 114956, 2020.
- [15] Y. Keskin, I. Caglar, and B. A. Koc, "Numerical solution of sine-Gordon equation by reduced differential transform method," in *Proceedings of the World Congress on Engineering*, pp. 109–113, London, UK, 2011.
- [16] X. Kang, W. Feng, K. Cheng, and C. Guo, "An efficient finite difference scheme for the 2D sine-Gordon equation," *Journal of Nonlinear Sciences and Applications*, vol. 10, no. 6, pp. 2998–3012, 2017.
- [17] R. J. Cheng and K. M. Liew, "Analyzing two-dimensional sine-Gordon equation with the mesh-free reproducing kernel particle Ritz method," *Computer Methods in Applied Mechanics and Engineering*, vol. 245–246, pp. 132–143, 2012.
- [18] Y. Ji, "Meshless singular boundary method for nonlinear sine-Gordon equation," *Mathematical Problems in Engineering*, vol. 2018, Article ID 6460480, 11 pages, 2018.
- [19] R. Jiwari, S. Pandit, and R. C. Mittal, "Numerical simulation of two-dimensional sine-Gordon solitons by differential quadrature method," *Computer Physics Communications*, vol. 183, no. 3, pp. 600–616, 2012.
- [20] W. X. Chen and J. Lin, "Some new exact solutions of (1+2)-dimensional sine-Gordon equation," *Abstract and Applied Analysis*, vol. 2014, Article ID 645456, 8 pages, 2014.
- [21] M. Hazewinkel, *Integral Transform First Edition*, Media B.V, 2001.
- [22] A. Babakhani and R. S. Dahiya, "Systems of multi-dimensional Laplace transforms and a heat equation," in *Proceedings of the 16th Conference on Applied Mathematics*, pp. 25–36, Edmond, Okla, USA, 2001.
- [23] Y. A. Brychkov, H. J. Glaeske, A. P. Prudnikov, and V. K. Tuan, *Multidimensional Integral Transformations*, Gordon and Breach Science Publishers, Philadelphia, Pa, USA, 1992.
- [24] A. Atangana and A. Kilicman, "A possible generalization of acoustic wave equation using the concept of perturbed derivative order," *Mathematical Problems in Engineering*, vol. 2013, Article ID 696597, 6 pages, 2013.
- [25] A. Atangana, "On multi-Laplace transform for solving nonlinear partial differential equations with mixed derivatives," *Mathematical Problems in Engineering*, vol. 2014, Article ID 267843, 9 pages, 2014.
- [26] L. Death, "The double Laplace transforms and their properties with applications to functional, integral and partial differential equations," *International Journal of Applied and Computational Mathematics*, vol. 2, no. 2, pp. 223–241, 2016.
- [27] H. Eltayeb and A. Kiliçman, "A note on double Laplace transform and telegraphic equations," *Abstract and Applied Analysis*, vol. 2013, Article ID 932578, 6 pages, 2013.
- [28] S. K. Elagan, M. Sayed, and M. Higazy, "An analytical study on fractional partial differential equations by Laplace transform operator method," *International Journal of Applied Engineering Research*, vol. 13, no. 1, pp. 545–549, 2018.

- [29] R. R. Dhunde and G. L. Waghmare, "Applications of double Laplace transform to boundary value problems," *IOSR Journal of Mathematics*, vol. 9, no. 2, pp. 57–60, 2013.
- [30] A. Khan, K. Khan, T. Khan, and G. Zaman, "Extension of triple Laplace transform for solving fractional differential equations," *Discrete and Continuous Dynamical Systems Series*, vol. 13, no. 3, pp. 755–768, 2020.
- [31] A. K. Thakur and S. Panda, "Some properties of triple Laplace transform," *Journal of Mathematics and Computer Applications Research*, vol. 2250-2408, 2015.
- [32] A. Mousa and T. M. Elzaki, "Solution of volterra integro-differential equations by triple Laplace transform," *Irish Interdisciplinary Journal of Science & Research*, vol. 3, no. 4, pp. 67–72, 2019.
- [33] A. K. Thakur, A. Kumar, and H. Suryavanshi, "The triple Laplace transforms and their properties," *International Journal of Applied Mathematics Statistical Sciences*, vol. 7, no. 4, pp. 23–34, 2018.
- [34] A. Atangana, "A note on the triple Laplace transform and its applications to some kind of third-order differential equation," *Abstract and Applied Analysis*, vol. 2013, Article ID 769102, 10 pages, 2013.
- [35] S. Bhalekar and V. Daftardar-Gejji, "Convergence of the new iterative method," *International Journal of Differential Equations*, vol. 2011, Article ID 989065, 10 pages, 2011.
- [36] M. A. AL-Jawary, G. H. Radhi, and J. Ravnik, "Daftardar-Jafari method for solving nonlinear thin film flow problem," *Arab Journal of Basic and Applied Sciences*, vol. 25, no. 1, pp. 20–27, 2018.
- [37] R. R. Dhunde and G. L. Waghmare, "Analytical solution of the non-linear Klein-Gordon equation using double Laplace transform and iterative method," *American Journal of Computational and Applied Mathematics*, vol. 6, no. 6, pp. 195–201, 2016.
- [38] V. Daftardar-Gejji and H. Jafari, "An iterative method for solving nonlinear functional equations," *Journal of Mathematical Analysis and Applications*, vol. 316, no. 2, pp. 753–763, 2006.
- [39] R. R. Dhunde and G. L. Waghmare, "Double Laplace transform combined with iterative method for solving non-linear telegraph equation," *Journal of the Indian Mathematical Society*, vol. 83, no. 3-4, pp. 221–230, 2016.
- [40] W. Liu, J. Sun, and B. Wu, "Space-time spectral method for the two-dimensional generalized sine-Gordon equation," *Journal of Mathematical Analysis and Applications*, vol. 427, no. 2, pp. 787–804, 2015.
- [41] A. T. Deresse, Y. O. Mussa, and A. K. Gizaw, "Analytic solution of two-dimensional sine-Gordon equation," *Advances in Mathematical Physics*, vol. 2021, Article ID 6610021, 15 pages, 2021.