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Research Article

Attribute Control Chart for Rayleigh Distribution Using Repetitive Sampling under Truncated Life Test

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A control chart is an important tool in statistical process monitoring that is useful to monitor and improve production process quality. In this article, an attribute control chart using repetitive sampling under a truncated life test is proposed for monitoring the mean life of the product where the lifetime follows the Rayleigh distribution. The repetitive sampling parameters and the control limit coefficients of the chart are determined so that the in-control average run length (ARL) is very close to the target ARL. Tables of ARL values for various shift sizes in the scale parameter were presented, and the performance of the proposed chart is compared with the existing attribute control charts using the out-of-control ARL. The proposed control chart is shown to outperform the existing control charts in terms of ARL. An illustrative example is given to demonstrate the application of the proposed chart.

1. Introduction

A control chart is an important statistical process control (SPC) tool used to improve the quality of processes. It has been widely used in manufacturing and nonmanufacturing environments to reduce variability by indicating the presence of a special cause of variation and helps an organization to improve products and services, hence, increasing their profit margins. The control chart is a graphical representation of the information collected about a process that consists of the upper control limit, lower control limit, and the central line. A process is said to be in-control state if the data point is within the control limits; otherwise, it is exhibiting out-of-control conditions.

The control chart is classified into attribute and variable control charts. An attribute control chart is used for the monitoring of processes where the data are classified as conforming or nonconforming, such as the number of nonconforming products in a production process, whereas the variable control charts such as the mean or dispersion charts are used for monitoring continuous data. The design and application of control charts for different attributes and variable control charts have attracted the attention of many authors. Wu et al. [1] studied an optimal attribute control chart with curtailment. Ho and Quinino [2] proposed an attribute control chart for monitoring process variability. Khan et al. [3] introduced the variable control chart under the truncated life test for Weibull distribution. Further details about attribute and variable control charts can be found in Epprecht et al. [4], Chiu and Kuo [5], De Araujo Rodrigues et al. [6], Joekes and Barbosa [7], Arif et al. [8], and Rao et al. [9] to mention but a few.

Generally, the design of control charts for process monitoring has been done on the assumption that the process characteristic follows the normal distribution and that one has to wait until the testing process of the entire product is completed. However, in practice, this may be difficult because the form of distribution of the process is unknown or deviates from normality, and the testing process is time-consuming and requires more cost. Hence, designing a control chart for monitoring nonnormal process variables under the time truncated test is preferred to inspect the lifetime of the product because only a few random sample items are inspected for a specified time [10] and have attracted the attention of many researchers. Aslam and Jun [11] designed an attribute control chart for Weibull distribution under a truncated life test. Aslam et al. [12] proposed a control chart under a truncated life test to monitor the lifetime of a product that follows the Pareto distribution of the second kind. Rao [13] proposed an attribute control chart for the exponentiated half logistic distribution under the time truncated life test. Rosaiah et al. [14] studied the attribute control chart for the exponentiated Frechet distribution under the time truncated life test. Rao et al. [9] introduced a control chart for the Dagum distribution under the truncated life test. Adeoti and Ogundipe [15] proposed an attribute chart under the time truncated life test for the generalized exponential distribution. Rosaiah et al. [16] proposed a control chart for the type II generalized log-logistic distribution. Jafarian-Namin et al. [17] investigated the efficient design of attribute control charts under a truncated life test for the Weibull distribution. All aforementioned control charts are designed for the single sampling (SS) scheme.

In designing control charts, the use of different sampling schemes has been undertaken by different researchers in the SPC literature towards increasing the sensitivity of control charts to detect small-to-moderate shifts. The sampling schemes such as repetitive sampling, double sampling, multiple dependent state sampling, and multiple dependent state repetitive sampling are superior to single sampling for quick detection of process shifts. Repetitive sampling was originally proposed by Sherman [18] and has been used in the design of control charts extensively. In repetitive sampling, the process is repeated until a decision is made based on the current sample if the person cannot reach a decision from the previous sample. Aslam et al. [19] designed an attribute control chart for the Birnbaum–Saunders distribution using repetitive sampling. Adeoti [20] introduced the repetitive sampling DEWMA control chart for process monitoring. Jeyadurga and Balamurali [21] designed an attribute control chart under a truncated life test for process monitoring based on repetitive group sampling.

Many researchers have studied control charts in the SPC literature for some exponential families of distributions in which the Rayleigh distribution is one of them; however, designing a variable control chart for a nonnormal distribution like the Rayleigh distribution may be difficult because the exact distribution of the associated statistics may not be known. Variable control charts for some exponential families of distributions have been studied by many researchers in the SPC literature; however, designing an attribute control chart for the Rayleigh distribution is yet to be studied. Hence, designing an attribute control chart for a product that follows the Rayleigh distribution under a truncated life test is desirable.

Rayleigh distribution, which is a special case of the Weibull distribution, is one of the statistical distributions which can be used effectively in reliability engineering and life testing. It is a skewed distribution that has been studied by many authors because of its applications in engineering-related studies. Raza et al. [22] investigated the performance of the Shewhart control chart for Rayleigh distribution under type I censored data. Hossain et al. [23] proposed the Shewhart type control chart to monitor the single scale parameter of the Rayleigh distributed process. Adeoti and Rao [24] studied the moving average control chart to monitor the number of failures where the lifetime of the product follows the Rayleigh and inverse Rayleigh distribution. Shafqat et al. [25] designed a Shewhart $\overline{X}$ control chart for inverse Rayleigh under repetitive group sampling at lower record values. The Rayleigh distribution can be used quite efficiently for modeling the lifetime of products in the marketplace made from a manufacturing process as well. However, none of these authors considered the case under the truncated life test.

Since no work is available in the literature on the attribute control chart using repetitive sampling for monitoring the mean life of product where the lifetime follows the Rayleigh distribution under truncated life test to the best of the authors' knowledge, in this paper, we proposed the attribute control chart using repetitive sampling to monitor the mean life of the product where the lifetime follows the Rayleigh distribution under the truncated life test.

The design of the attribute control chart for Rayleigh distribution using repetitive sampling under truncated life test and the performance of the proposed control chart evaluated in terms of in-control ARL and out-of-control ARL are given in Section 2. The application of the proposed chart is demonstrated with the simulated and real life data in Section 3. A comparative study of the proposed control chart and single sampling control chart is given in Section 4. Section 5 presents the conclusion of the study.

2. Design of Attribute Chart for Rayleigh Distribution under Truncated Life Test

The cumulative distribution function (CDF) of the Rayleigh distribution is given by

$$F(t,\theta) = 1 - e^{-\left(t^2/2\theta^2\right)}, \quad t > 0, \theta > 0, \tag{1}$$

and the probability density function (PDF) of the Rayleigh distribution is given by

$$f(t,\theta) = \frac{t}{\theta^2}\, e^{-\left(t^2/2\theta^2\right)}, \quad t > 0, \theta > 0. \tag{2}$$

Let T be the lifetime of the product which is distributed as a Rayleigh distribution with scale parameter θ. The mean life of the Rayleigh distribution is given by

$$\mu = \theta\sqrt{\frac{\pi}{2}}. \tag{3}$$

We are concerned with monitoring the shift in the process quality by observing the number of failed products before the truncated time t_0 where $t_0 = a\mu_0$ when the process is in-control and a is truncated constant. The probability that a product failed before time t_0 when the process is in-control is given as

$$p_0 = 1 - e^{-\left(t_0^2/2\theta^2\right)}. \tag{4}$$

Equation (4) can be rewritten if the scale parameter θ is obtained in terms of μ as

$$p_0 = 1 - e^{-\left(a^2\pi/4\right)}. \tag{5}$$

We propose an attribute control chart for the Rayleigh distribution under truncated life test based on the number of products as follows:

(1) Select a random sample of n products from the production process. Conduct the life test on the product and consider t_0 as the termination time. Count the number of failed products denoted D before t_0.
(2) Declare the process as out-of-control if $D > \mathrm{UCL}_1$ or $D < \mathrm{LCL}_1$. Otherwise, declare the process as in-control if $\mathrm{LCL}_2 \leq D \leq \mathrm{UCL}_2$. However, if $\mathrm{UCL}_2 \leq D \leq \mathrm{UCL}_1$ or $\mathrm{LCL}_1 \leq D \leq \mathrm{LCL}_2$, repeat step 1.

The above procedure is known as the np control chart because the number of failures rather than the fraction defective (p) is plotted. The number of the failed products follows a binomial distribution with parameters n and p_0 for an in-control process, where n is the sample size and p_0 is the probability that a product failed before t_0 given in equation (5).

The proposed control chart consists of two pairs of control limits called the outer and inner control limits. The outer (upper and lower) control limits of the proposed control chart are given as

$$\mathrm{UCL}_1 = np_0 + L_1\sqrt{np_0(1-p_0)}, \tag{6a}$$

$$\mathrm{LCL}_1 = \max\left[0, np_0 - L_1\sqrt{np_0(1-p_0)}\right]. \tag{6b}$$

The inner (upper and lower) control limits of the proposed control chart are given as

$$\mathrm{UCL}_2 = np_0 + L_2\sqrt{np_0(1-p_0)}, \tag{6c}$$

$$\mathrm{LCL}_2 = \max\left[0, np_0 - L_2\sqrt{np_0(1-p_0)}\right]. \tag{6d}$$

where L_1 and L_2 are the coefficients of the control limits selected to achieve a desired in-control average run length (ARL_0) and p_0 is obtained from equation (5).

Sometimes, the probability of failed product p_0 is unknown; hence, the control limits of the number of the failed product are obtained for practical purposes as

$$\mathrm{UCL}_1 = \overline{D} + L_1\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)}, \tag{7a}$$

$$\mathrm{LCL}_1 = \max\left[0, \overline{D} - L_1\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)}\right], \tag{7b}$$

$$\mathrm{UCL}_2 = \overline{D} + L_2\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)}, \tag{7c}$$

$$\mathrm{LCL}_2 = \max\left[0, \overline{D} - L_2\sqrt{\overline{D}\left(1-\frac{\overline{D}}{n}\right)}\right], \tag{7d}$$

where $\overline{D}$ is the mean number of failed products that occur in the production process over the entire subgroup.

2.1. In-Control ARL (IC ARL) of Proposed Control Chart.

For the proposed control chart, the probability of a process to be out-of-control when the process is in-control is given by

$$p_{\mathrm{out},0}^{(0)} = P\left(D > \mathrm{UCL}_1 | p_0\right) + P\left(D < \mathrm{LCL}_1 | p_0\right), \tag{8}$$

or

$$\begin{aligned} p_{\mathrm{out},0}^{(0)} &= \sum_{d=\mathrm{UCL}_1+1}^{n}\binom{n}{d}p_0^d(1-p_0)^{n-d} + \sum_{d=0}^{\mathrm{LCL}_1}\binom{n}{d}p_0^d(1-p_0)^{n-d} \\ &= \sum_{d=\mathrm{UCL}_1+1}^{n}\binom{n}{d}\left(1-e^{-\left(a^2\pi/4\right)}\right)^d\left(e^{-\left(a^2\pi/4\right)}\right)^{n-d} + \sum_{d=0}^{\mathrm{LCL}_1}\binom{n}{d}\left(1-e^{-\left(a^2\pi/4\right)}\right)^d\left(e^{-\left(a^2\pi/4\right)}\right)^{n-d}. \end{aligned} \tag{9}$$

The probability of repetition when the process is in-control is given by

$$p_{\mathrm{rep}}^{(0)} = P\left(\mathrm{UCL}_2 < D < \mathrm{UCL}_1 | p_0\right) + P\left(\mathrm{LCL}_1 < D < \mathrm{LCL}_2 | p_0\right), \tag{10}$$

or

$$p_{\text{rep}}^{(0)} = \sum_{d=\text{LCL}_1+1}^{\text{LCL}_2} \binom{n}{d} p_0^d (1-p_0)^{n-d} + \sum_{d=\text{UCL}_2+1}^{\text{UCL}_1} \binom{n}{d} p_0^d (1-p_0)^{n-d}$$
$$= \sum_{d=\text{LCL}_1+1}^{\text{LCL}_2} \binom{n}{d} \left(1-e^{-(a^2\pi/4)}\right)^d \left(e^{-(a^2\pi/4)}\right)^{n-d} + \sum_{d=\text{UCL}_2+1}^{\text{UCL}_1} \binom{n}{d} \left(1-e^{-(a^2\pi/4)}\right)^d \left(e^{-(a^2\pi/4)}\right)^{n-d}. \quad (11)$$

Therefore, the probability of the process to be out-of-control when the process is in-control under repetitive sampling is given as

$$p_{\text{out}}^{(0)} = \frac{p_{\text{out},0}^{(0)}}{1-p_{\text{rep}}^{(0)}}. \quad (12)$$

The performance of the proposed control chart when the process is in-control is evaluated based on the in-control average run length (ARL_0). The ARL_0 is the average number of samples to signal out-of-control when the process is in control, and it is given as

$$\text{ARL}_0 = \frac{1}{p_{\text{out}}^{(0)}}. \quad (13)$$

The in-control average sample size (ASS_0) of the proposed chart is given as

$$\text{ASS}_0 = \frac{n}{1-p_{\text{rep}}^{(0)}}. \quad (14)$$

2.2. Out-of-Control ARL (OOC ARL) of Proposed Control Chart. The process is declared as out-of-control when the process is shifted to a new scale parameter $\theta_1 = c\theta$ where c is a constant. The probability that a product failed before the time t_0 when the process is shifted denoted p_1 is given as

$$p_1 = 1 - e^{-(a^2\pi f^2/4)}. \quad (15)$$

The probability that a process is out-of-control when the process is shifted is given as

$$p_{\text{out},1}^{(1)} = P(D > \text{UCL}_1 | p_1) + P(D < \text{LCL}_1 | p_1), \quad (16)$$

or

$$p_{\text{out},1}^{(1)} = \sum_{d=\text{UCL}_1+1}^{n} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \sum_{d=0}^{\text{LCL}_1} \binom{n}{d} p_1^d (1-p_1)^{n-d}$$
$$= \sum_{d=\text{UCL}_1+1}^{n} \binom{n}{d} \left(1-e^{-(a^2\pi f^2/4)}\right)^d \left(e^{-(a^2\pi f^2/4)}\right)^{n-d} + \sum_{d=0}^{\text{LCL}_1} \binom{n}{d} \left(1-e^{-(a^2\pi f^2/4)}\right)^d \left(e^{-(a^2\pi f^2/4)}\right)^{n-d}. \quad (17)$$

The probability of repetition when the process is shifted is given by

$$p_{\text{rep}}^{(1)} = P(\text{UCL}_2 < D < \text{UCL}_1 | p_1) + P(\text{LCL}_1 < D < \text{LCL}_2 | p_1), \quad (18)$$

or

$$p_{\text{rep}}^{(1)} = \sum_{d=\text{LCL}_1+1}^{\text{LCL}_2} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \sum_{d=\text{UCL}_2+1}^{\text{UCL}_1} \binom{n}{d} p_1^d (1-p_1)^{n-d}$$
$$= \sum_{d=\text{LCL}_1+1}^{\text{LCL}_2} \binom{n}{d} \left(1-e^{-(a^2\pi f^2/4)}\right)^d \left(e^{-(a^2\pi f^2/4)}\right)^{n-d} + \sum_{d=\text{UCL}_2+1}^{\text{UCL}_1} \binom{n}{d} \left(1-e^{-(a^2\pi f^2/4)}\right)^d \left(e^{-(a^2\pi f^2/4)}\right)^{n-d}. \quad (19)$$

Therefore, the probability of the process to be out-of-control when the process has shifted to out-of-control under repetitive sampling is given as

$$p_{\text{out}}^{(1)} = \frac{p_{\text{out},1}^{(0)}}{1-p_{\text{rep}}^{(1)}}. \quad (20)$$

The out-of-control average run length of the proposed chart denoted ARL_1 is given as

$$\text{ARL}_1 = \frac{1}{p_{\text{out}}^{(1)}}, \quad (21)$$

where the ARL_1 is the average number of samples to signal out-of-control when there is a shift in the process quality characteristics. The out-of-control average sample size (ASS_1) of the proposed chart is given as

$$ASS_1 = \frac{n}{1 - p_{rep}^{(1)}}. \quad (22)$$

Now, let the specified IC ARL be denoted R_0. The values of control chart constants L_1 and L_2 are determined such that $ARL_0 \geq R_0$ where $L_1 > L_2$. The algorithm for calculating the ARL values for different shift sizes and determining the parameters of the control chart is given in the following steps:

(i) Specify the in-control ARL value, say R_0 and sample size n

(ii) Determine the values of L_1, L_2, and constant a for which $ARL_0 \geq R_0$

(iii) Using selected values of L_1, L_2, n, and a obtained in previous steps, determine the ARL_1 for different shift constants

The computer code is written in the R program. Tables 1–4 present the ARL values for specified $ARL_0 = 200, 250, 300,$ and 370 for different shifts. Moreover, Tables 1–4 provide the average sample size (ASS) of the proposed control chart. From Tables 1–4, we observe the following trends in the proposed control chart:

(i) The ARL_1 values of the proposed control chart decrease as the shift parameter c decreases while other parameters are fixed. For example, when $ARL_0 = 200$ and $n = 20$, then $ARL_1 = 52.54$ for shift size $c = 1.10$ and $ARL_1 = 2.36$ for $c = 1.40$.

(ii) The ARL_1 values decrease as the sample size n increases for small shift size c and fixed ARL_0. For example, when $n = 20$ and $ARL_0 = 370$, then $ARL_1 = 16.23$ for shift size $c = 1.25$ while for $n = 40$, $ARL_1 = 2.29$ for same shift size c.

3. Examples

3.1. Simulation Study. In this section, the application of the proposed control chart is demonstrated using simulated data. The data are generated from a Rayleigh distribution when the process is in-control with parameters $\theta = 1$. We consider a random sample of size $n = 40$ for each sample batch. The first twenty samples are generated from the in-control process, and the next ten samples are from a shifted process with $c = 1.50$ to achieve 30 sample batches. Then, from Table 4, when $n = 40$ and $ARL_0 = 370$, we have constants $k_1 = 3.138, k_2 = 1.187$, and $a = 0.785$. The values of the nonconforming items are plotted with four control limits in Figure 1. Similarly, we plot the values of the nonconforming item for the single sampling attribute control chart in Figure 2 and compare their performances. From Figure 1, we observed that the proposed control chart detects a shift at the 27th sample while in Figure 2, the existing attribute control chart did not detect any shift. This shows the superiority of the proposed chart using repetitive sampling over the existing control chart.

3.2. Real-Life Application. In this section, a design example is given to demonstrate a real-life application of the proposed control chart using repetitive sampling. Suppose that a manufacturer is interested in enhancing the quality of its product. It is known that the failure time of the product follows the Rayleigh distribution with parameter $\theta = 1$, and the target mean life of the product is 500 hours. Let a sample of size $n = 20$ be taken from each subgroup and subjected to a truncated life test. The target in-control ARL, R_0 is 300. From Table 3, the constant $a = 0.76$, $k_1 = 3.115$, $k_2 = 1.257$, $UCL_1 = 13$, $LCL_1 = 0$, $UCL_2 = 9$, and $LCL_2 = 4$. We obtain $p_0 = 0.3647$ from equation (5). Therefore, the control chart to be set up by the manufacturer is designed as follows:

Step 1: Select a sample of 20 products from each subgroup and conduct a life test for 380 hours. Record the number of failed products denoted D before 380 hours.

Step 2: Declare the process as out-of-control if $D > 13$ or $D < 0$. Declare the process as in-control if $4 \leq D \leq 9$.

Step 3: Repeat step 1 if $0 \leq D \leq 4$ or $9 \leq D \leq 13$.

For the single sampling control chart, the control chart to be set up by the manufacturer is designed as follows:

Step 1: Select a sample of 20 products from each subgroup and conduct a life test for 420 hours. Record the number of failed products denoted D before 420 hours

Step 2: Declare the process as out-of-control if $D > 16$ or $D < 3$. Declare the process as in-control if $3 \leq D \leq 16$.

4. Comparative Study

First, the efficiency of the proposed control chart is compared with the control chart based on single sampling (SS) for the Rayleigh distribution under truncated life. Note that the attribute control chart for the Rayleigh distribution under truncated life test based on the single sampling does not exist yet in the SPC literature; however, it is included for comparison purposes. To construct the chart for single sampling, the values of n, k, a, UCL, and LCL are determined to achieve an ARL_0 value of 370. The ARL values of the proposed control chart and the single sampling chart are given in Table 5. In addition, the performance of the proposed control chart is compared with another nonnormal skewed control chart. Since Rayleigh distribution belongs to the exponential family of distributions, but the field of application differs from each other, we compare the proposed control chart with one of the well-known exponential families named Weibull distribution. For comparison purposes, we consider the ARLs of the two control charts for different out-of-control ARL values. A chart with smaller out-of-control ARL values is considered superior to the

Table 1: ARL and ASS of the Rayleigh distribution under time truncated life test for $ARL_0 = 200$.

	$\mathbf{k}_1 = 2.949$, $k_2 = 2.028$		$\mathbf{k}_1 = 3.056$, $k_2 = 0.991$		$\mathbf{k}_1 = 2.944$, $k_2 = 2.207$		$\mathbf{k}_1 = 2.812$, $k_2 = 2.329$		$\mathbf{k}_1 = 2.843$, $k_2 = 1.951$	
a	0.7805		0.775		0.85		0.68		0.595	
$\mathbf{LCL_1}$	1		2		5		3		2	
$\mathbf{LCL_2}$	3		7		7		4		4	
$\mathbf{UCL_1}$	13		16		20		18		17	
$\mathbf{UCL_2}$	11		11		18		16		14	
n	20		25		30		35		40	
c	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS
1.00	200.44	21.23	200.87	42.21	200.10	31.12	200.88	35.77	200.11	42.50
1.05	106.18	21.55	105.61	43.26	110.32	31.48	115.76	36.28	100.81	43.61
1.10	52.54	22.24	44.59	47.20	46.60	32.51	51.49	37.28	43.97	45.78
1.15	26.91	23.31	18.83	54.12	20.59	34.22	23.56	38.85	20.01	49.06
1.20	14.56	24.74	8.41	63.70	9.96	36.57	11.68	40.97	9.83	53.32
1.25	8.35	26.45	4.13	73.95	5.32	39.22	6.33	43.44	5.28	57.98
1.30	5.11	28.24	2.34	80.37	3.16	41.52	3.77	45.76	3.14	61.87
1.35	3.34	29.82	1.58	78.75	2.10	42.65	2.47	47.27	2.10	63.59
1.40	2.36	30.80	1.25	70.08	1.57	42.19	1.80	47.46	1.57	62.45
1.45	1.80	30.93	1.11	59.12	1.30	40.40	1.43	46.30	1.29	59.05
1.50	1.47	30.21	1.05	49.35	1.15	38.01	1.23	44.26	1.15	54.71
1.55	1.27	28.88	1.02	41.82	1.08	35.67	1.13	41.96	1.08	50.54
1.60	1.16	27.28	1.01	36.39	1.04	33.75	1.07	39.87	1.04	47.11
1.65	1.09	25.67	1.00	32.57	1.02	32.33	1.03	38.19	1.02	44.55
1.70	1.05	24.25	1.00	29.94	1.01	31.37	1.02	36.97	1.01	42.76
1.75	1.03	23.07	1.00	28.16	1.00	30.76	1.01	36.15	1.00	41.60
1.80	1.02	22.15	1.00	26.97	1.00	30.40	1.00	35.63	1.00	40.88
1.85	1.01	21.47	1.00	26.19	1.00	30.20	1.00	35.33	1.00	40.45
1.90	1.00	20.97	1.00	25.70	1.00	30.09	1.00	35.16	1.00	40.22
1.95	1.00	20.62	1.00	25.39	1.00	30.04	1.00	35.07	1.00	40.10
2.00	1.00	20.39	1.00	25.21	1.00	30.02	1.00	35.03	1.00	40.05

Table 2: ARL and ASS of the Rayleigh distribution under time truncated life test for $ARL_0 = 250$.

	$\mathbf{k}_1 = 2.964$, $k_2 = 1.174$		$\mathbf{k}_1 = 3.03$, $k_2 = 2.722$		$\mathbf{k}_1 = 3.011$, $k_2 = 1.874$		$\mathbf{k}_1 = 2.958$, $k_2 = 2.234$		$\mathbf{k}_1 = 2.957$, $k_2 = 2.113$	
a	0.9653		0.655		0.76		0.745		0.985	
$\mathbf{LCL_1}$	3		0		3		4		12	
$\mathbf{LCL_2}$	7		1		6		5		14	
$\mathbf{UCL_1}$	16		13		18		20		30	
$\mathbf{UCL_2}$	12		13		15		18		27	
n	20		25		30		35		40	
c	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS
1.00	250.32	27.29	250.03	25.32	250.66	32.68	250.50	35.64	250.52	41.46
1.05	121.63	29.05	116.16	25.55	125.75	33.27	126.22	36.18	158.45	42.59
1.10	50.42	32.64	56.28	25.98	52.81	34.93	51.81	37.24	52.31	45.40
1.15	21.28	38.33	29.06	26.65	23.10	37.70	22.60	38.93	18.30	50.29
1.20	9.51	46.13	15.97	27.62	10.91	41.51	10.85	41.24	7.33	57.07
1.25	4.65	54.98	9.30	28.92	5.63	45.88	5.75	43.88	3.46	63.89
1.30	2.59	61.88	5.73	30.52	3.23	49.72	3.39	46.26	1.98	67.10
1.35	1.70	63.16	3.73	32.32	2.10	51.58	2.23	47.57	1.40	64.54
1.40	1.31	58.28	2.57	34.08	1.55	50.62	1.64	47.32	1.16	58.36
1.45	1.13	50.37	1.89	35.49	1.27	47.43	1.34	45.67	1.06	51.99
1.50	1.06	42.55	1.48	36.22	1.13	43.37	1.17	43.30	1.02	47.10
1.55	1.03	36.18	1.23	36.11	1.07	39.53	1.09	40.91	1.01	43.88
1.60	1.01	31.39	1.09	35.20	1.03	36.41	1.04	38.90	1.00	41.97
1.65	1.00	27.92	1.01	33.77	1.01	34.11	1.02	37.40	1.00	40.92
1.70	1.00	25.44	0.97	32.13	1.01	32.52	1.01	36.39	1.00	40.40
1.75	1.00	23.69	0.95	30.52	1.00	31.48	1.00	35.75	1.00	40.16
1.80	1.00	22.47	0.95	29.10	1.00	30.83	1.00	35.38	1.00	40.06

Table 2: Continued.

	$\mathbf{k}_1 = 2.964$ $k_2 = 1.174$		$\mathbf{k}_1 = 3.03$ $k_2 = 2.722$		$\mathbf{k}_1 = 3.011$ $k_2 = 1.874$		$\mathbf{k}_1 = 2.958$ $k_2 = 2.234$		$\mathbf{k}_1 = 2.957$ $k_2 = 2.113$	
1.85	1.00	21.62	0.96	27.93	1.00	30.44	1.00	35.18	1.00	40.02
1.90	1.00	21.05	0.96	27.03	1.00	30.22	1.00	35.08	1.00	40.01
1.95	1.00	20.66	0.97	26.36	1.00	30.11	1.00	35.03	1.00	40.00
2.00	1.00	20.41	0.98	25.88	1.00	30.05	1.00	35.01	1.00	40.00

Table 3: ARL and ASS of the Rayleigh distribution under time truncated life test for $ARL_0 = 300$.

	$\mathbf{k}_1 = 3.115$ $k_2 = 1.257$		$\mathbf{k}_1 = 3.089$ $k_2 = 1.855$		$\mathbf{k}_1 = 2.965$ $k_2 = 2.349$		$\mathbf{k}_1 = 3.28$ $k_2 = 2.409$		$\mathbf{k}_1 = 2.976$ $k_2 = 2.788$	
a	0.76		0.78		0.64		0.51		0.7	
$\mathbf{LCL_1}$	0		2		1		0		4	
$\mathbf{LCL_2}$	4		4		2		0		4	
$\mathbf{UCL_1}$	13		16		15		13		21	
$\mathbf{UCL_2}$	9		13		13		11		20	
n	20		25		30		35		40	
c	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS
1.00	300.15	26.45	300.77	26.71	300.27	30.64	300.03	35.86	300.42	40.30
1.05	131.46	27.70	149.16	27.61	151.14	31.07	159.45	36.25	146.01	40.40
1.10	58.26	30.13	64.67	29.23	69.21	31.83	77.85	36.98	58.56	40.77
1.15	26.90	33.85	29.09	31.67	33.15	33.00	39.22	38.09	25.43	41.43
1.20	13.05	38.88	13.98	34.96	17.00	34.59	20.90	39.61	12.31	42.39
1.25	6.74	44.84	7.23	38.90	9.35	36.55	11.82	41.49	6.63	43.56
1.30	3.79	50.61	4.09	42.80	5.53	38.64	7.10	43.59	3.96	44.71
1.35	2.37	54.26	2.56	45.55	3.53	40.51	4.54	45.66	2.61	45.52
1.40	1.68	54.20	1.80	46.09	2.44	41.69	3.10	47.31	1.89	45.73
1.45	1.34	50.57	1.41	44.26	1.83	41.86	2.27	48.20	1.50	45.29
1.50	1.17	45.05	1.21	40.93	1.48	41.00	1.77	48.11	1.28	44.38
1.55	1.09	39.35	1.11	37.23	1.28	39.40	1.47	47.09	1.15	43.28
1.60	1.04	34.41	1.05	33.87	1.16	37.49	1.29	45.44	1.08	42.25
1.65	1.02	30.47	1.03	31.17	1.09	35.62	1.17	43.50	1.04	41.42
1.70	1.01	27.48	1.01	29.14	1.05	34.01	1.10	41.58	1.02	40.82
1.75	1.00	25.25	1.01	27.68	1.03	32.73	1.06	39.87	1.01	40.44
1.80	1.00	23.63	1.00	26.68	1.01	31.77	1.04	38.46	1.00	40.22
1.85	1.00	22.47	1.00	26.02	1.01	31.11	1.02	37.37	1.00	40.10
1.90	1.00	21.65	1.00	25.59	1.00	30.66	1.01	36.56	1.00	40.04
1.95	1.00	21.07	1.00	25.33	1.00	30.38	1.01	35.99	1.00	40.02
2.00	1.00	20.69	1.00	25.18	1.00	30.21	1.00	35.61	1.00	40.01

other control charts. Table 6 displays the ARL values of the attribute control charts for the Rayleigh and Weibull distributions.

From Table 5, we can observe that the ARL values (boldface) of the proposed control chart are smaller than those of the control chart based on single sampling. For example, if $c = 1.30$, the ARL_1 of the proposed control chart for $n = 20$ is 9.82 while the ARL_1 is 13.51 for a single sampling control chart. This indicates that the proposed control chart is quicker at detecting process shifts than the control chart based on single sampling. From Table 6, it can be seen that the attribute control chart based on the Rayleigh distribution has smaller ARL values for each shift size compared to the Weibull distribution.

Table 4: ARL and ASS of the Rayleigh distribution under time truncated life test for ARL_0 = 370.

	$\mathbf{k}_1 = 3.216$, $k_2 = 3.215$		$\mathbf{k}_1 = 3.137$, $k_2 = 1.404$		$\mathbf{k}_1 = 3.167$, $k_2 = 1.941$		$\mathbf{k}_1 = 3.129$, $k_2 = 1.448$		$\mathbf{k}_1 = 3.138$, $k_2 = 1.187$	
a	0.695		0.775		0.64		0.705		0.785	
$\mathbf{LCL_1}$	0		1		0		2		5	
$\mathbf{LCL_2}$	0		6		3		7		11	
$\mathbf{UCL_1}$	12		16		15		19		24	
$\mathbf{UCL_2}$	12		12		12		15		18	
n	20		25		30		35		40	
c	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS	ARL	ASS
1.00	370.47	20.26	370.24	31.77	371.05	31.98	370.05	40.97	370.34	53.66
1.05	196.29	20.36	149.56	32.42	150.34	32.80	138.44	41.72	121.23	57.79
1.10	98.55	20.58	61.29	34.67	64.71	34.35	52.75	44.47	37.47	67.48
1.15	51.29	20.96	26.44	38.60	29.86	36.70	21.59	49.29	12.55	83.46
1.20	28.12	21.54	12.13	44.18	14.76	39.87	9.61	55.98	4.81	103.40
1.25	16.23	22.36	6.02	50.83	7.83	43.64	4.74	63.39	2.29	116.75
1.30	9.82	23.43	3.31	56.89	4.50	47.45	2.67	68.94	1.45	111.69
1.35	6.22	24.77	2.08	59.81	2.84	50.36	1.76	69.74	1.15	93.51
1.40	4.12	26.31	1.51	58.09	1.98	51.38	1.35	65.45	1.05	75.06
1.45	2.86	27.92	1.24	52.81	1.53	50.18	1.16	58.55	1.02	61.50
1.50	2.08	29.40	1.11	46.37	1.28	47.30	1.07	51.68	1.01	52.66
1.55	1.60	30.50	1.05	40.49	1.15	43.71	1.03	46.11	1.00	47.17
1.60	1.31	30.97	1.02	35.80	1.08	40.22	1.01	42.05	1.00	43.88
1.65	1.12	30.75	1.01	32.32	1.04	37.27	1.01	39.28	1.00	41.99
1.70	1.02	29.89	1.01	29.84	1.02	34.96	1.00	37.49	1.00	40.96
1.75	0.96	28.63	1.00	28.11	1.01	33.26	1.00	36.38	1.00	40.43
1.80	0.93	27.18	1.00	26.95	1.01	32.06	1.00	35.72	1.00	40.18
1.85	0.92	25.76	1.00	26.18	1.00	31.26	1.00	35.36	1.00	40.07
1.90	0.92	24.47	1.00	25.69	1.00	30.74	1.00	35.17	1.00	40.03
1.95	0.93	23.37	1.00	25.39	1.00	30.41	1.00	35.08	1.00	40.01
2.00	0.94	22.48	1.00	25.21	1.00	30.22	1.00	35.03	1.00	40.00

Table 5: ARL values of attribute control chart under repetitive sampling (RS) and single sampling (SS) when $ARL_0 = 370$.

Shift c	$n = 20$ RS $a = 0.695$ $k_1 = 3.216$ $k_2 = 3.215$ $LCL_1 = 0, LCL_2 = 0$ $UCL_1 = 12$ $UCL_2 = 12$	$n = 20$ SS $a = 0.9241$ $k = 3.035$ $LCL = 3$ $UCL = 16$	$n = 25$ RS $a = 0.775$ $k_1 = 3.137$ $k_2 = 1.404$ $LCL_1 = 1, LCL_2 = 6$ $UCL_1 = 16$ $UCL_2 = 12$	$n = 25$ SS $a = 0.7313$ $k = 3.141$ $LCL = 1$ $UCL = 15$	$n = 30$ RS $a = 0.64$ $k_1 = 3.167$ $k_2 = 1.941$ $LCL_1 = 0, LCL_2 = 3$ $UCL_1 = 15$ $UCL_2 = 12$	$n = 30$ SS $a = 0.8546$ $k = 2.983$ $LCL = 5$ $UCL = 21$	$n = 35$ RS $a = 0.705$ $k_1 = 3.129$ $k_2 = 1.448$ $LCL_1 = 2, LCL_2 = 7$ $UCL_1 = 19$ $UCL_2 = 15$	$n = 35$ SS $a = 0.8009$ $k = 3.052$ $LCL = 5$ $UCL = 22$	$n = 40$ RS $a = 0.785$ $k_1 = 3.138$ $k_2 = 1.187$ $LCL_1 = 5, LCL_2 = 11$ $UCL_1 = 24$ $UCL_2 = 18$	$n = 40$ SS $a = 0.58595$ $k = 3.139$ $LCL = 1$ $UCL = 13$
1.00	370.47	370.20	370.24	370.10	371.05	370.10	370.05	370.40	370.34	370.24
1.05	**196.29**	324.86	**149.56**	171.53	**150.34**	281.30	**138.44**	189.44	**121.23**	153.83
1.10	**98.55**	168.56	**61.29**	78.85	**64.71**	120.01	**52.75**	74.37	**37.47**	66.22
1.15	**51.29**	81.17	**26.44**	39.02	**29.86**	50.94	**21.59**	31.73	**12.55**	31.32
1.20	**28.12**	41.44	**12.13**	20.96	**14.76**	23.94	**9.61**	15.26	**4.81**	16.32
1.25	**16.23**	22.81	**6.02**	12.17	**7.83**	12.51	**4.74**	8.22	**2.29**	9.30
1.30	**9.82**	13.51	**3.31**	7.59	**4.50**	7.22	**2.67**	4.92	**1.45**	5.76
1.35	**6.22**	8.55	**2.08**	5.06	**2.84**	4.56	**1.76**	3.24	**1.15**	3.86
1.40	**4.12**	5.76	**1.51**	3.59	**1.98**	3.12	**1.35**	2.32	**1.05**	2.77
1.45	**2.86**	4.11	**1.24**	2.69	**1.53**	2.30	**1.16**	1.79	**1.02**	2.11
1.50	**2.08**	3.08	**1.11**	2.12	**1.28**	1.81	**1.07**	1.47	**1.01**	1.71
1.55	**1.60**	2.42	**1.05**	1.75	**1.15**	1.51	**1.03**	1.28	**1.00**	1.45
1.60	**1.31**	1.98	**1.02**	1.50	**1.08**	1.31	**1.01**	1.16	**1.00**	1.28
1.65	**1.12**	1.68	**1.01**	1.33	**1.04**	1.19	**1.01**	1.09	**1.00**	1.17
1.70	**1.02**	1.48	**1.01**	1.22	**1.02**	1.11	**1.00**	1.05	**1.00**	1.10
1.75	**1.00**	1.33	**1.00**	1.14	**1.01**	1.07	**1.00**	1.02	**1.00**	1.06
1.80	**1.00**	1.23	**1.00**	1.09	**1.01**	1.04	**1.00**	1.01	**1.00**	1.03
1.85	**1.00**	1.16	**1.00**	1.06	**1.00**	1.02	**1.00**	1.01	**1.00**	1.02
1.90	**1.00**	1.11	**1.00**	1.03	**1.00**	1.01	1.00	1.00	**1.00**	1.01
1.95	**1.00**	1.07	**1.00**	1.02	1.00	1.00	1.00	1.00	1.00	1.00
2.00	**1.00**	1.05	**1.00**	1.01	1.00	1.00	1.00	1.00	1.00	1.00

Table 6: ARL values of attribute control chart under repetitive sampling (RS) and single sampling (SS) for Rayleigh and Weibull distributions when $ARL_0 = 370$.

Shift c	$n = 20$				$n = 30$			
	Rayleigh		Weibull		Rayleigh		Weibull	
	RS	SS	RS	SS	RS	SS	RS	SS
	$a = 0.695$ $k_1 = 3.216$ $k_2 = 3.215$	$a = 0.9241$ $k = 3.035$	$a = 0.67595$ $k_1 = 3.105$ $k_2 = 1.357$	$a = 0.6959$ $k = 2.8495$	$a = 0.64$ $k_1 = 3.167$ $k_2 = 1.941$	$a = 0.8546$ $k = 2.983$	$a = 0.5424$ $k_1 = 3.325$ $k_2 = 1.407$	$a = 0.7084$ $k = 2.8607$
1.00	370.47	370.20	370.18	370.10	371.05	370.10	370.46	371.49
1.10	98.55	168.56	258.36	438.10	64.71	120.01	222.52	216.61
1.20	28.12	41.44	95.87	193.79	14.76	23.94	81.96	68.62
1.30	9.82	13.51	38.85	89.99	4.50	7.22	33.75	28.10
1.40	4.12	5.76	17.87	48.47	1.98	3.12	15.80	14.25
1.50	2.08	3.08	9.22	29.40	1.28	1.81	8.30	8.44
1.60	1.31	1.98	5.30	19.52	1.08	1.31	4.86	5.62
1.70	1.02	1.48	3.38	13.90	1.02	1.11	3.15	4.08
1.80	1.00	1.23	2.38	10.46	1.01	1.04	2.26	3.16
1.90	1.00	1.11	1.83	8.23	1.00	1.01	1.77	2.58
2.00	1.00	1.05	1.52	6.69	1.00	1.00	1.48	2.19

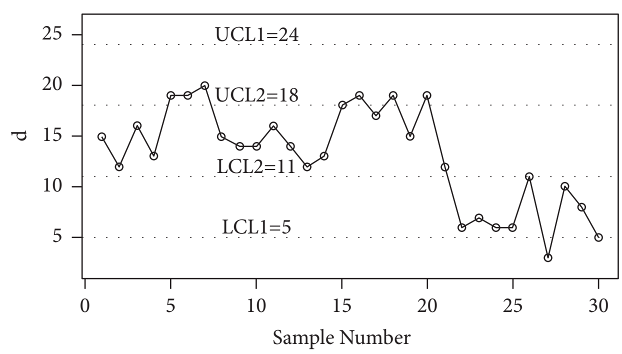

Figure 1: The proposed control chart for simulated data.

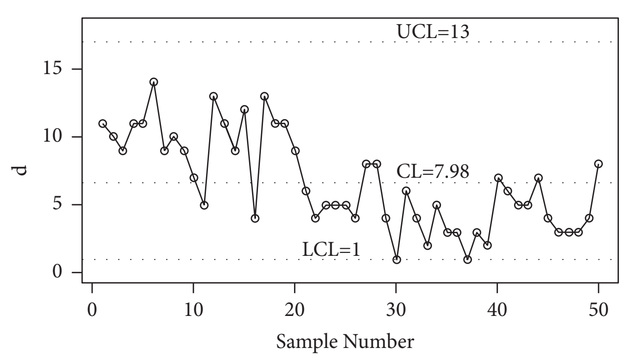

Figure 2: The single sampling control chart for simulated data.

5. Conclusion

In this paper, an attribute control chart for repetitive sampling using the Rayleigh distribution under a truncated life test is presented. The structure of the proposed control chart is developed to assess the performance of the control chart based on ARL values. Tables of control chart constants are given for different quality parameters for practical purposes. From the results of the study, the proposed control chart has smaller ARL values compared to the control chart based on single sampling for monitoring process shifts. This shows that the proposed control chart is more sensitive and detects shifts faster than a single sampling chart. The proposed control chart can be used to monitor the lifetime of the process quality characteristics that follows the nonnormal distribution. The proposed control chart can be extended for future research using some other sampling schemes. Similarly, the design of an attribute control chart under neutrosophic statistics can be considered a future study.

Data Availability

The used data are included in the paper.

Conflicts of Interest

The authors declare that they have no potential conflicts of interest.

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