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Research Article

Exponentially Weighted Moving Average Control Charts for the Process Mean Using Exponential Ratio Type Estimator

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This study proposes EWMA-type control charts by considering some auxiliary information. The ratio estimation technique for the mean with ranked set sampling design is used in designing the control structure of the proposed charts. We have developed EWMA control charts using two exponential ratio-type estimators based on ranked set sampling for the process mean to obtain specific ARLs, being suitable when small process shifts are of interest.

1. Introduction

To ensure the quality standards of products, every industry must develop some mechanism by adopting suitable statistical quality control techniques and procedures. The absence of a well-structured quality control system or a wrong choice of quality control methods may affect the ability of producing quality products. Control chart is an important statistical technique in statistical process control (SPC) that visually highlights the presence of special causes in a production process that causes deviations in quality standards.

It is generally observed that some variation always exists in the data. A control chart can easily detect or identify whether the variation is usual or unexpected for production process, because something special or unusual is happening. A control chart most widely uses statistical process control tool in determining the state of a manufacturing or a business process (whether in control or out of control). Control charts are typically used to observe changes in a process over time. The main purpose of using control charting technique in SPC is to attain and sustain continuous improvement in the quality and production of products by keenly observing changes in the manufacturing process. However the decision regarding the selection of suitable control chart depends on the type of data. In spite of the wide application and popularity, it has been observed that the traditional Shewhart control charts can only be helpful when the process suffers an out-of-control situation due to the presence of assignable causes, resulting in large shifts in the process. The reason is that Shewhart control chart only considers the information provided by the last sample and it does not pay any attention to the information about the process contained in the rest of the samples. This demerit makes Shewhart control chart insensitive towards small shift in the production process. Thus, this classical chart is not a good choice for SPC where the process seems to be in control state and assignable causes do not create disturbances in the process on considerably large scale.

The exponentially weighted moving average chart, a well-known control charting technique, is sensitive to the detection of control signals while small or moderate shifts occur in the production process. EWMA chart was first introduced by Roberts (1959) and it has gradually achieved a significant place in SPC. A lot of innovations and designs have been introduced in the structure of EWMA control charts for monitoring process mean and dispersion, by the researchers in different fields. For example, Haq et al. [1] have developed new EWMA control charts for controlling the procedure mean dispersion, using the ideas of ordered double ranked set sampling (ODRSS) and ordered imperfect double ranked set sampling in the new designed EWMA control charts. Abbasi et al. [2] constructed a set of EWMA control charts based on large range of dispersion estimates used for managing procedure dispersion based on normal and a range non-normal distribution. The investigated EWMA dispersion control charts were based on an extensive variety of dispersion estimators. Abbas et al. [3] recommended a new EWMA-type control chart that uses a single auxiliary variable known as $M_X EWMA$ control chart. For the estimation of mean in defining the control constitution of the designed control chart, regression estimator was used. Haq [4] recommended an enhanced mean deviation exponentially weighted moving average (IMDEWMA) control chart based on rank set sampling to control process dispersion. Haq et al. [5] studied the effect of measurements errors on the detection ability of EWMA control charts for controlling process mean under ranked set sampling (RSS), median ranked set sampling (MRSS), imperfect ranked set sampling (IRSS), and imperfect median ranked set sampling (IMRSS) schemes. Azam et al. [6] presented repetitive exponentially weighted moving average (EWMA) control chart using regression estimator based on ranked set sampling (RSS) design to examine and detect the changes in the manufacturing process. They studied the detection ability of the proposed control chart for monitoring shifts in the process mean. Ridwan et al. [7] developed an EWMA scheme using ratio estimator to increase the effectiveness of typical EWMA chart in monitoring the location parameter.

Although in efficiency comparison the EWMA charts are found parallel to the Shewhart control charts, these are said to be best alternatives to the Shewhart charts in monitoring small shifts in the process mean because they provide quick alarms when small shifts are introduced in the process.

2. Methodology

This study proposes EWMA-type control charts by considering some auxiliary information provided by an auxiliary variable. The ratio estimation technique for the mean with ranked set sampling design is used in designing the control structure of the proposed charts. Here we developed EWMA control charts using Bahl and Tuteja (1991) exponential ratio-type estimator and Khan et al. [8] exponential ratio-type estimator, based on ranked set sampling design for the process mean to obtain specific ARLs, being suitable when small process shifts are of interest.

2.1. Ranked Set Sampling (RSS). The idea of RSS was first given by Mcintyre (1952). The mechanism of RSS design is based on the ranking of sampling units within the samples, and it makes the collection of actual measurements of sampling units more feasible and reliable as compared to SRS design with respect to simplicity, time, cost, or other complicated factors.

This sampling technique obtains samples from a population in such a way that the extent of information covers the entire range of observations in the population. In this way ranked set sample becomes more representative than the simple random sample, obtained by identical number of observations from the same population [9]. The structure of RSS design is simply described as follows: Firstly n^2 sampling units are identified from a large or infinite population. These units are further assigned randomly to the n equal sized samples. After that ranking is imposed to the units within each sample with respect to some characteristics of interest or study variable. A mixture of mechanisms can be used to acquire this ranking, comprising visual comparison, expert judgment, or the use of auxiliary variables; however, it cannot include actual measurements of the characteristics of interest on the selected units [10].

Let $y_{11}.y_{12}, y_{13} \cdots y_{1n}, y_{21}, y_{22}, y_{23} \ldots y_{n1}, y_{n2}, y_{n3}, \ldots, y_{nn}$ be ‘n’ independent simple random samples, each of size n. Implement the RSS process to these n samples to obtain a ranked set sample (RSS) of size t defined by $y_{1(1,n)}, y_{2(2,n)}, y_{3(3,n)} \cdots\cdots\cdots y_{t(n,n)}$.

Now RSS estimator for $\overline{y}$ is defined as

$$\overline{y}_{RSS} = \frac{1}{tn}\sum_{i=1}^{t}\sum_{i=1}^{n}\left[y_{i(i:n)}\right] \tag{1}$$

with mean

$$\mu_y = E\left(\overline{y}_{RSS}\right) = \overline{Y} \tag{2}$$

and variance

$$\operatorname{var}\left(\overline{y}_{RSS}\right) = \frac{\sigma^2}{n} - \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i)} - \mu_y\right)^2. \tag{3}$$

$$\operatorname{var}\left(\overline{y}_{RSS}\right) = \operatorname{var}\left(\overline{y}_{SRS}\right) - \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i)} - \mu_y\right)^2 \tag{4}$$

This estimator is unbiased and is also more precise than simple random sampling estimator $\overline{y}_{SRS}$.

2.2. Ratio Estimator. In sampling surveys, while estimating the population parameters, role of auxiliary variable is very significant to increase its efficiency. Many authors have improved the precision of their estimators through the use of auxiliary variable. Auxiliary variable which is highly correlated to the variable of interest actually provides information intended to improve the efficiency of the estimator of population parameter of the study or main variable [11].

If y_i ($i = 1, 2, 3, \ldots, N$) is the observations for the variable of interest y from a finite population and if x_i ($i = 1, 2, 3, \ldots, N$) is the observations of auxiliary variable x whose information is completely known and is highly correlated to the variable under consideration, then the traditional ratio estimator for $\overline{Y}$ is defined as

$$\widehat{\overline{y}}_R = \overline{y}\left(\frac{\overline{X}}{\overline{x}}\right). \tag{5}$$

The complete information of parameters of the auxiliary variable x, such as C_x (coefficient of variation) and $\beta_2(x)$ (coefficient of kurtosis), basically help to increase efficiency of the estimator of the study variable (Yadav et al., 2014).

3. Proposed Repetitive Exponentially Weighted Moving Average (EWMA) Control Charts Based on Ranked Set Sampling

Here, we proposed two repetitive exponentially weighted moving average (EWMA) control charts using exponential ratio-type estimators, suggested by Bahl and Tuteja (1991) and Khan et al. [8], based on ranked set sampling (RSS) design. EWMA control charts using these ratio estimators are developed to observe process mean by obtaining specific ARLs using Monte Carlo simulation and, being suitable when small shifts are of interest. How to calculate probability of declaring the process out of control when shift is introduced in the process is also elaborated here.

3.1. Proposed Repetitive EWMA-RSS Control Charts Using Exponential Ratio-Type Estimators. As it is already mentioned, the assumed study variable y is difficult to measure directly but it is easy to measure with the corresponding auxiliary variable x. Let $\overline{y}_{i(i:n)t}^{RSS}$ be the sample mean under RSS resultant of $\overline{x}_{i(i:n)t}$ at time t ($t = 1, 2, 3, 4, \ldots$); then Bahl and Tuteja (1991) exponential ratio-type estimator and Khan et al. [8] exponential ratio-type estimator for mean under RSS, respectively, are defined as

$$M_{y1}^{RSS} = \overline{y}_{i(i:n)t}\left[\exp\left(\frac{\overline{X} - \overline{x}_{i(i:n)t}}{\overline{X} + \overline{x}_{i(i:n)t}}\right)\right] \tag{6}$$

with mean and variance (by using (4))

$$\begin{aligned} &\mathrm{E}\left(M_{y1}^{RSS}\right) = \overline{Y} = \mu_y \\ &\operatorname{var}\left(M_{y1}^{RSS}\right) = \psi\overline{Y}^2\left[C_y^2 + \frac{1}{4}C_x^2(1 - C)\right] \\ &\qquad - \frac{1}{n^2}\sum_{i=1}^{n}\left[\mu_{(i)} - \mu_y\right]^2 \quad \therefore \psi = \frac{N - n}{N.n} \end{aligned} \tag{7}$$

where constant 'C' = $\rho(C_y/C_x)$

$$M_{y2}^{RSS} = \overline{y}_{i(i:n)t}\exp\left[\Upsilon\left(\frac{\overline{X}^{1/h} - \overline{x}_{i(i:n)t}{}^{1/h}}{\overline{X}^{1/h} + (a-1)\,\overline{x}_{i(i:n)t}{}^{1/h}}\right)\right]. \tag{8}$$

Then the mean and the variance of this estimator are

$$\begin{aligned} &\mathrm{E}\left(M_{y2}^{RSS}\right) = \overline{Y} = \mu_y \\ &\operatorname{var}\left(M_{y2}^{RSS}\right) = \psi\overline{Y}^2\left[C_y^2 - C^2C_x^2\right] - \frac{1}{n^2}\sum_{i=1}^{n}\left[\mu_{(i)} - \mu_y\right]^2 \\ &\qquad \therefore \psi = \frac{N - n}{N.n} \end{aligned} \tag{9}$$

where constant C = $\rho(C_y/C_x)$.

3.2. Structure of the Proposed Repetitive EWMA-RSS Control Charts. Here a sequence M_{ki}^{RSS} based on M_{yk}^{RSS} (where k =1,2) using recurrence formula is described as

$$M_{ki}^{RSS} = \lambda M_{yk}^{RSS} + (1-\lambda)\, M_{k(i-1)}^{RSS}, \tag{10}$$

where $0 < \lambda \leq 1$.

Therefore, M_{ki}^{RSS} (k = 1,2) is the statistic of EWMA-RSS control chart based on Bahl and Tuteja (1991) and Khan et al. [8] exponential ratio-type estimators, respectively, for mean, and its initiatory value is $M_{k(0)}^{RSS} = \mu_0$. Thus, mean and variance of this EWMA statistic are

$$\begin{aligned} &\mathrm{E}\left(M_{ki}^{RSS}\right) = \mu_y, \quad \text{where, } k = 1, 2 \\ &\operatorname{var}\left(M_{ki}^{RSS}\right) = \frac{\lambda}{2-\lambda}\operatorname{var}\left(M_{yk}^{RSS}\right)\left[1 - (1-\lambda)^{2i}\right]. \end{aligned} \tag{11}$$

For large number of times, the variance of M_{ki}^{RSS} becomes

$$\operatorname{var}\left(M_{ki}^{RSS}\right) = \frac{\lambda}{2-\lambda}\operatorname{var}\left(M_{yk}^{RSS}\right). \tag{12}$$

Thus, variances of the statistic of EWMA-RSS control chart based on Bahl and Tuteja (1991) and Khan et al. [8] exponential ratio-type estimators are, respectively, described as

$$\begin{aligned} \operatorname{var}\left(M_{1i}^{RSS}\right) = \frac{\lambda}{2-\lambda}\Bigg[&\psi\overline{Y}^2\left\{C_y^2 + \frac{1}{4}C_x^2(1 - C)\right\} \\ &- \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i)} - \mu_y\right)^2\Bigg]. \end{aligned} \tag{13}$$

$$\begin{aligned} \operatorname{var}\left(M_{2i}^{RSS}\right) = \frac{\lambda}{2-\lambda}\Bigg[&\psi\overline{Y}^2\left[C_y^2 - C^2C_x^2\right] \\ &- \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i)} - \mu_y\right)^2\Bigg] \end{aligned} \tag{14}$$

Control Limits for the Proposed Repetitive EWMA-RSS Control Charts. Control Limits for the proposed repetitive EWMA-RSS control charts, for k = 1,2, are defined as

$$\begin{aligned} \mathrm{LCL}_1 &= \mu_y - L_1 sd\left(M_{ki}^{RSS}\right) \\ \mathrm{LCL}_2 &= \mu_y - L_2 sd\left(M_{ki}^{RSS}\right) \\ \mathrm{UCL}_1 &= \mu_y + L_1 sd\left(M_{ki}^{RSS}\right) \\ \mathrm{UCL}_2 &= \mu_y + L_2 sd\left(M_{ki}^{RSS}\right) \end{aligned} \tag{15}$$

where L_1 and L_2 are the control constant multipliers. The values of L_1 and L_2 are chosen such that the in-control ARLs of the proposed repetitive EWMA-RSS control chart reach a specific level of decided value of target ARL of the process.

Now the probability of reporting the process as out of control when actually the process is in control is obtained as

$$P_{out,1}^{o} = P\left(M_{ki}^{RSS} < LCL_1\right) + P\left(M_{ki}^{RSS} > UCL_1\right) \tag{16}$$

$$\begin{aligned} P_{out,1}^{o} = P&\left[\frac{M_{ki}^{RSS} - \mu_y}{\sigma} < \frac{\mathrm{LCL1} - \mu_y}{\sigma}\right] \\ &+ P\left[\frac{M_{ki}^{RSS} - \mu_y}{\sigma} > \frac{\mathrm{UCL1} - \mu_y}{\sigma}\right]. \end{aligned} \tag{17}$$

After simplification, the above equation becomes

$$P^{o}_{out,1} = P\left(Z < -L_1\right) + P\left(Z > L_1\right). \tag{18}$$

The term Ø(.) is cumulative distribution function of the standard normal distribution

$$\begin{aligned} P^{o}_{out,1} &= \varnothing\left(-L_1\right) + 1 - \varnothing\left(L_1\right) \\ P^{o}_{out,1} &= 2\left[1 - \varnothing\left(L_1\right)\right]. \end{aligned} \tag{19}$$

The probability of repetition P^{o}_{REP} for the planned control chart is given as follows:

$$\begin{aligned} P^{o}_{REP} = P\left\{UCL_2 < M^{RSS}_{ki} < UCL_1\right\} \\ + P\left\{LCL_1 < M^{RSS}_{ki} < LCL_2\right\}. \end{aligned} \tag{20}$$

After simplification, it becomes

$$\begin{aligned} P^{o}_{REP} &= P\left\{L_2 < Z < L_1\right\} + P\left\{-L_1 < Z < -L_2\right\} \\ P^{o}_{REP} &= \varnothing\left(L_1\right) - \varnothing\left(L_2\right) + \varnothing\left(-L_2\right) - \varnothing\left(-L_1\right) \\ P^{o}_{REP} &= 2\left[\varnothing\left(L_1\right) - \varnothing\left(L_2\right)\right] \end{aligned} \tag{21}$$

so

$$P^{o}_{out} = \frac{P^{o}_{out,1}}{1 - P^{o}_{REP}} \tag{22}$$

$$ARL_o = \frac{1}{P^{o}_{out}}. \tag{23}$$

For Shifted Process. If our mean is shift

$$\begin{aligned} &\mu_1 = \mu_0 + f\sigma \quad \therefore \mu_0 = \mu_y \\ &\mu_1 = \mu_y + f\sigma \\ &\mu_1 = \mu_y + f\sigma \sim \mathrm{N}\left(\mu_y + f\sigma, \frac{\lambda}{2-\lambda}\operatorname{var}\left(\overline{y}_{kSRS}\right)\right), \end{aligned} \tag{24}$$

where k = 1,2, now the probability of reporting the process as out of control when the shift will be introduced in the process mean is described as

$$\begin{aligned} P^{1}_{out,1} = P\left(M^{RSS}_{ki} < LCL_1 \mid \mu_1\right) \\ + P\left(M^{RSS}_{ki} > UCL_1 \mid \mu_1\right), \quad \text{where } \mathrm{k} = 1, 2. \end{aligned} \tag{25}$$

After simplification, the above equation becomes

$$\begin{aligned} P^{1}_{out,1} &= P\left(Z < -L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right) \\ &\quad + P\left(Z > L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right) \\ P^{1}_{out,1} &= \varnothing\left(-L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right) + 1 \\ &\quad - \varnothing\left(L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right). \end{aligned} \tag{26}$$

Similarly, the probability of repetition P^{1}_{REP} for the proposed control chart when the shift will be introduced in the process mean is given as

$$\begin{aligned} P^{1}_{REP} = P\left\{UCL_2 < M^{RSS}_{ki} < \frac{UCL_1}{\mu_1}\right\} \\ + P\left\{LCL_1 < M^{RSS}_{ki} < \frac{LCL_2}{\mu_1}\right\}. \end{aligned} \tag{27}$$

After simplification, it becomes

$$\begin{aligned} &P^{1}_{REP} \\ &= P\left[L_2 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)} < Z < L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right] \\ &\quad + P\left[-L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)} < Z < -L_2 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right] \end{aligned} \tag{28}$$

and then

$$\begin{aligned} P^{1}_{REP} &= \varnothing\left(L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right) \\ &\quad - \varnothing\left(L_2 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right) \\ &\quad + \varnothing\left(-L_2 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right) \\ &\quad - \varnothing\left(-L_1 - \frac{f\sigma_y}{sd\left(M^{RSS}_{ki}\right)}\right) \end{aligned} \tag{29}$$

so

$$P^{1}_{out} = \frac{P^{1}_{out,1}}{1 - P^{1}_{REP}} \tag{30}$$

$$ARL_1 = \frac{1}{P^{1}_{out}}. \tag{31}$$

4. Results and Interpretation

The proposed repetitive EWMA control charts are designed for the purpose of monitoring the shifts in the process mean using Bahl and Tuteja (1991) and Khan et al. [8] exponential ratio-type estimators under ranked set sampling RSS. We used ARLs as a performance criterion to compare the performance efficiency of repetitive EWMA control charts. Here the ARLs tables and graphs are constructed for frequently used values of in-control ARLs, such as 500 and 370, for the repetitive EWMA control charts. Different values of the smoothing constant λ ($0 < \lambda \leq 1$) are taken, with an interval of 0.1 to construct the ARLs tables. Three different datasets having different levels of correlation ρ, i.e., 0.25, 0.50, and 0.90, between the auxiliary variable and the variable of interest, are used to assess the performance of the proposed repetitive EWMA-RSS control charts. Here, the out-of-control ARLs are evaluated for the shifted process in

Table 1: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator, when r_o is 500.

shift	$k_1 = 3.0958, k_2 = 2.339, \lambda = 0.1$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	500.62	500.62	500.62
0.01	425.80	412.09	322.23
0.02	283.38	257.61	137.87
0.03	172.49	148.08	58.59
0.04	103.70	84.95	26.54
0.05	63.24	49.82	12.93
0.10	7.61	5.50	9.39
0.15	1.91	1.51	1.01
0.20	1.12	1.05	1.00
0.25	1.01	1.00	1.00
0.30	1.00	1.00	1.00
0.35	1.00	1.00	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Table 2: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator, when r_o is 500.

shift	$k_1 = 3.0958, k_2 = 2.339, \lambda = 0.2$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	500.62	500.62	500.62
0.01	464.17	456.48	400.22
0.02	372.17	352.05	237.32
0.03	274.82	248.83	130.27
0.04	195.57	170.11	72.01
0.05	137.70	115.66	40.96
0.10	26.49	19.71	4.15
0.15	6.78	4.81	1.31
0.20	2.48	1.87	1.02
0.25	1.39	1.19	1.00
0.30	1.09	1.04	1.00
0.35	1.02	1.00	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

mean, while maintaining the in-control ARLs at the same level, and compared with each other. The term r_o depicts the considered target values of in-control ARLs. By considering the subgroup size $n = 10$, the values of the control constants ($k's$) are determined for frequently used target values of average run lengths such as 500 and 370. Finally, ARLs based on different values of correlation ρ are obtained for the small and moderate shifts in the process average.

The ARLs tables of repetitive control charts that are arranged according to the various random small shifts from 0 to 0.5 are given in Tables 1–12.

In Table 1, out-of-control ARLs are calculated by using repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator, for various small shifts under different levels of ρ by taking $\lambda = 0.1$. We observed that for the fixed value of smoothing constant λ, the values of out-of-control ARLs decrease as the value of ρ increases. For example, when $r_o = 500$ and shift is 0.01, then the value of out-of-control ARL for $\rho = 0.25$ is 425.80, but when $\rho = 0.90$, the value of out-of-control ARL is 322.23, and when $r_o = 500$ and shift is 0.02, then the value of out-of-control ARL for $\rho = 0.25$ is 283.38, but when $\rho = 0.90$, the value of out-of-control ARL is 137.87. In the same way, Tables 2 and 3 show a decreasing tendency in ARLs as the value of ρ increases. Also, the trend of ARLs shows that the ARLs decrease rapidly with the increasing values of shift. When we compared Tables 1–3, an increasing trend in ARLs is observed as the value of smoothing constant λ increases with the interval of 0.1, but at the same time a quick decreasing trend in ARLs is also found when the value of ρ increases.

Table 3: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator, when r_o is 500.

shift	$\boldsymbol{k_1 = 3.0958, k_2 = 2.339, \lambda = 0.3}$		
f	$\boldsymbol{\rho = 0.25}$	$\boldsymbol{\rho = 0.50}$	$\boldsymbol{\rho = 0.90}$
0	500.62	500.62	500.62
0.01	478.40	473.24	433.91
0.02	413.12	397.69	299.94
0.03	333.88	310.58	189.33
0.04	**259.47**	**233.21**	**117.32**
0.05	**197.80**	**172.26**	**73.38**
0.10	**50.70**	**39.32**	**9.52**
0.15	**15.21**	**11.01**	**2.30**
0.20	**5.53**	**3.93**	**1.21**
0.25	**2.54**	**1.91**	**1.03**
0.30	**1.53**	**1.28**	**1.00**
0.35	**1.18**	**1.08**	**1.00**
0.40	**1.06**	**1.02**	**1.00**
0.5	**1.00**	**1.00**	**1.00**

Table 4: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator, when r_o is 370.

shift	$k_1 = 3.0025, k_2 = 2.6008, \lambda = 0.1$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	370.98	370.98	370.98
0.01	317.51	307.89	244.08
0.02	216.08	197.39	108.91
0.03	134.78	116.57	48.40
0.04	83.08	68.76	23.06
0.05	52.01	41.55	11.88
0.10	7.33	5.39	1.51
0.15	2.08	1.65	1.02
0.20	1.19	1.09	1.00
0.25	1.02	1.01	1.00
0.30	1.00	1.00	1.00
0.35	1.00	1.00	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Table 5: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator, when r_o is 370.

shift	$k_1 = 3.0025, k_2 = 2.6008, \lambda = 0.2$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	370.98	370.98	370.98
0.01	344.26	338.92	299.54
0.02	279.70	265.39	182.59
0.03	209.89	190.99	102.19
0.04	151.89	133.01	58.80
0.05	108.78	92.16	34.57
0.10	23.02	17.51	4.24
0.15	6.62	4.85	1.42
0.20	2.65	2.03	1.04
0.25	1.51	1.28	1.00
0.30	1.15	1.06	1.00
0.35	1.04	1.01	1.00
0.40	1.01	1.00	1.00
0.5	1.00	1.00	1.00

Table 6: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator, when r_o is 370.

shift	$k_1 = 3.0025, k_2 = 2.6008, \lambda = 0.3$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	370.98	370.98	370.98
0.01	354.10	350.54	323.19
0.02	308.61	279.75	228.05
0.03	252.42	235.71	147.27
0.04	198.73	179.57	93.41
0.05	153.54	134.61	59.86
0.10	42.24	33.27	8.99
0.15	13.80	10.26	2.48
0.20	5.51	4.05	1.29
0.25	2.72	2.07	1.05
0.30	1.67	1.38	1.00
0.35	1.26	1.13	1.00
0.40	1.09	1.04	1.00
0.5	1.01	1.00	1.00

Table 7: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] exponential ratio-type estimator, when r_o is 500.

shift	$k_1 = 3.0958, k_2 = 2.339, \lambda = 0.1$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	500.62	500.62	500.62
0.01	393.36	374.56	177.62
0.02	226.36	198.83	41.57
0.03	121.12	99.58	11.81
0.04	65.61	51.20	4.23
0.05	36.70	27.46	2.02
0.10	3.61	2.57	1.00
0.15	1.23	1.10	1.00
0.20	1.01	1.00	1.00
0.25	1.00	1.00	1.00
0.30	1.00	1.00	1.00
0.35	1.00	1.00	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Table 8: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] exponential ratio-type estimator, when r_o is 500.

shift	$k_1 = 3.0958, k_2 = 2.339, \lambda = 0.2$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	500.62	500.62	500.62
0.01	445.62	434.29	280.03
0.02	325.79	300.73	101.11
0.03	217.52	190.15	38.07
0.04	141.41	118.00	15.79
0.05	92.10	73.90	7.27
0.10	13.58	9.62	1.11
0.15	3.23	2.32	1.00
0.20	1.43	1.21	1.00
0.25	1.07	1.03	1.00
0.30	1.01	1.00	1.00
0.35	1.00	1.00	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Table 9: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] exponential ratio-type estimator, when r_o is 500.

shift	$k_1 = 3.0958, k_2 = 2.339, \lambda = 0.3$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	500.62	500.62	500.62
0.01	465.88	458.08	338.43
0.02	376.84	356.15	154.65
0.03	281.12	253.98	68.99
0.04	201.98	175.04	32.43
0.05	143.42	119.84	16.22
0.10	28.42	20.91	1.60
0.15	7.38	5.15	1.02
0.20	2.67	1.97	1.00
0.25	1.45	1.22	1.00
0.30	1.11	1.04	1.00
0.35	1.02	1.01	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Table 10: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] exponential ratio-type estimator, when r_o is 370.

shift	$k_1 = 3.0025, k_2 = 2.6008, \lambda = 0.1$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	370.98	370.98	370.98
0.01	274.70	187.39	138.59
0.02	174.55	114.29	35.05
0.03	96.29	79.95	10.94
0.04	53.85	42.63	4.32
0.05	31.20	23.80	2.19
0.10	3.74	2.75	1.00
0.15	1.32	1.16	1.00
0.20	1.03	1.01	1.00
0.25	1.00	1.00	1.00
0.30	1.00	1.00	1.00
0.35	1.00	1.00	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Table 11: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] exponential ratio-type estimator, when r_o is 370.

shift	$k_1 = 3.0025, k_2 = 2.6008, \lambda = 0.2$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	370.98	370.98	370.98
0.01	331.37	323.45	213.66
0.02	246.63	228.62	81.11
0.03	168.07	147.88	32.28
0.04	111.57	93.93	14.27
0.05	74.24	62.26	7.05
0.10	12.43	9.08	1.17
0.15	3.38	2.50	1.00
0.20	1.55	1.30	1.00
0.25	1.12	1.05	1.00
0.30	1.02	1.00	1.00
0.35	1.00	1.00	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Table 12: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] exponential ratio-type estimator, when r_o is 370.

shift	$k_1 = 3.0025, k_2 = 2.6008, \lambda = 0.3$		
f	$\rho = 0.25$	$\rho = 0.50$	$\rho = 0.90$
0	370.98	370.98	370.98
0.01	345.45	340.04	255.68
0.02	283.01	268.31	121.48
0.03	214.45	194.74	56.47
0.04	156.62	136.67	27.79
0.05	113.07	95.32	14.63
0.10	24.57	18.49	1.75
0.15	7.14	5.16	1.04
0.20	2.84	2.13	1.00
0.25	1.58	1.31	1.00
0.30	1.17	1.08	1.00
0.35	1.05	1.01	1.00
0.40	1.00	1.00	1.00
0.5	1.00	1.00	1.00

Moreover, in Tables 4–6 for $r_o = 370$ the same trends in ARLs are observed as for $r_o = 500$. Similarly, Tables 7–12 for the proposed repetitive EWMA-RSS control chart based on Khan et al. [8] exponential ratio-type estimator show a rapid decreasing pattern in the values of ARLs as the level of correlation increases, and besides this an increasing trend in ARLs values is also observed in these tables while the value of smoothing constant λ increases with the interval of 0.1. As per these tables compared in terms of the values of smoothing constant λ, an increasing trend in ARLs is observed as the value of smoothing constant λ increases with the gap of 0.1 for the proposed repetitive EWMA-RSS control charts under two different exponential ratio-type estimators. It means that with small value of λ, it detects the smaller shifts in the process quickly as compared to the large values of λ, or we can say that the small value of λ is good to detect smaller shifts in the process mean.

4.1. Performance Efficiency Comparison between the Proposed Repetitive EWMA-RSS Control Charts based on Ratio Estimators. The efficiency comparison between the performances of the proposed repetitive EWMA-RSS charts, based on exponential ratio-type estimators developed by Bahl and Tuteja (1991) and Khan et al. [8], is made here on the basis of ARLs. The ARLs tables are set up for comparing the performance of the proposed repetitive EWMA control charts based on Bahl and Tuteja (BT) and Khan et al. (HK) exponential ratio-type estimators under ranked set sampling.

In each given table (Tables 13–18), the values of ARLs, for the proposed repetitive EWMA-RSS charts based on BT and HK ratio estimators, are arranged according to the various random small shifts from 0 to 0.5 for a specified value of ρ at the value of smoothing constant $\lambda = 0.1$. The values of L_1 *and* L_2 determined for $r_o = 500, 370$ are also specified in the tables.

From Tables 12–18, it is observed that the repetitive EWMA-RSS control chart using Khan et al. (HK) estimator outperforms the proposed repetitive EWMA-RSS control chart based on Bahl and Tuteja (BT) by detecting small shifts much earlier, as it produces considerably smaller values of ARLs for different small shifts 0 to 0.5. For example, in Table 13, the ARL value produced by EWMA-RSS chart using Khan et al. [8] exponential ratio-type estimator is 393.36 for shift 0.01, which is much smaller than the values of ARLs produced by EWMA-RSS chart using Bahl and Tuteja (1991) exponential ratio-type estimator for the same shift 0.01.

Table 13: ARLs for proposed repetitive EWMA-RSS control charts using exponential ratio-type estimators, for $\rho = 0.25$ when r_o is 500 at $\lambda = 0.1$.

Shift	$k_1 = 3.0958$	$k_2 = 2.339$
f	BT	HK
0	500.62	500.62
0.01	425.80	393.36
0.02	283.38	226.36
0.03	172.49	121.12
0.04	103.70	65.61
0.05	63.24	36.70
0.10	7.61	3.61
0.15	1.91	1.23
0.20	1.12	1.01
0.25	1.01	1.00
0.30	1.00	1.00
0.35	1.00	1.00
0.40	1.00	1.00
0.5	1.00	1.00

4.2. ARLs Graphs for the Proposed Repetitive EWMA-RSS Control Charts. ARLs graphs for the proposed repetitive EWMA-RSS control charts based on Bahl and Tuteja (1991) and Khan et al. [8] exponential ratio-type estimators, based

Table 14: ARLs for proposed repetitive EWMA-RSS control charts using exponential ratio-type estimators, for $\rho = 0.50$ when r_o is 500 at $\lambda = 0.1$.

Shift	$k_1 = 3.0958$	$k_2 = 2.339$
f	BT	HK
0	500.62	500.62
0.01	412.09	374.56
0.02	257.61	198.83
0.03	148.08	99.58
0.04	84.95	51.20
0.05	49.82	27.46
0.10	5.50	2.57
0.15	1.51	1.10
0.20	1.05	1.00
0.25	1.00	1.00
0.30	1.00	1.00
0.35	1.00	1.00
0.40	1.00	1.00
0.5	1.00	1.00

Table 15: ARLs for proposed repetitive EWMA-RSS control charts using exponential ratio-type estimators, for $\rho = 0.90$ when r_o is 500 at $\lambda = 0.1$.

Shift	$k_1 = 3.0958$	$k_2 = 2.339$
f	BT	HK
0	500.62	500.62
0.01	322.23	177.62
0.02	137.87	41.57
0.03	58.59	11.81
0.04	26.54	4.23
0.05	12.93	2.02
0.10	9.39	1.00
0.15	1.01	1.00
0.20	1.00	1.00
0.25	1.00	1.00
0.30	1.00	1.00
0.35	1.00	1.00
0.40	1.00	1.00
0.5	1.00	1.00

Table 16: ARLs for proposed repetitive EWMA-RSS control charts using exponential ratio-type estimators, for $\rho = 0.25$ when r_o is 370 at $\lambda = 0.1$.

Shift	$k_1 = 3.0025$	$k_2 = 2.6008$
f	BT	HK
0	370.98	370.98
0.01	317.51	274.70
0.02	216.08	174.55
0.03	134.78	96.29
0.04	83.08	53.85
0.05	52.01	31.20
0.10	7.33	3.74
0.15	2.08	1.32
0.20	1.19	1.03
0.25	1.02	1.00
0.30	1.00	1.00
0.35	1.00	1.00
0.40	1.00	1.00
0.5	1.00	1.00

Table 17: Average run lengths for proposed repetitive EWMA-RSS control charts using exponential ratio-type estimators, for $\rho = 0.50$ when r_o is 370 at $\lambda = 0.1$.

Shift	$k_1 = 3.0025$	$k_2 = 2.6008$
f	BT	HK
0	370.98	370.98
0.01	307.89	187.39
0.02	197.39	114.29
0.03	116.57	79.95
0.04	68.76	42.63
0.05	41.55	23.80
0.10	5.39	2.75
0.15	1.65	1.16
0.20	1.09	1.01
0.25	1.01	1.00
0.30	1.00	1.00
0.35	1.00	1.00
0.40	1.00	1.00
0.5	1.00	1.00

on ranked set sampling (for Tables 1–12), are given in Appendix A. Figures 1–12 show that, under both proposed charts, the ARLs at rho = 0.90 are decreased sharply from the considered target value as compared to the ARLs at different values of rho (in all the cases), but for the increasing value of λ, the gradual decreasing pattern in ARLs is observed in the graphs. Thus, the graphical behavior of ARLs also gives an idea about the detected small shifts of mean in the process sooner for larger value of rho (ρ) and smaller value of (λ).

ARLs graphs for performance efficiency comparison between the proposed repetitive EWMA-RSS control charts (for Tables 13–18) are provided in Appendix B. Figures 13–18 show that the proposed EWMA-RSS control chart based on Khan et al. [8] exponential ratio-type estimator performs more efficiently, generally in all the cases, than the proposed repetitive EWMA-RSS control chart based on Bahl and Tuteja (1991) exponential ratio-type estimator, by detecting shifts in the process mean sooner and faster than the rest of the charts.

4.3. Industrial Application. Here we have used the industrial data of Brinell hardness and tensile strength for the real bivariate process considered by Chen (1994), Wang and Chen (1998), and Sultan (1986). The variable of interest, Brinell hardness, is denoted by Y and the variable tensile strength is considered as an auxiliary variable, denoted by X with

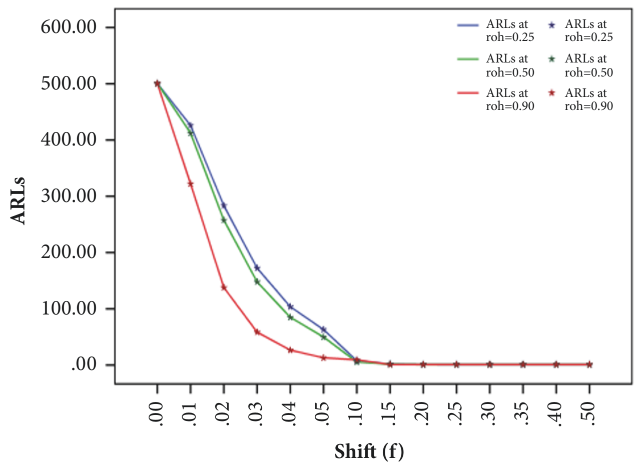

Figure 1: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) ratio estimator, when $r_o = 500$ at $\lambda = 0.1$.

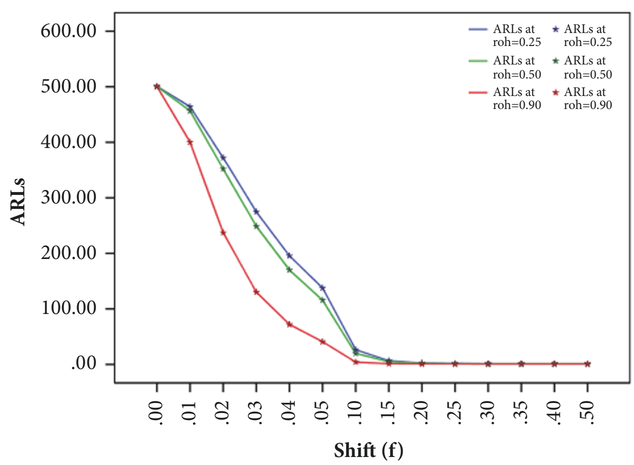

Figure 2: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) ratio estimator, when $r_o = 500$ at $\lambda = 0.2$.

average value $\mu = 50$. Data set of size 25 for both variables is as follows:

$$\begin{gathered}\mathbf{Y}: 143, 200, 168, 181, 148, 178, 162, 215, 161, 141, 175, 187, 187, 186, 172, 182, 177, 204, 178, 196, 160, 183, 179, 194, 181 \\ \mathbf{X}: 34.2, 57.0, 47.5, 53.4, 47.8, 51.5, 45.9, 59.1, 48.4, 47.3, 57.3, 58.5, 58.2, 57.0, 49.4, 57.2, 50.6, 55.1, 50.9, 57.9, 45.5, 53.9, 51.2, 57.5, 18.5.\end{gathered} \tag{32}$$

By using this data, we prepared the proposed repetitive EWMA-RSS control chart based on Singh and Tailor [12] ratio-type estimator, as this control chart has been found relatively more efficient among all the proposed charts in our study.

For the above data we have $\rho = 0.50$, and by considering the target value of ARL (r_o) 500 at $\lambda = 0.1$, we have determined the values of k_1 *and* k_2; i.e., $k_1 = 3.09$, $k_2 = 2.33$. After computation the outer and inner control limits of the proposed repetitive EWMA-RSS control chart (Figure 19) using Khan et al. [8] exponential ratio-type estimator for the above data are

$$\begin{aligned} LCL_1 &= 50.0413 \\ UCL_1 &= 51.5586 \\ LCL_2 &= 50.2279 \\ UCL_2 &= 51.3721. \end{aligned} \tag{33}$$

Since the above industrial data lies within inner and outer control limits and no point exists beyond the control limits, the process is in control.

5. Conclusion

Both the proposed repetitive EWMA-RSS charts, designed using Bahl and Tuteja (1991) and Khan et al. [8] exponential

Table 18: Average run lengths for proposed repetitive EWMA-RSS control charts using exponential ratio-type estimators, for $\rho = 0.90$ when r_o is 370 at $\lambda = 0.1$.

Shift	$k_1 = 3.0025$	$k_2 = 2.6008$
f	BT	HK
0	370.98	370.98
0.01	244.08	138.59
0.02	108.91	35.05
0.03	48.40	10.94
0.04	23.06	4.32
0.05	11.88	2.19
0.10	1.51	1.00
0.15	1.02	1.00
0.20	1.00	1.00
0.25	1.00	1.00
0.30	1.00	1.00
0.35	1.00	1.00
0.40	1.00	1.00
0.5	1.00	1.00

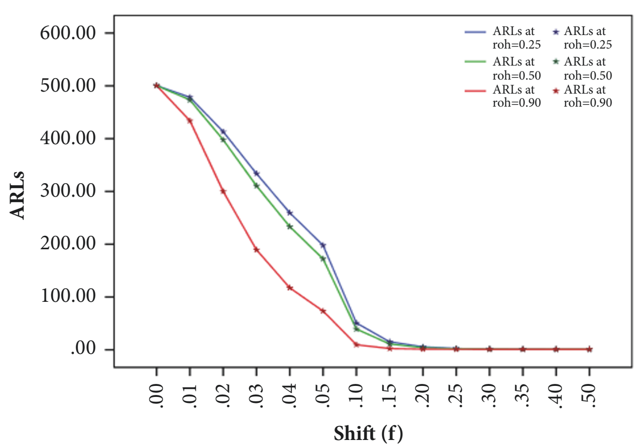

Figure 3: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) estimator, when $r_o = 500$ at $\lambda = 0.3$.

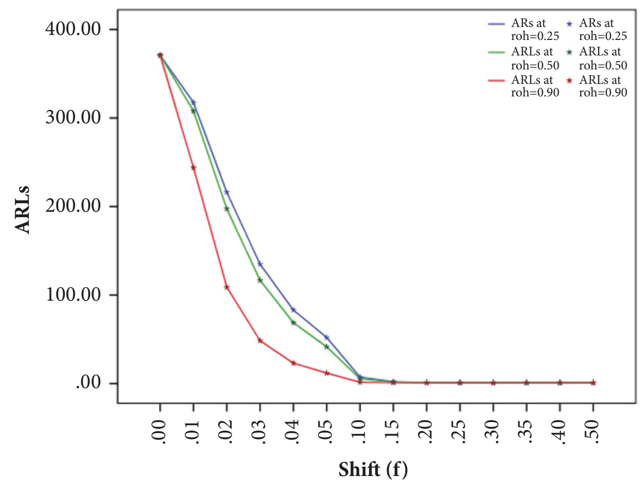

Figure 4: For proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) ratio estimator, when $r_o = 370$ at $\lambda = 0.1$.

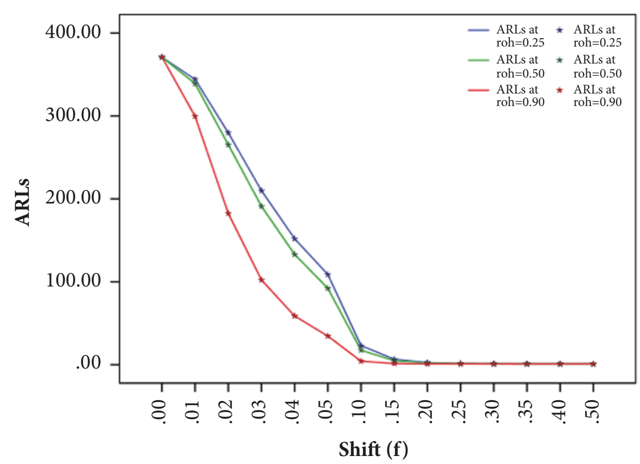

Figure 5: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) ratio estimator, when $r_o = 370$ at $\lambda = 0.2$.

ratio-type estimators, based on ranked set sampling RSS, are examined individually and collectively in order to observe their performance efficiencies on the basis of ARLs. The ARLs of both the proposed repetitive EWMA-RSS control charts have been evaluated using R codes. The two proposed repetitive EWMA-RSS control charts show sensitivity towards small or gradual changes in the process by the choice of the weighting factor (λ) and the level of correlation (ρ). The ARLs tables for the small random shifts in the process mean are set up by considering preconsidered target values of in-control ARLs, i.e., 500 and 370. Both the proposed repetitive EWMA-RSS control charts proved efficient because they detect shifts in the process mean earlier and quicker at high level of correlation as compared to the other levels of correlation, between the study and the auxiliary variables.

While examining the proposed repetitive EWMA-RSS control charts independently, it is observed that in all cases, as the value of ρ increases, the pattern of out-of-control ARLs tends to decrease rapidly, and when the values of smoothing constant λ increase, this leads to the increasing trend in the values of out-of-control ARLs. This can also be observed through the graphs of ARLs; that is, when ρ increases, the ARLs curves rapidly decrease downward.

While comparing the proposed repetitive EWMA-RSS control charts, based on Bahl and Tuteja (1991) and Khan et al. [8] exponential ratio-type estimators, collectively, it is revealed that in all cases, the proposed repetitive EWMA-RSS control chart using Khan et al. [8] exponential ratio-type estimator outperformed the proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) exponential ratio-type estimator chart, by means of detecting small shifts in the process mean much earlier.

Both the proposed repetitive EWMA-RSS control charts based on exponential ratio-type estimators have proven capable of monitoring process mean efficiently by keeping the

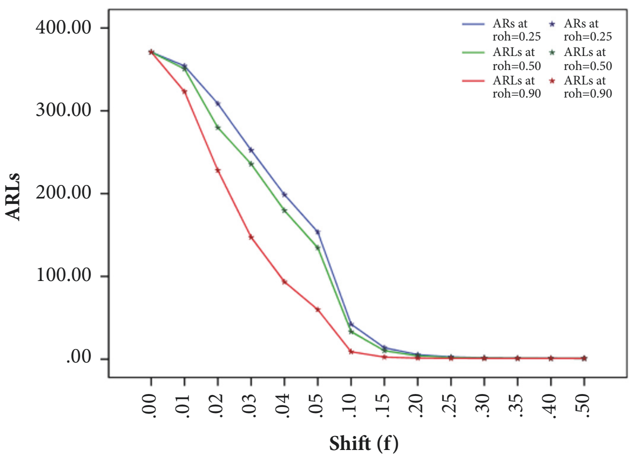

Figure 6: ARLs for proposed repetitive EWMA-RSS control chart using Bahl and Tuteja (1991) ratio estimator, when $r_o = 370$ at $\lambda = 0.3$.

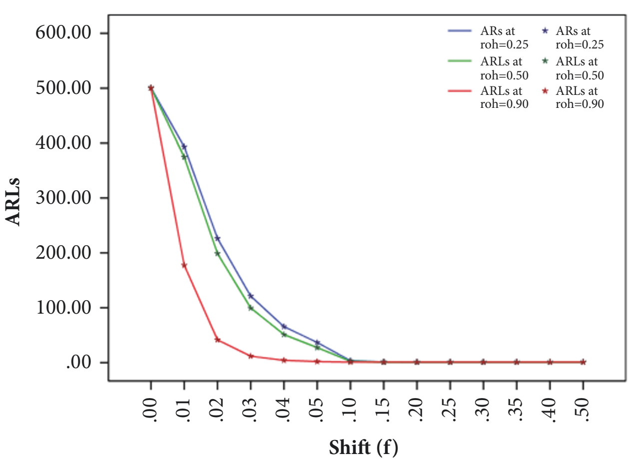

Figure 7: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] ratio estimator, when $r_o = 500$ at $\lambda = 0.1$.

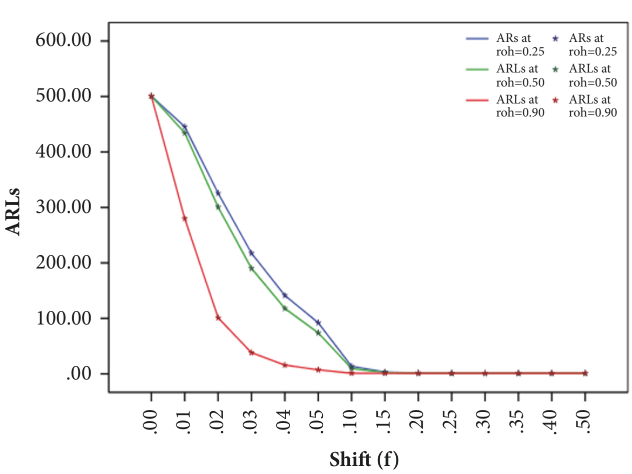

Figure 8: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] ratio estimator, when $r_o = 500$ at $\lambda = 0.2$.

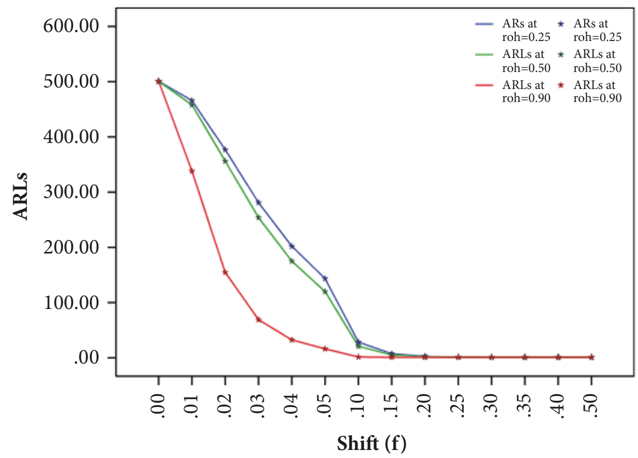

Figure 9: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] ratio estimator, when $r_o = 500$ at $\lambda = 0.3$.

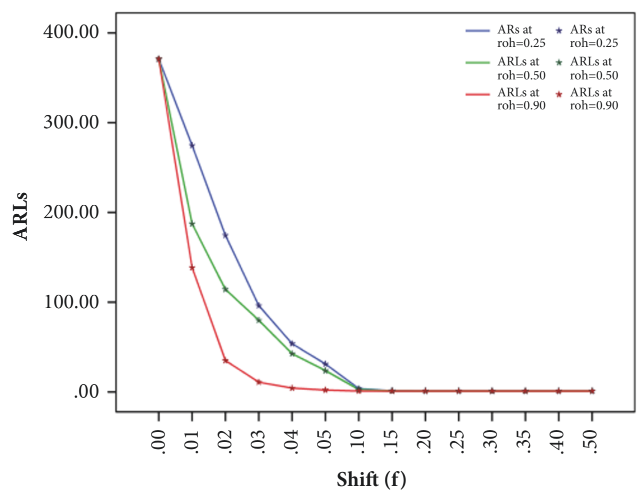

Figure 10: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] ratio estimator, when $r_o = 370$ at $\lambda = 0.1$.

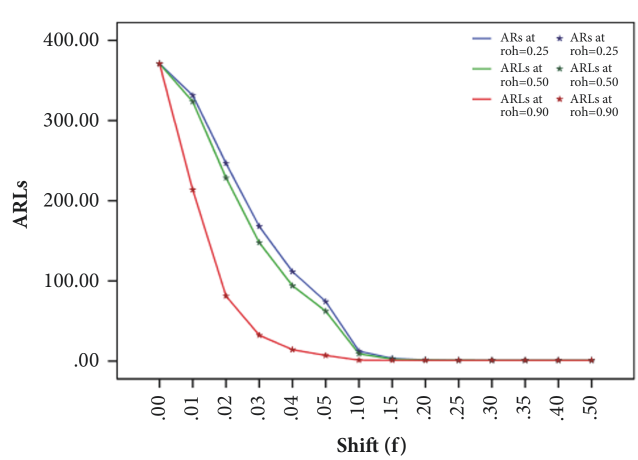

Figure 11: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] ratio estimator, when $r_o = 370$ at $\lambda = 0.2$.

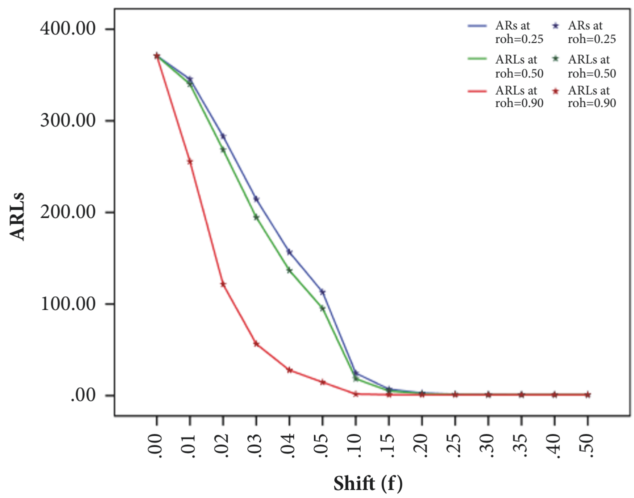

Figure 12: ARLs for proposed repetitive EWMA-RSS control chart using Khan et al. [8] ratio estimator, when $r_o = 370$ at $\lambda = 0.3$.

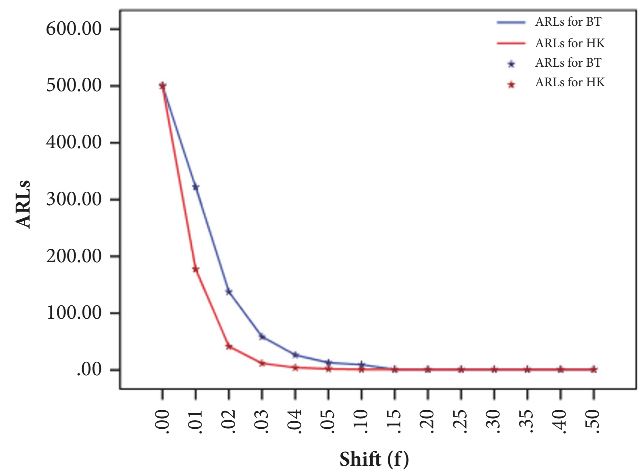

Figure 15: ARLs for proposed repetitive EWMA-RSS control charts using ratio estimators, for $\rho = 0.90$ when $r_o = 500$ at $\lambda = 0.1$.

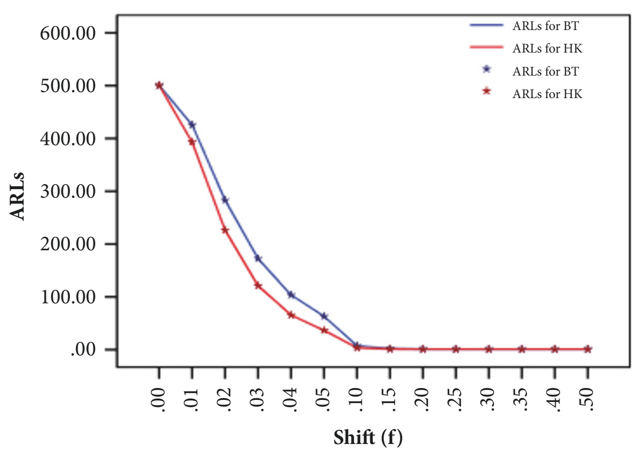

Figure 13: ARLs for proposed repetitive EWMA-RSS control charts using ratio estimators, for $\rho = 0.25$ when $r_o = 500$ at $\lambda = 0.1$.

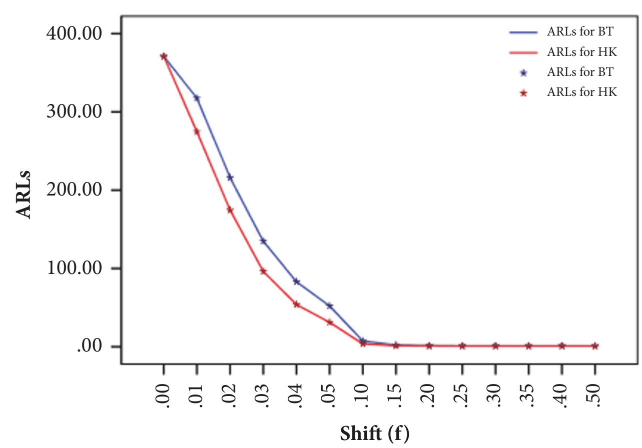

Figure 16: ARLs for proposed repetitive EWMA-RSS control charts using ratio estimators, for $\rho = 0.25$ when $r_o = 370$ at $\lambda = 0.1$.

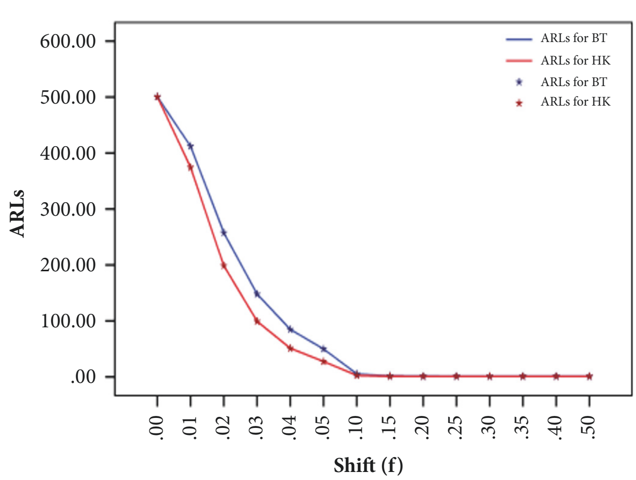

Figure 14: ARLs for proposed repetitive EWMA-RSS control charts using ratio estimators, for $\rho = 0.50$ when $r_o = 500$ at $\lambda = 0.1$.

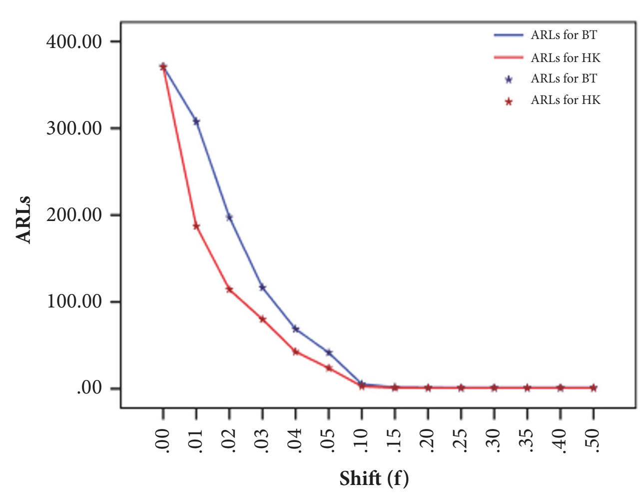

Figure 17: ARLs for proposed repetitive EWMA-RSS control charts using ratio estimators, for $\rho = 0.50$ when $r_o = 370$ at $\lambda = 0.1$.

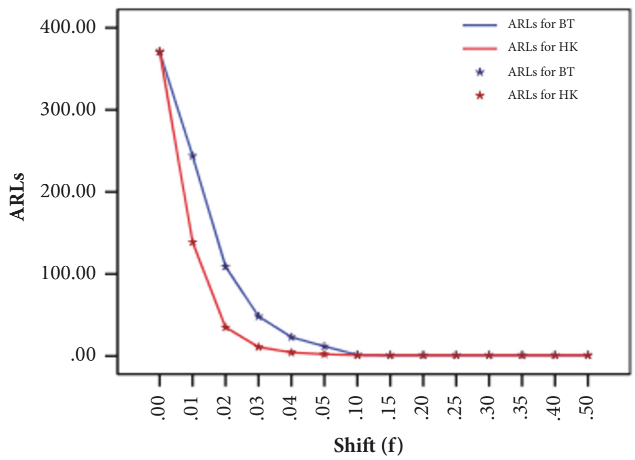

FIGURE 18: ARLs for proposed repetitive EWMA-RSS control charts using ratio estimators, for $\rho = 0.90$ when $r_o = 370$ at $\lambda = 0.1$.

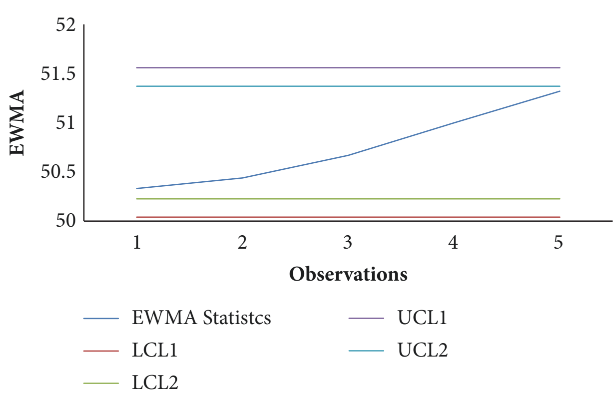

FIGURE 19: Proposed repetitive EWMA-RSS control chart for Industrial data.

process on target through quick detection of the small shifts in the process mean. Hence, it is recommended that these control charts must be used for future research work in order to get proficient results that would help to ensure the quality products.

Appendix

A. ARLs Graphs for the Proposed Repetitive EWMA-RSS Control Charts Based on Exponential Ratio-type Estimators (for Tables 1–12)

See Figures 1–12.

B. ARLs Graphs for Performance Efficiency Comparison between the Proposed Repetitive EWMA-RSS Control Charts (for Tables 19–24)

See Figures 13–18.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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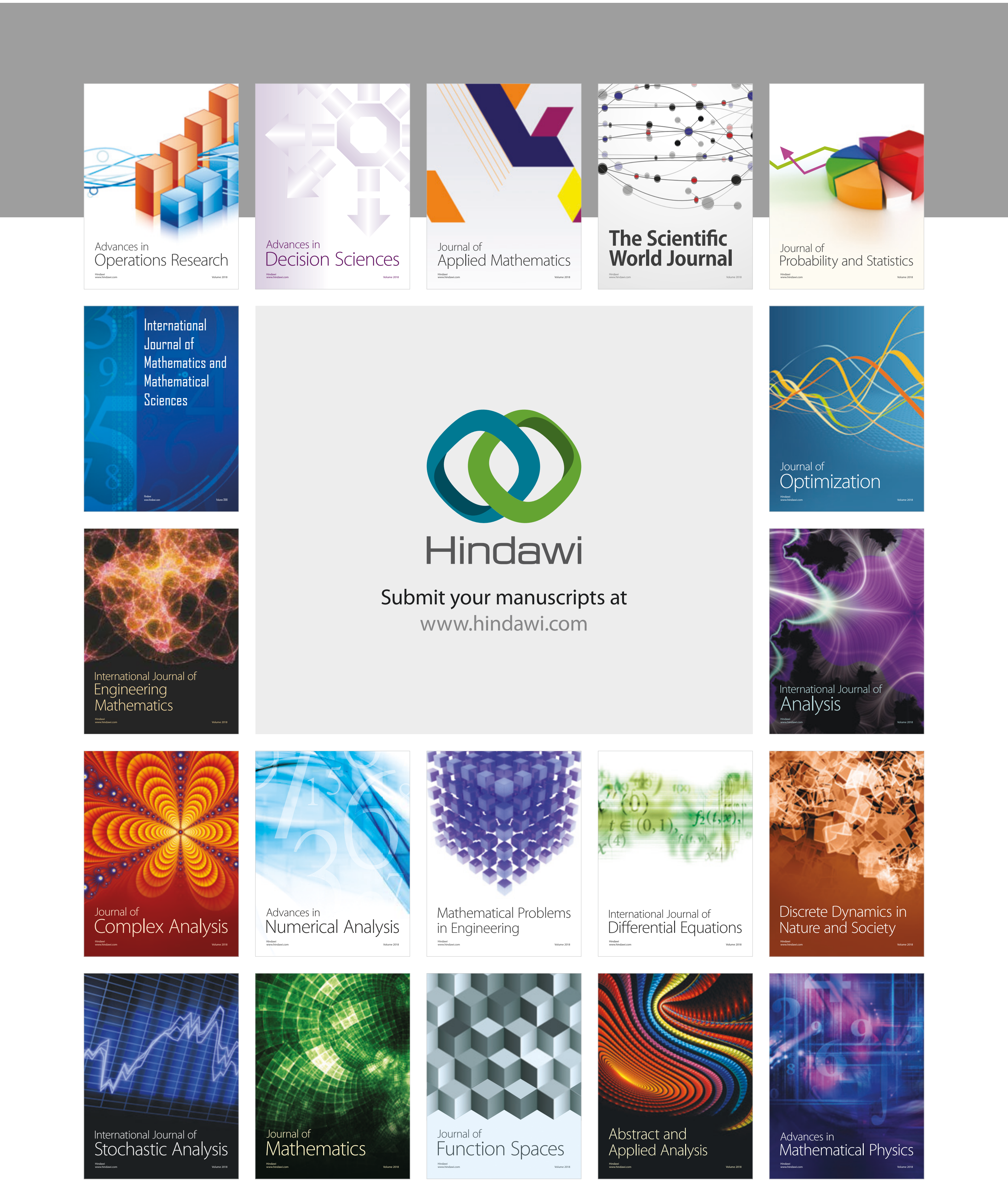
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