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Research Article

Theoretical Analysis of an Imprecise Prey-Predator Model with Harvesting and Optimal Control

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In our present paper, we formulate and study a prey-predator system with imprecise values for the parameters. We also consider harvesting for both the prey and predator species. Then we describe the complex dynamics of the proposed model system including positivity and uniform boundedness of the system, and existence and stability criteria of various equilibrium points. Also the existence of bionomic equilibrium and optimal harvesting policy are thoroughly investigated. Some numerical simulations have been presented in support of theoretical works. Further the requirement of considering imprecise values for the set of model parameters is also highlighted.

1. Introduction

The eternal relationship between prey and predators is one of the major topics to be discussed in recent science. Scientists from various fields are currently engaged in finding out different interactions among prey populations and predator populations. With the help of mathematical modeling, one can describe the strong and competitive relationship between these two types of creatures. However the discussion on this fascinating topic, with the help of some mathematical tool, was started during the first quarter of the twentieth century, thanks to the age-breaking works of Lotka [1] and Volterra [2]. Influenced by those works, researchers are still engaged in theoretical study of the ecological system with the help of mathematical modeling. In their book, Kot [3], Britton (2003) described some ecological phenomena on various ecological interactions including prey-predator interactions. Smith [4] has demonstrated various aspects in theoretical ecology with the help of some basic mathematical models. May [5] has also considered and analyzed some other types of ecological systems including prey-predator dynamics, with the help of some sophisticated mathematical models, which are comparatively complex in nature. Some other research works on theoretical ecology including predator-prey dynamics can be found, like Cushing [6], Hadeler and Freedman [7], Chen et al. [8], Kar [9, 10], Kar et al. [11], Chakraborty et al. [12], and references therein.

Harvesting is however a common and quite natural phenomenon. In fishery harvesting is used frequently as the biological resources are mostly renewable resources. On an exploited fishery system with interacting prey and predator species, researchers are considering harvesting on either prey species or predator species or harvesting on both prey and predator species. Martin and Ruan [13] discussed the dynamics of harvesting of prey populations whereas, in his article, Kar [9] describes phenomenon on selective harvesting on a prey-predator system. Further harvesting in predator species or both of prey and predator species can be found in literature also (Kar and Pahari [14], Zhang and Zhang [15], Jana et al. [16–18], Pal et al. [19], Walters et al. [20], Liu and Zhang [21], etc.). From a bioeconomic point of view, harvesting on a species should be in a balance to both keep the resource live and keep fishermen in profitable mode. In his two books Clark [22, 23] describes different harvesting policies in some realistic ecological systems with optimal outcome.

In this regard, in our present article, we consider a prey-predator type ecological system with harvesting on both the species. However, till now most of the models are proposed by considering only precise set of parameters but the natural world may not be precise every time. In many situations at experimental field like birth and death rate of different individuals of the same species, interaction between two different species, etc., may be imprecise. For this purpose introduction of fuzzy sets (Zadeh [24]) is now considered as a revolutionary work. However, consideration of interval-valued parameters was due to the broad application of fuzzy sets. The imprecise parameters set may not always belong to the interval $[0, 1]$ but they may belong to any interval of positive number. Hence interval-valued parameter set of an imprecise mathematical model would be regardlessly better from a realistic point of view.

The rest of the paper is organized in the following manner: In Section 2, we put some preliminaries on interval value numbers. In Section 3, on the basis of some realistic assumptions we formulate our predator-prey system and then convert it to an imprecise parametric system whose dynamical behavior is thoroughly discussed in Section 4. Section 5 is devoted to discussing the existence of bionomic equilibrium, and optimal harvesting policies are studied in Section 6 keeping harvesting parameter as the control variable. In Section 7, we validate our theoretical results through some numerical simulation works, and in the last section we present some key findings.

2. Preliminaries

Here we give the definition of interval numbers with some operations. We use interval-valued function in lieu of interval number.

Definition 1 (interval number). We denote interval number A as $[\underline{a}, \overline{a}]$ and define it as $A = [\underline{a}, \overline{a}] = \{x : \underline{a} \le x \le \overline{a}, x \in \mathfrak{R}\}$ where $\mathfrak{R}$ is called the set of all real numbers and $\underline{a}, \overline{a}$ are the lower and upper limits of the interval number, respectively.

A real number a can also be used in form of interval number as $[a, a]$.

The basic operations between any two interval numbers are as follows:

(i) $[\underline{a_1}, \overline{a_1}] + [\underline{a_2}, \overline{a_2}] = [\underline{a_1} + \underline{a_2}, \overline{a_1} + \overline{a_2}]$.

(ii) $[\underline{a_1}, \overline{a_1}] - [\underline{a_2}, \overline{a_2}] = [\underline{a_1} - \underline{a_2}, \overline{a_1} - \overline{a_2}]$.

(iii) $c[\underline{a_1}, \overline{a_1}] = [c\underline{a_2}, c\overline{a_2}]$ where c is a real number.

(iv) $[\underline{a_1}, \overline{a_1}].[\underline{a_2}, \overline{a_2}] = [\min\{\underline{a_1 a_2}, \overline{a_1}\underline{a_2}, \underline{a_1}\overline{a_2}, \overline{a_1 a_2}\}, \max\{\underline{a_1 a_2}, \overline{a_1}\underline{a_2}, \underline{a_1}\overline{a_2}, \overline{a_1 a_2}\}]$.

(v) $[\underline{a_1}, \overline{a_1}]/[\underline{a_2}, \overline{a_2}] = [\underline{a_1}, \overline{a_1}].[1/\overline{a_2}, 1/\underline{a_2}]$.

Definition 2 (interval-valued function). For an interval $[a, b]$ the interval-valued function can be created as $f(p) = a^{(1-p)} b^p$ for $p \in [0, 1]$.

3. Predator-Prey Model with Harvesting with Imprecise Parameter

3.1. Crisp Model. In this article, we consider only two species, namely, prey species and predator species. Let $x(t)$ denote the prey biomass and $y(t)$ the predator class at any time t. Let the prey population grow logistically with intrinsic growth rate r and environmental carrying capacity k. Also let the predator attack the prey in the predation rate $\alpha(> 0)$ following the mass action law. Thus the differential equation for the prey population becomes

$$\frac{dx(t)}{dt} = rx\left(1 - \frac{x}{k}\right) - \alpha xy. \tag{1}$$

Let $m(> 0)$ be the conversion factor from the prey population to the matured predator population, d be the natural death rate, and δ be the intraspecific competition rate for the predator populations (due to Ruan et al. [25]). Then the differential equation of the predator population $y(t)$ reduces to

$$\frac{dy(t)}{dt} = m\alpha xy - dy - \delta y^2. \tag{2}$$

Here $m, r, k, \alpha, \delta, d$ are all positive parameters.

Next if we consider that both the species are harvested, this is carried out on assuming the demand in the market of both species (prey and predator). Taking E as the harvesting effort for both species and q_1 & q_2 as the catchability coefficient of the prey species and predator species, respectively, then our system reduces to

$$\begin{aligned} \frac{dx(t)}{dt} &= rx\left(1 - \frac{x}{k}\right) - \alpha xy - q_1 Ex \\ \frac{dy(t)}{dt} &= m\alpha xy - dy - \delta y^2 - q_2 Ey \end{aligned} \tag{3}$$

subject to the initial conditions

$$\begin{aligned} x(0) &\ge 0, \\ y(0) &\ge 0. \end{aligned} \tag{4}$$

3.2. Fuzzy Model. The environment and other factors including temperature and food habits caused the parameters to be imprecise. So they should be taken as interval number rather than a single value. Let $\widehat{r}, \widehat{k}, \widehat{\alpha}, \widehat{m}, \widehat{d}, \widehat{\delta}$ be the corresponding interval numbers for $r, k, \alpha, m, d, \delta$, respectively. Then the prey-predator model with combined harvesting effort E becomes

$$\begin{aligned} \frac{dx}{dt} &= \widehat{r}x\left(1 - \frac{x}{\widehat{k}}\right) - \widehat{\alpha}xy - q_1 Ex \\ \frac{dy}{dt} &= \widehat{m}\widehat{\alpha}xy - \widehat{d}y - \widehat{\delta}y^2 - q_2 Ey \end{aligned} \tag{5}$$

where $\widehat{r} = [\underline{r}, \overline{r}], \widehat{k} = [\underline{k}, \overline{k}], \widehat{\alpha} = [\underline{\alpha}, \overline{\alpha}], \widehat{m} = [\underline{m}, \overline{m}], \widehat{d} = [\underline{d}, \overline{d}]$, $\widehat{\delta} = [\underline{\delta}, \overline{\delta}]$.

For the interval number $[\underline{r}, \overline{r}]$ we consider the interval-valued function $r_p = (\underline{r})^{(1-p)}(\overline{r})^p$ for $p \in [0,1]$. Similarly taking the other interval numbers in the same way as function form, we get the model as

$$\frac{dx}{dt} = (\underline{r})^{(1-p)} (\overline{r})^p x \left(1 - \frac{x}{(\underline{k})^{(1-p)} (\overline{k})^p}\right) - (\underline{\alpha})^{(1-p)} (\overline{\alpha})^p xy - q_1 Ex$$
$$\frac{dy}{dt} = (\underline{m})^{(1-p)} (\overline{m})^p (\underline{\alpha})^{(1-p)} (\overline{\alpha})^p xy - (\underline{d})^{(1-p)} (\overline{d})^p y - (\underline{\delta})^{(1-p)} (\overline{\delta})^p y^2 - q_2 Ey \quad (6)$$

subject to the initial conditions

$$x(0) \geq 0, \quad y(0) \geq 0. \quad (7)$$

Here $p \in [0,1]$, where the value of p depends on the underlying environment.

4. Dynamical Behavior

In this section, we describe a thorough dynamical behavior of the proposed model system. To do so, we first check the positivity of the solutions of crisp system and uniform boundedness of the solution of the same system. Now, it can also be concluded that uniform boundedness and positivity in the solutions also hold for the corresponding fuzzy systems, if these things hold in crisp system.

4.1. Positivity. First we consider the corresponding crisp system in following form.

$$\frac{dx}{x} = \left[r\left(1 - \frac{x}{k}\right) - \alpha y - q_1 E\right] dt$$
$$\frac{dy}{y} = \left[\widehat{m}\widehat{\alpha}x - \widehat{d} - \widehat{\delta}y - q_2 E\right] dt \quad (8)$$

Now, on integration, we have, from above system of equations,

$$x(t) = x(0) \exp\left(\left[\widehat{r}\left(1 - \frac{x}{\widehat{k}}\right) - \widehat{\alpha}y - q_1\right] E\right) \quad (\geq 0) \quad (9)$$

and

$$y(t) = y(0) \exp\left(\left[\widehat{m}\widehat{\alpha}x - \widehat{d} - \widehat{\delta}y - q_2 E\right]\right) \quad (\geq 0). \quad (10)$$

Hence from above, two expressions related to two state variables will always be positive. Thus the solution of corresponding crisp problem will be nonnegative and so the solution of the corresponding fuzzy system will also be nonnegative.

4.2. Uniform Boundedness. In this section we now study the uniform boundedness of the proposed imprecise system. Now from the first expression of system (5), we have

$$\frac{dx}{dt} + q_1 Ex \leq \widehat{r}x\left(1 - \frac{x}{\widehat{k}}\right). \quad (11)$$

Now, by simple mathematics, it can be concluded that $\widehat{r}x(1 - x/\widehat{k})$ has maximum value $\widehat{r}\widehat{k}/4$, which is obtained for $x = \widehat{k}/2$. Thus from above, we have

$$\frac{dx}{dt} + q_1 Ex \leq \frac{\widehat{r}\widehat{k}}{4}. \quad (12)$$

Now Integrating both sides of the above inequality and then applying the theory of differential inequality due to (see Birkhoff and Rota [26]), we have

$$0 < x(t) \leq \frac{\widehat{r}\widehat{k}}{4q_1 E}\left(1 - e^{-q_1 Et}\right) + x(0). \quad (13)$$

Now on letting $t \longrightarrow \infty$, we have

$$0 < x(t) \leq \frac{\widehat{r}\widehat{k}}{4q_1 E} + \epsilon_1. \quad (14)$$

Hence the biomass density of prey population $x(t)$ is uniformly bounded with an upper and lower limit $\widehat{r}\widehat{k}/4q_1 E + \epsilon_1$ and 0, respectively.

Next we are targeting to show that the biomass of predator population $y(t)$ is uniformly bounded. In this regard from (5), we have

$$\frac{dy}{dt} + q_2 Ey \leq \widehat{m}\widehat{\alpha}\left(\frac{\widehat{r}\widehat{k}}{4q_1 E} + \epsilon_1\right) y - \widehat{\delta}y^2. \quad (15)$$

Thus similarly to the above, it is to be claimed that the right hand side of the above expression has maximum value at $y = (\widehat{m}\widehat{\alpha}/2\widehat{\delta})(\widehat{r}\widehat{k}/4q_1 E + \epsilon_1)$ and this maximum value is $((\widehat{m}\widehat{\alpha})^2/4\widehat{\delta})(\widehat{r}\widehat{k}/4q_1 E + \epsilon_1)^2$.

Thus proceeding in the same way as prey populations and with the help of Birkhoff and Rota [26], we can write

$$0 < y(t) \leq \frac{(\widehat{m}\widehat{\alpha})^2}{4q_2 E\widehat{\delta}}\left(\frac{\widehat{r}\widehat{k}}{4q_1 E} + \epsilon_1\right)^2 \left(1 - e^{-q_2 Et}\right) + y(0). \quad (16)$$

Now on letting $t \longrightarrow \infty$, we have

$$0 < y(t) \leq \frac{(\widehat{m}\widehat{\alpha})^2}{4q_2 E\widehat{\delta}}\left(\frac{\widehat{r}\widehat{k}}{4q_1 E} + \epsilon_1\right)^2 + \epsilon_2. \quad (17)$$

So the biomass density of predator populations y is also uniformly bounded with lower and upper bound, respectively, 0 and $((\widehat{m}\widehat{\alpha})^2/4q_2 E\widehat{\delta})(\widehat{r}\widehat{k}/4q_1 E + \epsilon_1)^2 + \epsilon_2$.

Hence the biomass density of both the population species is uniformly bounded.

4.3. Existence of Equilibria. The equilibrium points of this system are given below.

(1) Trivial equilibrium: $E_T(0,0)$.

(2) Axial equilibrium: $E_A(x_1,0)$ [where $x_1 = (\underline{k})^{(1-p)}(\overline{k})^p(1-q_1E/(\underline{r})^{(1-p)}(\overline{r})^p)$] exists if $(\underline{r})^{(1-p)}(\overline{r})^p > q_1E$.

(3) Interior equilibrium: $E_I(x^*, y^*)$ where

$$x^* = \frac{(\underline{k})^{(1-p)}(\overline{k})^p\left\{(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p(\underline{d})^{(1-p)}(\overline{d})^p + (\underline{r})^{(1-p)}(\overline{r})^p(\underline{\delta})^{(1-p)}(\overline{\delta})^p + (\underline{\alpha})^{(1-p)}(\overline{\alpha})^p q_2E - (\underline{\delta})^{(1-p)}(\overline{\delta})^p q_1E\right\}}{(\underline{r})^{(1-p)}(\overline{r})^p(\underline{\delta})^{(1-p)}(\overline{\delta})^p + (\underline{m})^{(1-p)}(\overline{m})^p(\underline{k})^{(1-p)}(\overline{k})^p(\underline{\alpha})^{2(1-p)}(\overline{\alpha})^{2p}}$$

$$y^* = \frac{(\underline{m})^{(1-p)}(\overline{m})^p(\underline{k})^{(1-p)}(\overline{k})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p\left\{(\underline{r})^{(1-p)}(\overline{r})^p - q_1E\right\} - (\underline{r})^{(1-p)}(\overline{r})^p\left\{(\underline{d})^{(1-p)}(\overline{d})^p + q_2E\right\}}{(\underline{r})^{(1-p)}(\overline{r})^p(\underline{\delta})^{(1-p)}(\overline{\delta})^p + (\underline{m})^{(1-p)}(\overline{m})^p(\underline{k})^{(1-p)}(\overline{k})^p(\underline{\alpha})^{2(1-p)}(\overline{\alpha})^{2p}}. \quad (18)$$

The interior equilibrium exists if

$$(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p(\underline{d})^{(1-p)}(\overline{d})^p + (\underline{r})^{(1-p)}(\overline{r})^p(\underline{\delta})^{(1-p)}(\overline{\delta})^p + (\underline{\alpha})^{(1-p)}(\overline{\alpha})^p q_2E > (\underline{\delta})^{(1-p)}(\overline{\delta})^p q_1E \quad (19)$$

and

$$(\underline{r})^{(1-p)}(\overline{r})^p(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p > r\left\{(\underline{d})^{(1-p)}(\overline{d})^p + q_2E\right\} + (\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p q_1E \quad (20)$$

hold if

$$E < \min(E_1, E_2) \quad (21)$$

where

$$E_1 = \frac{(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p(\underline{d})^{(1-p)}(\overline{d})^p + (\underline{r})^{(1-p)}(\overline{r})^p(\underline{\delta})^{(1-p)}(\overline{\delta})^p}{(\underline{\delta})^{(1-p)}(\overline{\delta})^p q_1 - (\underline{\alpha})^{(1-p)}(\overline{\alpha})^p q_2} \quad (22)$$

and

$$E_2 = \frac{(\underline{r})^{(1-p)}(\overline{r})^p\left\{(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - (\underline{d})^{(1-p)}(\overline{d})^p\right\}}{(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p q_1 + (\underline{r})^{(1-p)}(\overline{r})^p q_2}. \quad (23)$$

4.4. Local Asymptotic Stability. In this section we state and prove the local asymptotic stability criteria at different equilibrium points. Also the corresponding conditions for which the system is stable at different equilibria are given below.

Case 1. For trivial equilibrium the variational matrix at $E_T(0,0)$ is given by the following.

$$V(E_T) = \begin{pmatrix} (\underline{r})^{(1-p)}(\overline{r})^p - q_1E & 0 \\ 0 & -(\underline{d})^{(1-p)}(\overline{d})^p - q_2E \end{pmatrix} \quad (24)$$

Therefore, the eigenvalues are given by $\lambda_1 = (\underline{r})^{(1-p)}(\overline{r})^p - q_1E$, $\lambda_2 = -(\underline{d})^{(1-p)}(\overline{d})^p - q_2E$.

Here $\lambda_2 < 0$; then $E_T(0,0)$ is asymptotically stable if $\lambda_1 < 0$, i.e., if $(\underline{r})^{(1-p)}(\overline{r})^p - q_1E < 0$ which implies $E > (1/q_1)(\underline{r})^{(1-p)}(\overline{r})^p$.

In the next theorem, we state the stability criteria of trivial equilibrium point

Theorem 3. *Trivial equilibrium point $E_T(0,0)$ of the system is locally asymptotically stable if $E > (1/q_1)(\underline{r})^{(1-p)}(\overline{r})^p$ holds.*

Case 2. At axial equilibrium $E_A(x_1,0)$ the variational matrix is

$$V(E_A) = \begin{pmatrix} E_{A11} & E_{A12} \\ E_{A21} & E_{A22} \end{pmatrix} \quad (25)$$

where

$$E_{A11} = q_1E - (\underline{r})^{(1-p)}(\overline{r})^p,$$
$$E_{A12} = -(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p x_1, \quad (26)$$
$$E_{A21} = 0,$$

and $E_{A22} = (\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p\{1 - q_1E/(\underline{r})^{(1-p)}(\overline{r})^p\} - (\underline{d})^{(1-p)}(\overline{d})^p - q_2E$. Then the eigenvalues of the characteristic equation of $V(E_A)$ are $q_1E - (\underline{r})^{(1-p)}(\overline{r})^p$ and $(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p\{1 - q_1E/(\underline{r})^{(1-p)}(\overline{r})^p\} - (\underline{d})^{(1-p)}(\overline{d})^p - q_2E$. The first one of them is negative since $(\underline{r})^{(1-p)}(\overline{r})^p > q_1E$. Now E_A is asymptotically stable if the second one is negative, i.e.,

$$(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p \cdot \left\{1 - \frac{q_1E}{(\underline{r})^{(1-p)}(\overline{r})^p}\right\} - (\underline{d})^{(1-p)}(\overline{d})^p - q_2E < 0 \quad (27)$$

which implies

$$E > \frac{(\underline{r})^{(1-p)}(\overline{r})^p\left\{(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - (\underline{d})^{(1-p)}(\overline{d})^p\right\}}{q_1(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + q_2(\underline{r})^{(1-p)}(\overline{r})^p}. \quad (28)$$

In the next theorem we will state the local asymptotic stability criteria of the axial equilibrium or the predator free equilibrium $E_A(x_1, 0)$.

Theorem 4. *The axial equilibrium $E_A(x_1, 0)$ is locally asymptotically stable if*

$$\frac{1}{q_1}(\underline{r})^{(1-p)}(\overline{r})^p > E > \frac{(\underline{r})^{(1-p)}(\overline{r})^p\left\{(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - (\underline{d})^{(1-p)}(\overline{d})^p\right\}}{q_1(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + q_2(\underline{r})^{(1-p)}(\overline{r})^p}. \quad (29)$$

In this condition the trivial equilibrium becomes unstable.

Case 3. The variational matrix for interior equilibrium $E_I(x^*, y^*)$ is written below.

$$V(E_I) = \begin{pmatrix} \frac{-(\underline{r})^{(1-p)}(\overline{r})^p}{(\underline{k})^{(1-p)}(\overline{k})^p}x^* & -(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p x^* \\ (\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p y^* & -(\underline{\delta})^{(1-p)}(\overline{\delta})^p y^* \end{pmatrix} \quad (30)$$

The characteristic equation of $V(E_I)$ is given by

$$\lambda^2 + S\lambda + M = 0 \quad (31)$$

where

$$S = \frac{(\underline{r})^{(1-p)}(\overline{r})^p}{(\underline{k})^{(1-p)}(\overline{k})^p}x^* + (\underline{\delta})^{(1-p)}(\overline{\delta})^p y^* \quad (32)$$

and

$$M = \left\{\frac{(\underline{r})^{(1-p)}(\overline{r})^p(\underline{\delta})^{(1-p)}(\overline{\delta})^p}{(\underline{k})^{(1-p)}(\overline{k})^p} + (\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{2(1-p)}(\overline{\alpha})^{2p}\right\}x^*y^*. \quad (33)$$

Here $S > 0$ and $M > 0$ since $x^* > 0$ and $y^* > 0$. Then the values of λ are negative.

Therefore, The system is locally asymptotically stable at (x^*, y^*) and we state this criteria in the following theorem.

Theorem 5. *The interior equilibrium $E_I(x^*, y^*)$ of the system exists and is locally asymptotically stable if*

$$E < \min(E_1, E_2), \quad (34)$$

where

$$E_1 = \frac{(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p(\underline{d})^{(1-p)}(\overline{d})^p + (\underline{r})^{(1-p)}(\overline{r})^p(\underline{\delta})^{(1-p)}(\overline{\delta})^p}{(\underline{\delta})^{(1-p)}(\overline{\delta})^p q_1 - (\underline{\alpha})^{(1-p)}(\overline{\alpha})^p q_2} \quad (35)$$

and

$$E_2 = \frac{(\underline{r})^{(1-p)}(\overline{r})^p\left\{(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - (\underline{d})^{(1-p)}(\overline{d})^p\right\}}{(\underline{k})^{(1-p)}(\overline{k})^p(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p q_1 + (\underline{r})^{(1-p)}(\overline{r})^p q_2}. \quad (36)$$

4.5. Global Stability. Here we will discuss the global asymptotic stability criteria of the system around its interior equilibrium point. In next theorem we study the criteria.

Theorem 6. *The interior equilibrium $E_I(x^*, y^*)$ of the system is globally asymptotically stable provided it is locally asymptotically stable there.*

Proof. A Lyapunov function is constructed here as follows

$$V(x, y) = \int_{x^*}^{x} \frac{x - x^*}{x} dx + P \int_{y^*}^{y} \frac{y - y^*}{y} dy \quad (37)$$

where P is suitable positive constant to be determined in the subsequent steps.

Taking derivative with respect to t along the solutions of the system, we have

$$\frac{dV}{dt} = \frac{x - x^*}{x}\frac{dx}{dt} + P\frac{y - y^*}{y}\frac{dy}{dt}. \quad (38)$$

Now

$$\begin{aligned}
\frac{1}{x}\frac{dx}{dt} &= (\underline{r})^{(1-p)} (\overline{r})^{p} \left(1 - \frac{x}{(\underline{k})^{(1-p)} (\overline{k})^{p}}\right) \\
&\quad - (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} y - q_1 E \\
\frac{1}{x}\frac{dx^*}{dt} &= (\underline{r})^{(1-p)} (\overline{r})^{p} \left(1 - \frac{x^*}{(\underline{k})^{(1-p)} (\overline{k})^{p}}\right) \\
&\quad - (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} y^* - q_1 E \\
\frac{1}{y}\frac{dy}{dt} &= (\underline{m})^{(1-p)} (\overline{m})^{p} (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} x \\
&\quad - (\underline{d})^{(1-p)} (\overline{d})^{p} - (\underline{\delta})^{(1-p)} (\overline{\delta})^{p} y - q_2 E.
\end{aligned} \quad (39)$$

Then

$$\begin{aligned}
\frac{dV}{dt} &= (x - x^*) \left[-\frac{(\underline{r})^{(1-p)} (\overline{r})^{p}}{(\underline{k})^{(1-p)} (\overline{k})^{p}} (x - x^*) \right. \\
&\quad \left. - (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} (y - y^*) \right] + P (y - y^*) \\
&\quad \cdot \left[(\underline{m})^{(1-p)} (\overline{m})^{p} (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} (x - x^*) \right. \\
&\quad \left. - (\underline{\delta})^{(1-p)} (\overline{\delta})^{p} (y - y^*) \right] = -\frac{(\underline{r})^{(1-p)} (\overline{r})^{p}}{(\underline{k})^{(1-p)} (\overline{k})^{p}} (x \\
&\quad - x^*)^2 + \left(P (\underline{m})^{(1-p)} (\overline{m})^{p} - 1\right) (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} (x \\
&\quad - x^*) (y - y^*) - P (\underline{\delta})^{(1-p)} (\overline{\delta})^{p} (y - y^*)^2.
\end{aligned} \quad (40)$$

If we consider $P = 1/m$, then dV/dt reduces to the following.

$$\begin{aligned}
\frac{dV}{dt} &= -\frac{(\underline{r})^{(1-p)} (\overline{r})^{p}}{(\underline{k})^{(1-p)} (\overline{k})^{p}} (x - x^*)^2 \\
&\quad - \frac{(\underline{\delta})^{(1-p)} (\overline{\delta})^{p}}{(\underline{m})^{(1-p)} (\overline{m})^{p}} (y - y^*)^2
\end{aligned} \quad (41)$$

It is seen from the above that $dV/dt \leq 0$.

That is, the system is globally stable around its interior equilibrium $E_I(x^*, y^*)$. □

5. Bionomic Equilibrium

In this section we study the bionomic equilibrium of the competitive predator-prey model. Here we consider the following parameters: (1) c: fishing cost per unit effort, (2) p_1: price per unit biomass of the prey, (3) p_2: price per unit biomass of the predator. The net revenue at any time is given by

$$R = (p_1 q_1 x + p_2 q_2 y - c) E. \quad (42)$$

The interior equilibrium point of the system is on the line given below

$$\begin{aligned}
& x \left((\underline{k})^{(1-p)} (\overline{k})^{p} q_1 (\underline{m})^{(1-p)} (\overline{m})^{p} (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} \right. \\
&\quad \left. + (\underline{r})^{(1-p)} (\overline{r})^{p} q_2 \right) \\
&\quad + y \left((\underline{k})^{(1-p)} (\overline{k})^{p} q_2 (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} \right. \\
&\quad \left. - (\underline{k})^{(1-p)} (\overline{k})^{p} q_1 (\underline{\delta})^{(1-p)} (\overline{\delta})^{p} \right) = (\underline{k})^{(1-p)} (\overline{k})^{p} \\
&\quad \cdot \left((\underline{r})^{(1-p)} (\overline{r})^{p} q_2 + d q_1 \right).
\end{aligned} \quad (43)$$

This biological equilibrium line meets x-axis at $(x_R, 0)$ and y-axis at $(0, y_R)$, where

$$\begin{aligned}
\hat{x} &= \frac{(\underline{k})^{(1-p)} (\overline{k})^{p} \left((\underline{r})^{(1-p)} (\overline{r})^{p} q_2 + (\underline{d})^{(1-p)} (\overline{d})^{p} q_1 \right)}{(\underline{k})^{(1-p)} (\overline{k})^{p} q_1 (\underline{m})^{(1-p)} (\overline{m})^{p} (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} + (\underline{r})^{(1-p)} (\overline{r})^{p} q_2}
\end{aligned} \quad (44)$$

and

$$\begin{aligned}
\hat{y} &= \frac{(\underline{k})^{(1-p)} (\overline{k})^{p} \left((\underline{r})^{(1-p)} (\overline{r})^{p} q_2 + (\underline{d})^{(1-p)} (\overline{d})^{p} q_1 \right)}{(\underline{k})^{(1-p)} (\overline{k})^{p} \left(q_2 (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} - q_1 (\underline{\delta})^{(1-p)} (\overline{\delta})^{p} \right)}.
\end{aligned} \quad (45)$$

It is seen that always $\hat{x} > 0$ but $\hat{y}$ is feasible if $q_2 (\underline{\alpha})^{(1-p)} (\overline{\alpha})^{p} > q_1 (\underline{\delta})^{(1-p)} (\overline{\delta})^{p}$.

The 'zero-profit line' is given by

$$R = (p_1 q_1 x + p_2 q_2 y - c) E = 0. \quad (46)$$

Equation (6) together with the above condition represents the bionomic equilibrium of prey-predator harvesting system.

For the points on the equilibrium line where $(p_1q_1x + p_2q_2y - c) < 0$, the fishery becomes useless. Because it cannot produce any positive economic revenue.

These three cases may arise in bionomic equilibrium.

Case 1. When fishing or harvesting of predator species is not possible, then $x_R = c/p_1q_1$ gives that $c = (p_1q_1(\underline{k})^{(1-p)}(\overline{k})^p((\underline{r})^{(1-p)}(\overline{r})^p q_2 + (\underline{d})^{(1-p)}(\overline{d})^p q_1))/((\underline{k})^{(1-p)}(\overline{k})^p q_1(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + (\underline{r})^{(1-p)}(\overline{r})^p q_2)$.

Case 2. When harvesting of prey is not possible, then $y_R = c/p_2q_2$ gives that $c = (p_2q_2(\underline{k})^{(1-p)}(\overline{k})^p((\underline{r})^{(1-p)}(\overline{r})^p q_2 + (\underline{d})^{(1-p)}(\overline{d})^p q_1))/((\underline{k})^{(1-p)}(\overline{k})^p(q_2(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p))$ with $q_2(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p > q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p$.

Case 3. When the bionomic equilibrium is at a point (x_R, y_R) where both $x_R > 0$ and $y_R > 0$, then the fishing of prey and predator is possible. Here

$$x_R = \frac{x_{11}}{x_{12}}, \quad y_R = \frac{y_{11}}{x_{12}}, \tag{47}$$

where

$$\begin{aligned} x_{11} &= (\underline{k})^{(1-p)}(\overline{k})^p p_2q_2\left((\underline{r})^{(1-p)}(\overline{r})^p q_2 + (\underline{d})^{(1-p)} \cdot (\overline{d})^p q_1\right) - c(\underline{k})^{(1-p)}(\overline{k})^p\left(q_2(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - q_1\delta\right), \\ x_{12} &= p_2q_2\left((\underline{k})^{(1-p)}(\overline{k})^p q_1(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + (\underline{r})^{(1-p)}(\overline{r})^p q_2\right) - (\underline{k})^{(1-p)}(\overline{k})^p p_1q_1\left(q_2(\underline{\alpha})^{(1-p)} \cdot (\overline{\alpha})^p - q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p\right), \\ y_{11} &= c\left((\underline{k})^{(1-p)}(\overline{k})^p q_1(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + (\underline{r})^{(1-p)}(\overline{r})^p q_2\right) - (\underline{k})^{(1-p)}(\overline{k})^p p_2q_2\left(q_2\alpha - q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p\right). \end{aligned} \tag{48}$$

Since $x_R > 0$ and $y_R > 0$, then the following two conditions hold.

$$\text{(i)} \quad c > \frac{c_{11}}{c_{12}} \tag{49}$$

$$\text{or } c < \frac{c_{11}}{c_{12}}, \tag{50}$$

where

$$\begin{aligned} c_{11} &= (\underline{k})^{(1-p)}(\overline{k})^p(p_2)^2(q_2)^2\left((\underline{r})^{(1-p)}(\overline{r})^p q_2 + (\underline{d})^{(1-p)}(\overline{d})^p q_1\right) \\ &\quad \cdot\left((\underline{k})^{(1-p)}(\overline{k})^p q_1(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + (\underline{r})^{(1-p)}(\overline{r})^p q_2\right), \end{aligned} \tag{51}$$

$$c_{12} = \left\{(\underline{k})^{(1-p)}(\overline{k})^p\right\}^2 p_1q_1\left(q_2(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p\right)^2$$

$$\text{(ii)} \quad c > \frac{(\underline{k})^{(1-p)}(\overline{k})^p p_1p_2q_1q_2\left((\underline{r})^{(1-p)}(\overline{r})^p q_2 + (\underline{d})^{(1-p)}(\overline{d})^p q_1\right)\left(q_2(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p\right)}{p_2q_2\left((\underline{k})^{(1-p)}(\overline{k})^p q_1(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + (\underline{r})^{(1-p)}(\overline{r})^p q_2\right)^2} \tag{52}$$

$$\text{or } c < \frac{(\underline{k})^{(1-p)}(\overline{k})^p p_1p_2q_1q_2\left((\underline{r})^{(1-p)}(\overline{r})^p q_2 + (\underline{d})^{(1-p)}(\overline{d})^p q_1\right)\left(q_2(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p - q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p\right)}{p_2q_2\left(kq_1(\underline{m})^{(1-p)}(\overline{m})^p(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p + (\underline{r})^{(1-p)}(\overline{r})^p q_2\right)^2}. \tag{53}$$

Then we conclude the bionomic equilibrium shorty in the following theorem.

Theorem 7. *The bionomic equilibrium $(x_R, 0)$ always exists, $(0, y_R)$ exists when $q_2(\underline{\alpha})^{(1-p)}(\overline{\alpha})^p > q_1(\underline{\delta})^{(1-p)}(\overline{\delta})^p$, and (x_R, y_R) exists when conditions (49), (50) and (52), (53) hold simultaneously.*

6. Optimal Harvesting Policy

Here both prey and predator populations are considered as fish populations. The optimal net profit is obtained from fishing. We discuss in this section the optimal harvesting policy. We consider the profit gained from harvesting taking the cost as a quadratic function and focusing on the conservation of fish population. The price assumed here is inversely proportional to the available biomass of fish (prey and predator); i.e., if the biomass increases, the price decreases (see Chakraborty et al. (2011)). Let $\check{c}$ be the constant harvesting cost per unit effort and p_1 and p_2 be, respectively, the constant price per unit biomass of the prey and predator. Now our target is to get the maximum net revenues from fishery. Then the optimal control problem can be created in the following way:

$$J(E) = \int_{t_0}^{t_1} e^{-\sigma t} [(p_1 - v_1 q_1 E x) q_1 E x + (p_2 - v_2 q_2 E y) q_2 E y - \check{c} E] \, dt, \tag{54}$$

subject to the system of differential equations (6) and the initial conditions (7). v_1 and v_2 are economic constants and σ is the instantaneous discount rate.

Here the control E is bounded in $0 \le E \le E_{max}$ and our object is to find an optimal control E_o such that

$$J(E_o) = \max_{E \in U} J(E) \tag{55}$$

where U is the control set defined by

$$U = \{E : E \text{ is measurable and } 0 \le E \le E_{max}, \text{ for all } t\}. \tag{56}$$

Here the convexity of the objective functional with respect to the control variable E along with the compactness of the range values of the state variables can be combined to give the existence of the optimal control E_o. Now the optimal control can be found by using Pontryagin's maximum principle (Pontryagin et al. (1962)). To optimize the objective functional $J(E)$, we construct the Hamiltonian H of the system as follows:

$$\begin{aligned} H &= (p_1 - v_1 q_1 E x) q_1 E x - (p_2 - v_2 q_2 E y) q_2 E y - \check{c} E \\ &+ \lambda_1 \left((\underline{r})^{(1-p)} (\overline{r})^p x \left(1 - \frac{x}{(\underline{k})^{(1-p)} (\overline{k})^p} \right) - (\underline{\alpha})^{(1-p)} (\overline{\alpha})^p xy - q_1 E x \right) \\ &+ \lambda_2 \left((\underline{m})^{(1-p)} (\overline{m})^p (\underline{\alpha})^{(1-p)} (\overline{\alpha})^p xy - (\underline{d})^{(1-p)} (\overline{d})^p y - (\underline{\delta})^{(1-p)} (\overline{\delta})^p y^2 - q_2 E y \right). \end{aligned} \tag{57}$$

Here the variables λ_1 and λ_2 are adjoint variables and the transversality conditions are as follows.

$$\lambda_i(t_1) = 0, \quad i = 1, 2 \tag{58}$$

First we use the optimality condition $\partial H / \partial E = 0$ to obtain the optimal effort which is as follows:

$$E_\sigma = \frac{p_1 q_1 x - p_2 q_2 x - \check{c} - q_1 \lambda_1 x - q_2 \lambda_2 y}{2 (v_1 q_1^2 x^2 + v_2 q_2^2 y^2)}. \tag{59}$$

The adjoint equations are

$$\begin{aligned} \frac{d\lambda_1}{dt} &= \sigma \lambda_1 - \frac{\partial H}{\partial x} = -q_1 E (p_1 - 2 v_1 q_1^2 E x) \\ &+ \left((\underline{\alpha})^{(1-p)} (\overline{\alpha})^p y + q_1 E - (\underline{r})^{(1-p)} (\overline{r})^p \left(1 - \frac{2x}{K} \right) + \sigma \right) \lambda_1 - (\underline{m})^{(1-p)} (\overline{m})^p (\underline{\alpha})^{(1-p)} (\overline{\alpha})^p y \lambda_2, \\ \frac{d\lambda_2}{dt} &= \sigma \lambda_2 - \frac{\partial H}{\partial y} = -q_2 E (p_2 - 2 v_2 q_2^2 E y) \\ &+ x (\underline{\alpha})^{(1-p)} (\overline{\alpha})^p \lambda_1 \\ &- \left((\underline{m})^{(1-p)} (\overline{m})^p (\underline{\alpha})^{(1-p)} (\overline{\alpha})^p x - (\underline{d})^{(1-p)} (\overline{d})^p \right. \\ &\left. - 2 (\underline{\delta})^{(1-p)} (\overline{\delta})^p y - q_2 E - \sigma \right) \lambda_2, \end{aligned} \tag{60}$$

and, therefore, we have the following theorem regarding the optimal value of the harvesting effort.

Theorem 8. *There exists an optimal control E_σ, corresponding to the optimal solutions for the state variables as x_σ and y_σ such that this control E_σ optimizes the objective functional J over the region U. Moreover, there exist adjoint variables λ_1 and λ_2 satisfying the first order differential equations given in (60) with the transversality conditions given in (58), where, at the optimal harvesting level, the values of the state variables x and y are, respectively, x_σ and y_σ.*

7. Numerical Simulation

In this section, we analyze our mathematical model through some simulation works. The main difference of our proposed model compared to other models of the same type is the consideration of interval-valued parameters instead of fixed-valued parameters. Inclusion of the parameter p assumes the value corresponding to the parameters of the system as an interval. In this regard we first analyze the importance of considering the parameter p in Figure 1. For simulation purpose we consider the following parametric values: $\hat{r} = [4, 6]$, $\hat{k} = [800, 1000]$, $\hat{\alpha} = [0.50, 0.75]$, $\hat{m} = [0.6, 0.8]$, $\hat{d} = [0.1, 0.2]$, $\hat{\delta} = [0.0001, 0.0020]$, $q_1 = 1.2, q_2 = 0.001$, $E = 1.4$. For different parametric values of p $(0 \le p \le 1)$, we have obtained various types of dynamical behavior of the proposed prey-predator system. From Figure 1, it can be said that lower values of p make the system unstable at the interior equilibrium point whereas the higher values of p gradually make the system locally asymptotically stable around the interior equilibrium point. As the numerical value of p increases, the instability solutions slowly become stable (unstable branches at $p = 0.7$ are less than the number of unstable branches at $p = 0.5$, but still at $p = 0.7$ the system is unstable but asymptotically stable at a higher value ($p = 0.9$)).

Next we describe optimal control theory to simulate the optimal control problem numerically. We consider the same parametric values as above and find the solution of optimal control problem numerically. For this purpose we solve the system of differential equations of the state variables (6) and corresponding initial conditions (7) with the help of forward Runge-Kutta forth order procedure. Also the differential equations of adjoint variables (50) and corresponding transversality conditions (52) are solved with the help of backward Runge-Kutta forth order procedure

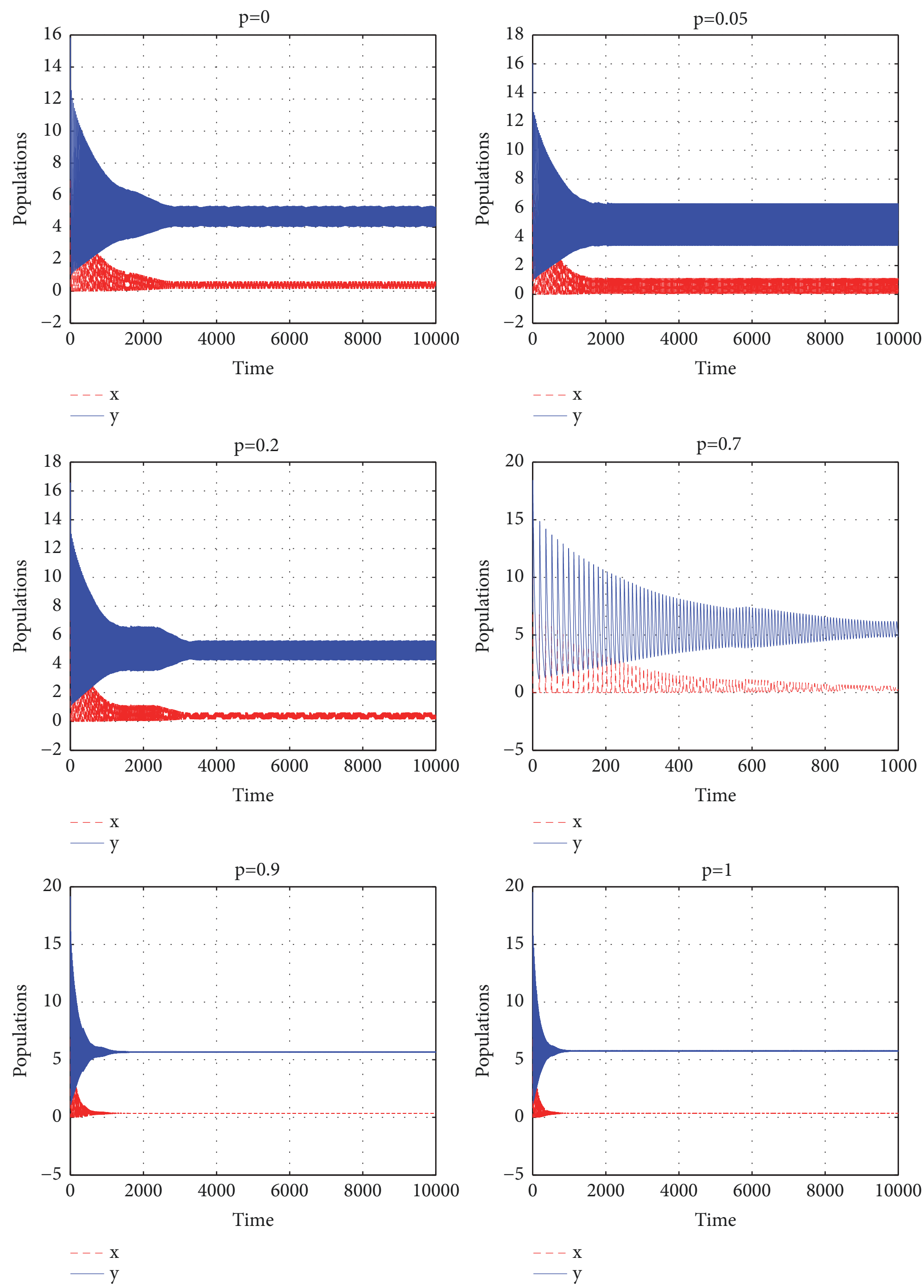

FIGURE 1: Stability of interior equilibrium depends on the numerical value of p. A higher value of p makes the interior equilibrium a stable equilibrium, whereas a lower value makes it unstable one.

for the time interval $[0, 100]$ (see, Jung et al. [27], Lenhart and Workman [28], etc.). Considering harvesting parameter E as the control variable, in Figure 2 and in Figure 3, we, respectively, plot the changes of prey biomass with respect to time and those of predator biomass with respect to time both in presence of control and in absence of control parameter. It is observed that when harvesting control is applied optimally, then the biomass of both prey species and predator species diminishes which is in accord with our expectations. Further in Figure 4, we plot the variation of control parameter (here harvesting effort is the control parameter), and in Figure 5, we plot variations of adjoint variables. It is also to be observed that the level of optimal harvesting effort always belongs to the range $[0.10, 0.45]$. Further according to the transversality conditions, both the adjoint variables λ_1 and λ_2 vanished at the final time (see Figure 5).

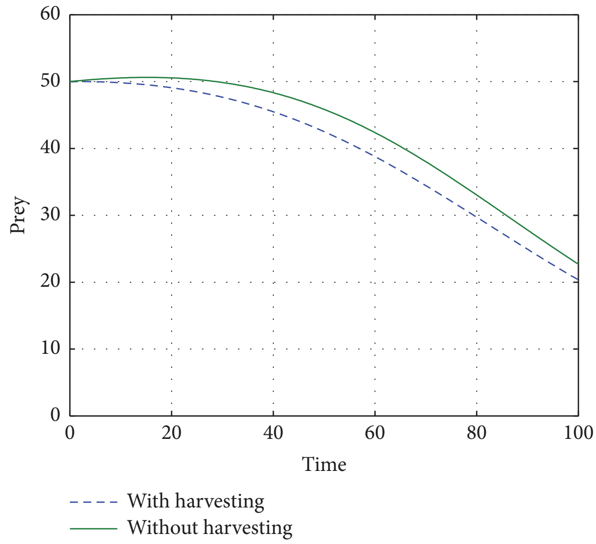

FIGURE 2: Variation of prey biomass both with control and without control cases.

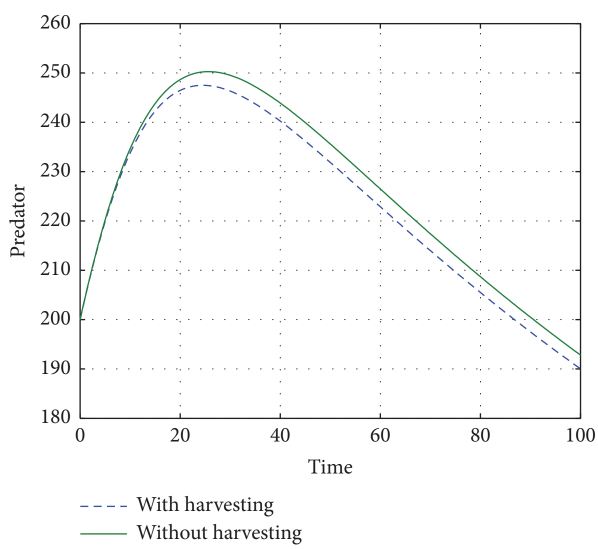

FIGURE 3: Variation of predator biomass both with control and without control cases.

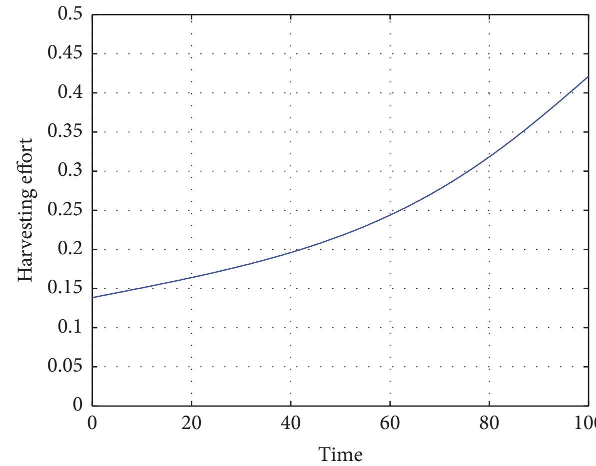

FIGURE 4: Variation of harvesting effort with time.

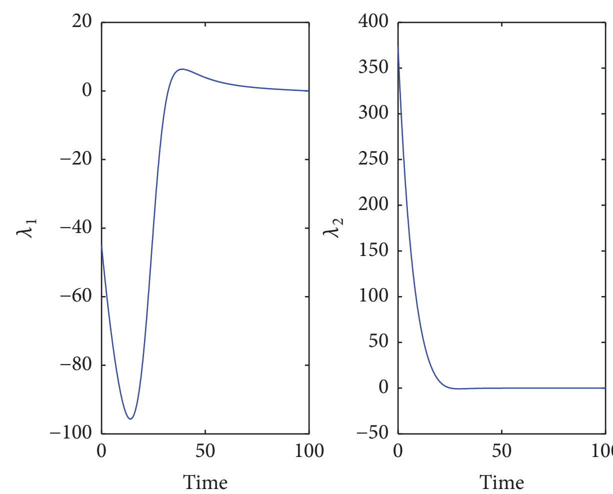

FIGURE 5: Variation of adjoint variables with time when harvesting control is applied optimally.

8. Discussions and Conclusions

The interactions between prey species and their predator species is an important topic to be analyzed. In present era, many experts are still analyzing the different aspects on this relationship. For this purpose in our present paper, we formulate and analyze a mathematical model on prey-predator system with harvesting on both prey and predator species. Further the model system is improved with the consideration of system parameters assuming an interval value instead of considering a single value. In reality due to various uncertainty aspects in nature, the parameters associated with a model system should not be considered a single value. But often this scenario has been neglected although some recent works considered these types of phenomena (see the works of Pal et al. [19, 29], Sharma and Samanta [30], Das and Pal [31], etc.). Influenced by those works, we also consider that all the parameters associated with our system are of interval value. Further harvesting of both prey and predator species is considered with catch per unit biomass in unit time with harvesting effort E.

The proposed model is analyzed for both crisp and interval-valued parametric cases. Different dynamical behavior of the system, including uniform boundedness, and existence and feasibility criteria of all the equilibria and both their local and global asymptotic stability criteria, has been described. It is found that the system may possess three equilibria, namely, the vanishing equilibrium point, the predator free equilibrium point, and the interior equilibrium point. Theoretical analysis shows that all of these three equilibria may be conditionally locally asymptotically stable depending on the numerical value of the harvesting parameter E. The classical prey-predator model with harvesting effort and without imprecise parametric space in general enables the vanishing equilibrium or trivial equilibrium point as an unstable equilibrium point, but the consideration of imprecise parametric space makes the trivial equilibrium a conditionally stable equilibrium. This phenomenon would surely describe the simultaneous extinction of a single species or both species although the crisp model failed to analyze it.

Next we study explicitly the existence criteria of bionomic equilibrium considering E as harvesting effort. Further considering harvesting effort as the control parameter, we form an optimal control problem with the objective of maximizing the profit due to harvesting in a finite horizon of time and solve that problem both theoretically and numerically. The objective functional considered in optimal control problem is also of both innovative and realistic type, as we consider here that the prices of biomass for both prey and predator species inversely depend upon their corresponding demands.

Consideration of imprecise parameters set makes the model more close to a realistic system which can be well explained with the help of Figure 1. It is shown that for different values of the parameter p, associated with the imprecise values, we obtain different nature of the coexisting equilibrium point. As the numeric value of the associated parameter p increases, the amount of unstable branches for both the species reduces and ultimately becomes a stable system for a higher value of $p(\geq 0.9)$. As the nature of the interior or coexisting equilibrium is one of the most important objects to study, we may claim that the different values of the imprecise parameter p are able to make the proposed prey-predator system understandable and reflect the real world problem.

However, in the present work we consider only a single prey species interacting with a single predator species which makes the model a quite simple one. For our future work we preserve the option of considering more than one type of prey species interacting with more than one type of predator species with imprecise set of parameters. Further due to unavailability of real world data, to simulate our theoretical works, we consider a hypothetical set for the parameters and obtain the result. However, we mainly aim to study the qualitative behavior of the system (not quantitative behavior) which would not be hampered at all due to the consideration of a simulated parametric set.

Data Availability

The data used in the manuscript are hypothetical data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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