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Transmissibility matrix in harmonic and random processes

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Abstract. The transmissibility concept may be generalized to multi-degree-of-freedom systems with multiple random excitations. This generalization involves the definition of a transmissibility matrix, relating two sets of responses when the structure is subjected to excitation at a given set of coordinates. Applying such a concept to an experimental example is the easiest way to validate this method.

Keywords: Transmissibility, random vibration, spectral density, modal analysis, structural dynamics

1. Introduction

Structural dynamics involving modal analysis techniques allows for the prediction of the dynamic response of a structure from the knowledge of its modal characteristics, applied forces and applied moments. However, in the case of unknown inputs, the available techniques suffer from serious limitations, since it is only possible to predict the dynamic behavior in quite a limited number of cases, often using assumed distributions of forces in a more or less approximate way.

This work intends to contribute to the reduction of those limitations, stepping forward in the study of the transmissibility in real structures, with multiple degrees of freedom (MDOF), and study its implications and possible practical uses. In this line of thought, our aim is to develop algorithms which, using a generalized transmissibility concept to MDOF and multiple generalized forces, allow foreseeing the responses of structures in certain coordinates from the knowledge of the responses in other coordinates whenever that is possible.

This concept was presented, in 1998, by Ewins and Liu [1] and, in the same year, Varoto and McConnell [2], dealt with the relationships among displacements for certain specific cases.

The present concept of transmissibility for MDOF systems can be found in previous papers by the authors [3–6], where the transmissibility matrix was calculated using either the Frequency Response Functions (FRFs) matrix of the structure, or the measured responses only. In the former case, one can calculate the transmissibility matrix using the FRFs of the system; it was proven that one can relate the ‘unknown’ responses to the ‘known’ ones using such a matrix. In the latter case one can calculate the transmissibility matrix using measured – only responses, building a matrix of ‘known’ responses and another one of ‘unknown’ responses for various tests. In any case, it was always assumed that the applied forces were periodic. A short review of this concept is presented in the Annex.

This paper is a continuation of those previous works, proposing a method for evaluating the transmissibility matrix when the applied forces are of a random kind. The transmissibility will therefore be evaluated by relating the spectral densities of the responses. The result will be compared with the one obtained by using the transmissibility matrix based on the FRFs. In fact, it will be proven that the transmissibility matrix is the same, independently of the applied forces.

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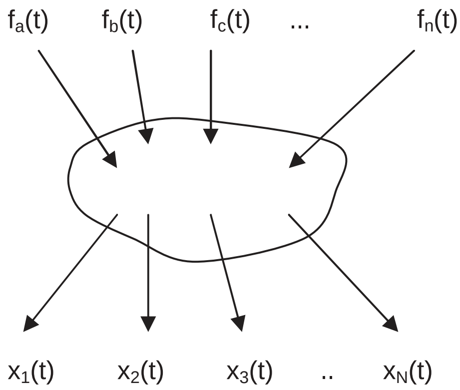

Fig. 1. Stable linear system.

2. Transmissibility matrix in random processes (a new statistical approach)

Considering now the stable linear system, presented in Fig. 1, subjected to multiple random excitations at certain locations, $f_a(t)$, $f_b(t)$,..., $f_r(t)$,..., $f_s(t)$,..., $f_n(t)$ and considering the responses of the system, at positions, $x_1(t)$, $x_2(t)$,..., $x_K(t)$,..., $x_J(t)$,..., $x_N(t)$, the impulse response functions relating the outputs (responses) to the inputs (forces) are $h_{x_1 f_a}(t), \ldots, h_{x_1, f_n}(t), \ldots, h_{x_N f_a}(t), \ldots, h_{x_N f_n}(t)$, and the corresponding FRFs are $H_{x_1 f_a}(\omega), \ldots, H_{x_1 f_n}(\omega), \ldots, H_{x_N f_a}(\omega), \ldots, H_{x_N f_n}(\omega)$ An arbitrary response of the system, $x_K(t)$, may be expressed as [7]:

$$x_K(t) = \int_{-\infty}^{+\infty} (h_{x_K f_a}(\omega) \cdot f_a(t-\omega) + h_{x_K f_b}(\omega) \cdot f_b(t-\omega) + \ldots + h_{x_K f_n}(\theta) \cdot f_n(t-\theta)) d\theta \quad (1)$$

As the autocorrelation function of this response is [7]:

$$R_{x_K x_K}(\tau) = E[x_K(t) \cdot x_K(t+\tau)] \quad (2)$$

the autocorrelation function for the output $x_K(t)$ becomes:

$$\begin{aligned} R_{x_K x_K}(\tau) = & \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (h_{x_K f_a}(\theta_1) \cdot h_{x_K f_a}(\theta_2) \cdot R_{f_a f_a}(\tau - \theta_2 + \theta_1) \\ & + \ldots + h_{x_K f_n}(\theta_1) \cdot h_{x_K f_n}(\theta_2) \cdot R_{f_n f_n}(\tau - \theta_2 + \theta_1)) d\theta_1 d\theta 2 \end{aligned} \quad (3)$$

Also, the auto-spectral density for the same output process $x_K(t)$ is:

$$S_{x_K x_K}(\omega) = \frac{1}{2 \cdot \pi} \cdot \int_{-\infty}^{+\infty} R_{x_K x_K}(\tau) \cdot e^{-i \cdot \omega \cdot \tau} d\tau \quad (4)$$

and after some mathematical manipulations, it is possible to write the above relation in the following form:

$$S_{x_K x_K}(\omega) = \sum_{r=1}^{n} \sum_{s=1}^{n} H^*_{x_K f_r}(\omega) \cdot H_{x_K f_s}(\omega) \cdot S_{f_r f_s}(\omega) \quad (5)$$

where the asterisk denotes the complex conjugate. In the same way, the cross-spectral density between two different responses $x_K(t)$ and $x_J(t)$, can be expressed as:

$$S_{x_K x_J}(\omega) = \sum_{r=1}^{n} \sum_{s=1}^{n} H^*_{x_K f_r}(\omega) \cdot H_{x_J f_s}(\omega) \cdot S_{f_r f_s}(\omega) \quad (6)$$

The cross-spectral density between the input, $f_r(t)$, and an arbitrary output, $x_J(t)$, may be written in the following form:

$$S_{f_r x_J}(\omega) = H_{x_J f_a}(\omega) \cdot S_{f_r f_a}(\omega) + \ldots + H_{x_J f_n}(\omega) \cdot S_{f_r f_n}(\omega) = \sum_{s=1}^{n} H_{x_J f_s}(\omega) \cdot S_{f_r f_s}(\omega) \quad (7)$$

Let us compare now expressions Eqs (6) and (7). It can easily be observed that the last part of relation Eq. (6) represents, in fact, the r.h.s. member of relation Eq. (7). Thus, the cross-spectral density between two arbitrary responses $x_K(t)$ and $x_J(t)$, can be expressed as:

$$S_{x_K x_J}(\omega) = \sum_{r=1}^{n} H^*_{x_K f_r}(\omega) \cdot S_{f_r x_J}(\omega) \quad (8)$$

Using now the properties of cross-spectral densities [7] in relation Eq. (8), the cross-spectral density between two arbitrary responses, $x_J(t)$ and $x_K(t)$, may be expressed in the following form:

$$S^*_{x_K x_J}(\omega) = S_{x_J x_K}(\omega) = \sum_{r=1}^{n} S_{x_J f_r}(\omega) \cdot H_{x_K f_r}(\omega) \quad (9)$$

Returning now to the system shown in Fig. 1, Eq. (9) can be used for any pair of responses, and then it is possible to build the following system of equations, expressed in matrix form:

$$[S_{xx}(\omega)] = [S_{xf}(\omega)] \cdot [H_{xf}(\omega)]^T \quad (10)$$

From Eq. (10) it is possible to write:

$$[S_{xx}(\omega)]^T = [H_{xf}(\omega)] \cdot [S_{xf}(\omega)]^T \quad (11)$$

Assuming now that the applied forces are only at a given subset "A" of co-ordinates, that means that there are no applied forces except at co-ordinates A, and from "N" responses of the system we shall consider two subsets, "U" for unknown responses and "K" for known responses, and taking into account relation Eq. (11) one can write the following equations:

$$[S_{KK}(\omega)]^T = [H_{KA}(\omega)] \cdot [S_{KA}(\omega)]^T \quad (12)$$

$$[S_{KU}(\omega)]^T = [H_{UA}(\omega)] \cdot [S_{KA}(\omega)]^T \quad (13)$$

In the above relations the following simplifications were used: $x_K = K$; $x_U = U$; $f = A$. Assuming that the number of known responses co-ordinates is equal to the number of applied forces co-ordinates, from Eq. (12), it turns out that:

$$[S_{KA}(\omega)]^T = [H_{KA}(\omega)]^{-1} \cdot [S_{KK}(\omega)]^T \quad (14)$$

Combining Eqs (13) and (14) leads to:

$$[S_{KU}(\omega)]^T = [H_{UA}(\omega)] \cdot [H_{KA}(\omega)]^{-1} \cdot [S_{KK}(\omega)]^T \quad (15)$$

Expression Eq. (15) relates the cross-spectral density matrix among known and unknown responses with cross-spectral density matrix among known responses. Taking into account relation (A7), it is possible to conclude that the same transmissibility matrix, obtained in the harmonic case, appears also in the random case:

$$[S_{KU}(\omega)]^T = [T^{(A)}_{UK}(\omega)] \cdot [S_{KK}(\omega)]^T \quad (16)$$

From Eq. (16), it is possible to calculate the cross-spectral density matrix between known and unknown responses, when the transmissibility matrix is known and the cross-spectral density matrix among the known responses is known too.

In the laboratory it is easy to measure the FRFs or the responses of a structure and after that, using the methods described for the case of a system subjected to periodic excitations, it is easy to determine the transmissibility matrix. After that, when the structure is in field service, it is possible to predict the cross-spectral density matrix between the known and unknown responses, based on the knowledge of the cross-spectral density matrix among the known responses and on the transmissibility matrix.

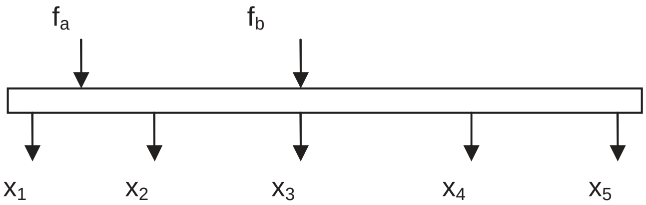

Fig. 2. Experimental beam.

The following alternative method, to obtain the transmissibility matrix using only spectral densities between responses, is presented. Once more the structure should be considered available for the measurements at every co-ordinate (K and U). To calculate the cross-spectral densities between two random responses $x_K(t)$ and $x_J(t)$ the following relationships may be used [7]:

$$S_{x_K x_J}(\omega) = X_K^*(\omega) \cdot X_J(\omega) \tag{17}$$

$$S_{x_J x_K}(\omega) = X_J^*(\omega) \cdot X_K(\omega) \tag{18}$$

In the above relations, $X_K(\omega)$ and $X_J(\omega)$ represent the Fourier transforms of $x_K(t)$ and $x_J(t)$ respectively.

In the case of a system subjected to periodic excitations, expression (A6) relates a subset of responses "U" to another subset of responses "K", through the transmissibility matrix, that depends on the set of co-ordinates "A" where the applied forces may be non-zero.

When the excitation is random, a similar reasoning may be followed. Applying a random dynamic load to the structure, at a given set of locations "A"(the vector $\{F_A^{(1)}(\omega)\}$, from relation (A12)), as in equation (A6), the responses $\{X_K^{(1)}(\omega)\}$ and $\{X_U^{(1)}(\omega)\}$ are related to such an excitation through the transmissibility matrix, as in equation (A10). It is then possible to write the following:

$$X_{K_1}^{*(1)}(\omega) \cdot \begin{Bmatrix} X_{U_1}^{(1)}(\omega) \\ X_{U_2}^{(1)}(\omega) \\ \vdots \\ X_{U_U}^{(1)}(\omega) \end{Bmatrix} = X_{K_1}^{*(1)}(\omega) \cdot [T_{UK}^{(A)}(\omega)] \cdot \begin{Bmatrix} X_{K_1}^{(1)}(\omega) \\ X_{k_2}^{(1)}(\omega) \\ \vdots \\ X_{K_K}^{(1)}(\omega) \end{Bmatrix} \tag{19}$$

Taking into account relations Eqs (17) and (18), the above equation can be written in the following form:

$$\begin{Bmatrix} S_{K_1U_1}^{(1)}(\omega) \\ S_{K_1U_2}^{(1)}(\omega) \\ \vdots \\ S_{K_1U_U}^{(1)}(\omega) \end{Bmatrix} = [T_{UK}^{(A)}(\omega)] \cdot \begin{bmatrix} S_{K_1K_1}^{(1)}(\omega) \\ S_{K_1K_2}^{(1)}(\omega) \\ \vdots \\ S_{K_1K_K}^{(1)}(\omega) \end{bmatrix} \tag{20}$$

Following the same method, if the structure is tested under as many different load vectors as the numbers of known responses, the last applied force vector is $\{F_A^{(K)}(\omega)\}$ and it is possible to write the following equation:

$$\begin{Bmatrix} S_{K_KU_1}^{(K)}(\omega) \\ S_{K_KU_2}^{(K)}(\omega) \\ \vdots \\ S_{K_KU_U}^{(K)}(\omega) \end{Bmatrix} = [T_{UK}^{(A)}(\omega)] \cdot \begin{Bmatrix} S_{K_KK_1}^{(K)}(\omega) \\ S_{K_KK_2}^{(K)}(\omega) \\ \vdots \\ S_{K_KK_K}^{(K)}(\omega) \end{Bmatrix} \tag{21}$$

Using Eqs (20) and (21), the following relationship can be written:

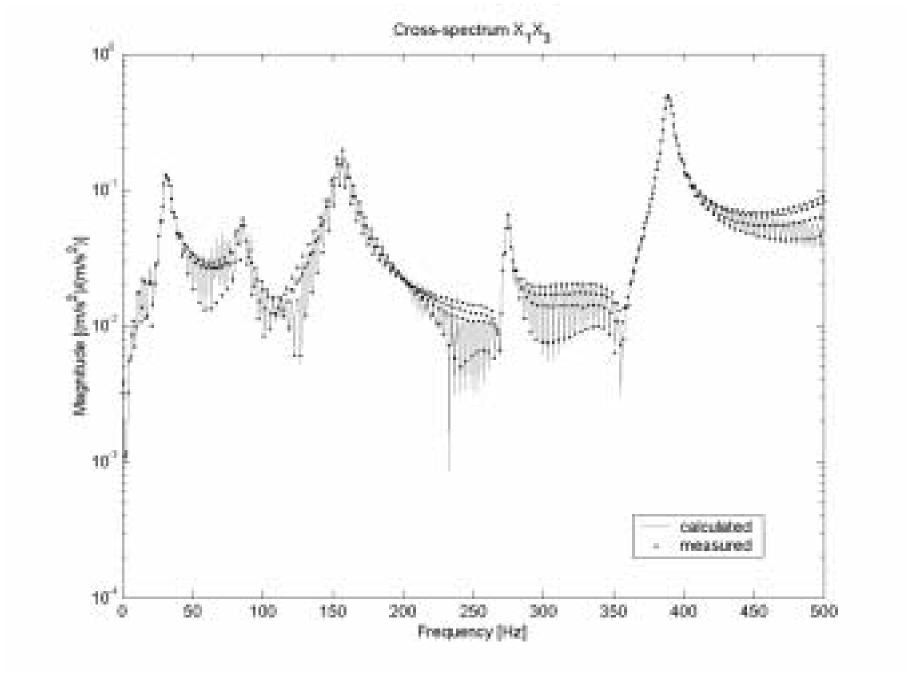

Fig. 3. Calculated and measured spectral densities.

$$[T_{UK}^{(A)}(\omega)] = \begin{bmatrix} S_{K_1U_1}^{(1)}(\omega) & S_{K_2U_1}^{(2)}(\omega) & \dots & S_{K_KU_1}^{(K)}(\omega) \\ S_{K_1U_2}^{(1)}(\omega) & S_{K_2U_2}^{(2)}(\omega) & \dots & S_{K_KU_2}^{(K)}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{K_1U_U}^{(1)}(\omega) & S_{K_2U_U}^{(2)}(\omega) & \dots & S_{K_KU_U}^{(K)}(\omega) \end{bmatrix} \begin{bmatrix} S_{K_1K_1}^{(1)}(\omega) & S_{K_2K_1}^{(2)}(\omega) & \dots & S_{K_KK_1}^{(K)}(\omega) \\ S_{K_1K_2}^{(1)}(\omega) & S_{K_2K_2}^{(2)}(\omega) & \dots & S_{K_KK_2}^{(K)}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{K_1K_K}^{(1)}(\omega) & S_{K_2K_K}^{(2)}(\omega) & \dots & S_{K_KK_K}^{(K)}(\omega) \end{bmatrix}^{-1} \quad (22)$$

Using Eq. (22) it is possible to calculate the transmissibility matrix using only the cross-spectral densities between responses. The number of needed measurements is exactly the same as in the case of obtaining the transmissibility matrix from measured responses or from frequency response functions.

3. Example

The easiest way to illustrate the method is through an experimental example. In Fig. 2 we consider a freely suspended simple beam subjected to random excitations at locations $f_a(t)$ and $f_b(t)$, and the responses of the system at positions $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$ and $x_5(t)$. From all the responses we consider that $x_1(t)$ and $x_2(t)$ are known (have been "measured") and all the others are unknown. So, in this case, the known responses, unknown responses and applied forces vectors are, respectively:

$$X_K(\omega) = \begin{Bmatrix} X_1(\omega)) \\ X_2(\omega) \end{Bmatrix}; \quad X_U(\omega) = \begin{Bmatrix} X_3(\omega) \\ X_4(\omega) \\ X_5(\omega) \end{Bmatrix}; \quad F_A(\omega) = \begin{Bmatrix} F_a(\omega) \\ F_b(\omega) \end{Bmatrix} \quad (23)$$

The spectral density matrix between the known responses, $[S_{KK}(\omega)]$, and the spectral density matrix between known and unknown responses, $[S_{KU}(\omega)]$, are:

$$[S_{KK}(\omega) = \begin{bmatrix} S_{x_1x_1}(\omega) & S_{x_1x_2}(\omega) \\ S_{x_2x_1}(\omega) & S_{x_2x_2}(\omega) \end{bmatrix} \quad (24)$$

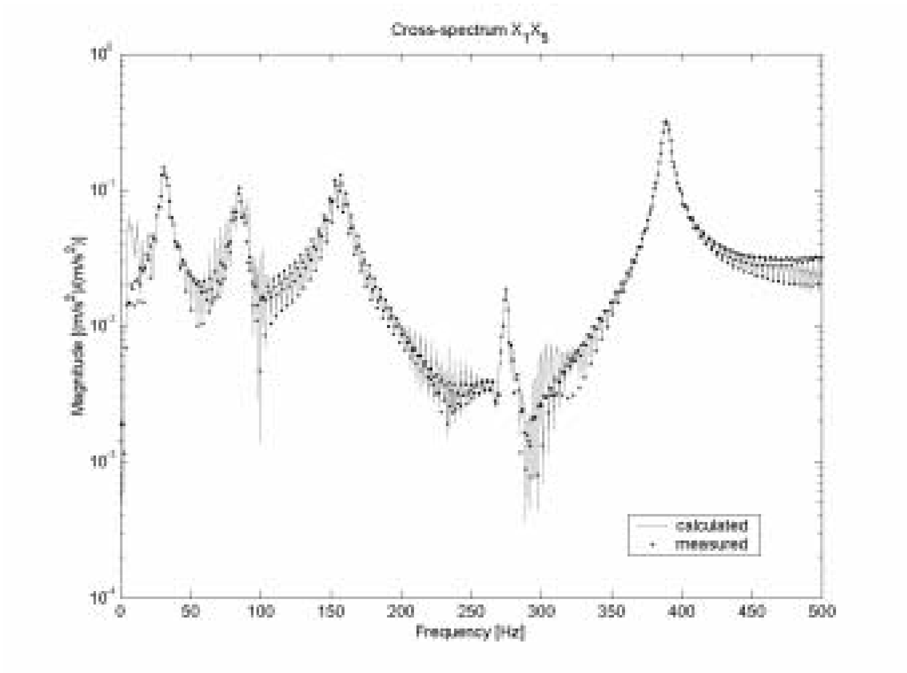

Fig. 4. Calculated and measured spectral densities.

$$[S_{KU}(\omega)] = \begin{bmatrix} S_{x_1x_3}(\omega) \; S_{x_1x_4}(\omega) \; S_{x_1x_5}(\omega) \\ S_{x_2x_3}(\omega) \; S_{x_2x_4}(\omega) \; S_{x_2x_5}(\omega) \end{bmatrix} \tag{25}$$

It is necessary to perform three tests, with three different applied force vectors, such as, for example:

$$1.\ F_a(\omega) \neq 0;\ \ F_b(\omega) = 0;\ \ \Rightarrow \{F_A^a(\omega)\} = \begin{Bmatrix} F_a(\omega) \\ 0 \end{Bmatrix} \tag{26}$$

$$2.\ F_a(\omega) = 0;\ \ F_b(\omega) \neq 0;\ \ \Rightarrow \{F_A^b(\omega)\} = \begin{Bmatrix} 0 \\ F_b(\omega) \end{Bmatrix} \tag{27}$$

$$3.\ F_a(\omega) \neq;\ \ F_b(\omega) \neq 0;\ \ \Rightarrow \{F_A^{ab}(\omega)\} = \begin{Bmatrix} F_a(\omega) \\ F_b(\omega) \end{Bmatrix} \tag{28}$$

From the first test, the following spectral densities are measured:

$$S^a_{x_1x_1}(\omega);\ \ S^a_{x_1x_2}(\omega);\ \ S^a_{x_1x_3}(\omega);\ \ S^a_{x_1x_4}(\omega);\ \ S^a_{x_1x_5}(\omega) \tag{29}$$

From the second test, the following spectral densities are measured:

$$S^b_{x_2x_1}(\omega);\ \ S^b_{x_2x_2}(\omega);\ \ S^b_{x_2x_3}(\omega);\ \ S^b_{x_2x_4}(\omega);\ \ S^b_{x_2x_5}(\omega) \tag{30}$$

From the last test, the following spectral densities are measured:

$$\begin{aligned} &S^{ab}_{x_1x_1}(\omega);\ \ S^{ab}_{x_1x_2}(\omega;\ \ S^{ab}_{x_1x_3}(\omega);\ \ S^{ab}_{x_1x_4}(\omega); S^{ab}_{x_1x_5}(\omega) \\ &S^{ab}_{x_2x_1}(\omega);\ \ S^{ab}_{x_2x_2}(\omega);\ \ S^{ab}_{x_2x_3}(\omega);\ \ S^{ab}_{x_2x_5}(\omega) \end{aligned} \tag{31}$$

According to Eq. (22) and using the spectral densities Eqs (29) and (30), the transmissibility matrix can be calculated:

$$[T^A_{UK}(\omega)] = \begin{bmatrix} S^a_{x_1x_3}(\omega) \; S^b_{x_2x_3}(\omega) \\ S^a_{x_1x_4}(\omega) \; S^b_{x_2x_4}(\omega) \\ S^a_{x_1x_5}(\omega) \; S^b_{x_2x_5}(\omega) \end{bmatrix} \cdot \begin{bmatrix} S^a_{x_1x_1}(\omega) \; S^b_{x_2x_1}(\omega) \\ S^a_{x_1x_2}(\omega) \; S^b_{x_2x_2}(\omega) \end{bmatrix}^{-1} \tag{32}$$

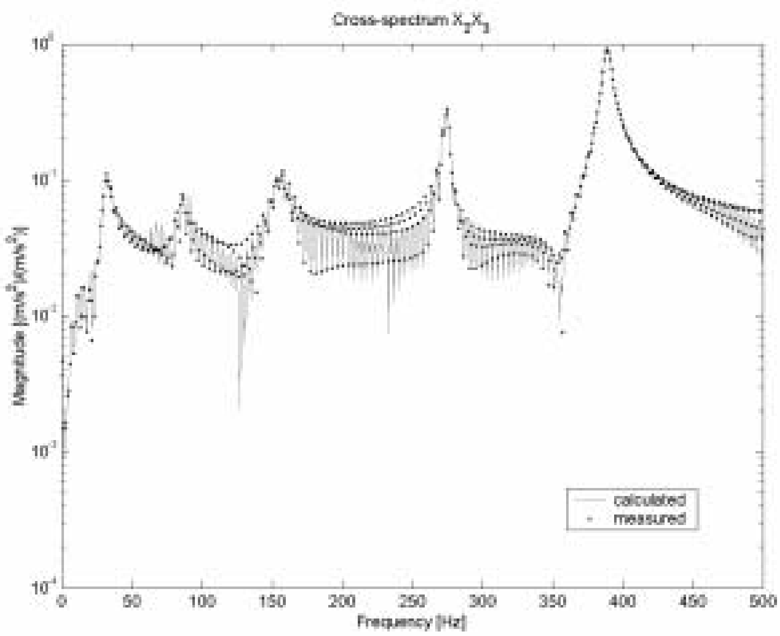

Fig. 5. Calculated and measured spectral densities.

Using the transmissibility matrix derived by Eq. (32) and the spectral densities between known responses in Eq. (31), it is possible to calculate the spectral densities matrix among known and unknown responses, according to Eq. (16):

$$\begin{bmatrix} S^c_{x_1x_3}(\omega) & S^c_{x_2x_3}(\omega) \\ S^c_{x_1x_4}(\omega) & S^c_{x_2x_4}(\omega) \\ S^c_{x_1x_5}(\omega) & S^c_{x_2x_5}(\omega) \end{bmatrix} = [T^A_{UK}(\omega)] \cdot \begin{bmatrix} S^{ab}_{x_1x_1}(\omega) & S^{ab}_{x_2x_1}(\omega) \\ S^{ab}_{x_1x_2}(\omega) & S^{ab}_{x_2x_2}(\omega) \end{bmatrix} \quad (33)$$

Where the superscript "c" means calculated. Now it is possible to compare the calculated spectral densities from Eq. (33) with the same measured spectral densities from Eq. (31); Figs 3, 4, 5 and 6.

As it is possible to observe, from the above graphics, there are some differences between the calculated and measured spectral densities, as a result of error propagation. The input data used to calculate the transmissibility matrix and, after that, the cross-spectrum, contains a lot of noise, so it was expected that the results would exhibit such a behavior.

4. Conclusion

It has been proven that the same transmissibility matrix appears in both periodical and random excitation cases. It has been confirmed once more that the transmissibility matrix does not depend on the applied type of forces exciting the system, but only on the chosen sets of co-ordinates for the applied forces and for the 'unknown' and 'known' responses. A simple method to calculate the spectral densities between the 'unknown' and 'known' responses and spectral densities between 'known' responses using the transmissibility matrix was presented. A simple example illustrated the goodness of the procedure. This work should clear any doubts about the validity of the transmissibility matrix when relating random signals. In fact it was shown that the statistical relationships between the 'known' and 'unknown' random responses are kept the same as the deterministic relationships already established for the periodic signals.

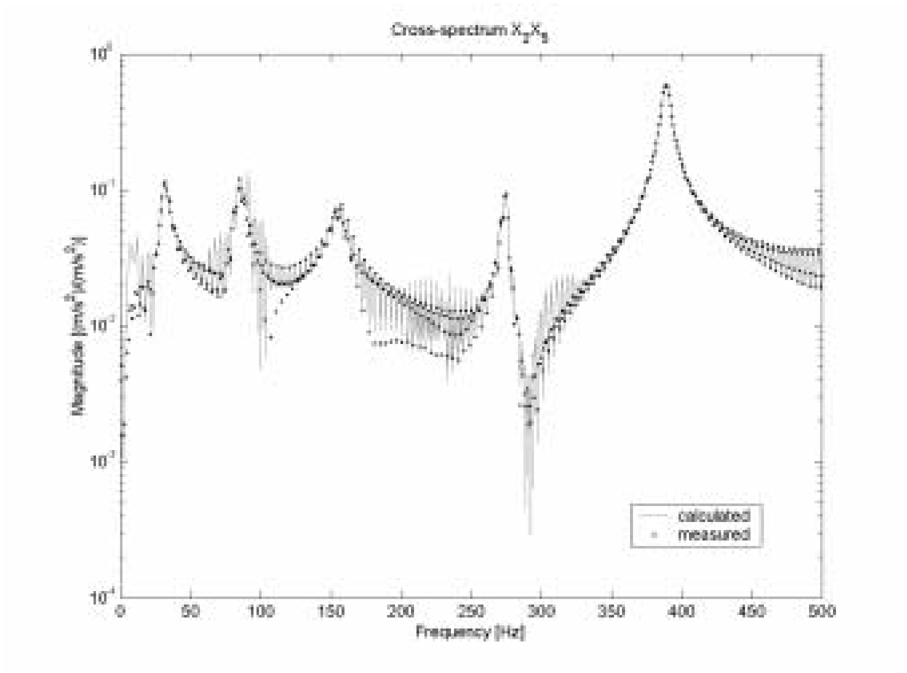

Fig. 6. Calculated and measured spectral densities.

Annex

Transmissibility matrix in harmonicaly processes (short review of previous work):

The dynamic behavior of a stable linear system, as shown in Fig. 1, can be described as:

$$\{X(\omega)\} = [H(\omega)] \cdot \{F(\omega)\} \tag{A1}$$

$[H(\omega)]$ – FRF matrix; $\{X(\omega)\}$ – the vector of dynamic responses; $\{F(\omega)\}$ – the vector of applied forces.

The forces are applied only at a given subset "A" of co-ordinates; from the "N" responses of the system we shall consider two subsets: subset "U" for unknown responses and subset "K" for known responses.

$$\{X_K(\omega)\} = [H_{KA}(\omega)] \cdot \{F_A(\omega)\} \tag{A2}$$

$$\{X_U(\omega)\} = [H_{UA}(\omega)] \cdot \{F_A(\omega)\} \tag{A3}$$

If the number of the known response co-ordinates is equal to the number of the applied forces [3]:

$$\{F_A(\omega)\}[H_{KA}(\omega)]^{-1} \cdot \{X_K(\omega)\} \tag{A4}$$

Combining now relations Eqs (A3) and (A4):

$$\{X_U(\omega)\} = [H_{UA}(\omega)] \cdot [H_{KA}(\omega)]^{-1} \cdot \{X_K(\omega)\} \tag{A5}$$

The transmissibility matrix is then defined [1] as:

$$\{X_U(\omega)\} = [T_{UK}^{(A)}(\omega)] \cdot \{X_K(\omega)\} \tag{A6}$$

$$[T_{UK}^{(A)}(\omega)] = [H_{UA}(\omega)] \cdot [H_{KA}(\omega)]^{-1} \tag{A7}$$

Applying a dynamic load at a given set of locations "A" on the structure, say, the vector $\{F_A^{(1)}(\omega)\}$:

$$\{X_U^{(1)}(\omega) = [T_{UK}^{(A)}(\omega)] \cdot \{X_K^{(1)}(\omega)\} \tag{A8}$$

Performing now another test with applied forces, say vector $\{F_A^{(2)}(\omega)\}$ at the same set of given locations "A":

$$\{X_U^{(2)}(\omega)\} = [T_{UK}^{(A)}(\omega)] \cdot \{X_K^{(2)}(\omega)\} \tag{A9}$$

The transmissibility matrix is the same since the sets of locations "A", "K" and "U" have not changed. If the structure is tested under as many different load vectors as the number of known responses "K", it is possible to write:

$$[\{X_U^{(1)}(\omega)\}\{X_U^{(2)}(\omega)\ldots\{X_U^{(K)}(\omega)\}] = [T_{UK}^{(A)}(\omega)] \cdot [\{X_K^{(1)}(\omega)\}\{X_K^{(2)}(\omega)\}\ldots\{X_K^{(K)}(\omega)\}] \tag{A10}$$

If the applied force vectors are all linearly independent, it is possible to calculate the transmissibility matrix, from relation (A10), using only the previously measured responses:

$$[T_{UK}^{(A)}(\omega)] = \begin{bmatrix} X_{U_1}^{(1)}(\omega) & X_{U_1}^{(2)}(\omega) & \ldots & X_{U_1}^{(K)}(\omega) \\ X_{U_2}^{(1)}(\omega) & X_{U_2}^{(2)}(\omega) & \ldots & X_{U_2}^{(K)}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ X_{U_U}^{(1)}(\omega) & X_{U_U}^{(2)}(\omega) & \ldots & X_{U_U}^{(K)}(\omega) \end{bmatrix} \cdot \begin{bmatrix} X_{K_1}^{(1)}(\omega) & X_{K_1}^{(2)}(\omega) & \ldots & X_{K_1}^{(K)}(\omega) \\ X_{K_2}^{(1)}(\omega) & X_{K_2}^{(2)}(\omega) & \ldots & X_{K_2}^{(K)}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ X_{K_K}^{(1)}(\omega) & X_{K_K}^{(2)}(\omega) & \ldots & X_{K_K}^{(K)}(\omega) \end{bmatrix}^{-1} \tag{A11}$$

The vectors $\{F_A^{(1)}(\omega)\}, \{F_A^{(2)}(\omega)\}, \ldots, \{F_A^{(K)}(\omega)\}$ can be chosen to be the following:

$$\{F_A^{(1)}(\omega)\} = \begin{Bmatrix} F_{A_1}(\omega) \\ 0 \\ \vdots \\ 0 \end{Bmatrix}, \quad \{F_A^{(2)}(\omega)\} = \begin{Bmatrix} 0 \\ F_{A_2}(\omega) \\ \vdots \\ 0 \end{Bmatrix}, \quad \ldots \quad \{F_A^{(K)}(\omega)\} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ F_{A_K}(\omega) \end{Bmatrix} \tag{A12}$$

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