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Research Article

SIR Meta Distribution in the Heterogeneous and Hybrid Networks

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With the development of the technology, the wireless systems are becoming more heterogeneous with the introduction of various power nodes including femtocells, relays, or distributed antennas. Among the research of wireless network performance, the meta distribution of the signal-to-interference ratio (SIR) has attracted significant attention. Compared to the standard success (coverage) probability, the meta distribution provides much more fine-grained information about the network performance. In this paper, we analyze the meta distribution of the SIR in the multi-tier heterogeneous and hybrid networks, where each tier is based on a homogeneous independent Poisson point process model. For the open tiers (the users can associate with any tier) and the closed tiers (the users can only associate with a certain tier), we study the bth moment of the conditional success probability for the typical user and give the beta approximation of the meta distribution from analysis and simulations. Furthermore, we analyze the per-link rate control for open tiers and closed tiers, which answers the question: "how to set the SIR threshold to meet a target reliability?". We give the approximate value of the SIR threshold to meet a target reliability and show how the value is related to the path loss exponent and densities.

1. Introduction

1.1. Motivation. The developing technology makes the wireless communication influence more and more on the daily life of human beings. In 5G, the objective of the wireless technology is to support three generic services with vastly heterogeneous requirements: enhanced mobile broadband (eMBB), mass machine-type communications (mMTC), and ultra-reliable and low latency communications (URLLC). Under such a background, the structure of the network is becoming more and more heterogeneous. Meanwhile, direct communication between mobile devices can save transmit power and help utilize the network resources more efficiently, such as the device-to-device (D2D), machine-to-machine (M2M), and vehicle-to-vehicle (V2V), which makes the network more hybrid. A modern wireless network usually consists of some open tiers of nodes that can be accessed by users through its association rule, such as the heterogeneous cellular networks (HCNs), and some closed tiers that users are served only by partial tiers, such as some direct communications. More heterogeneous and hybrid is becoming one of the characteristics of the future wireless networks. In this paper, a heterogeneous network with the direct communications is called a het-hybrid network.

The conventional SIR analysis or the mean success probability provides limited information about the network performance [1]. It is defined as the complementary cumulative distribution function (CCDF) of the SIR evaluated at the typical link. Such a performance metric is merely a macroscopic quantity by averaging the conditional success probability (CSP) $P_s(\theta) \triangleq \mathbb{P}(\mathrm{SIR} > \theta \mid \Phi)$ over the underlying point process Φ. Hence, it provides no information about the difference between links. It cannot answer such questions as "How are the link reliability distributed among users in different tiers ?" or "What is the reliability level that the '5% user' can achieve in each tier?". To obtain a fine-grained information on the SIR performance, the notion of meta distribution, as the distribution of CSP was introduced in [2]. The concept characterizes the distribution of the CSP of the individual links given the point process and is widely researched in many network scenarios, such as the cellular networks [3–5], the HCNs [6–9], the millimeter wave networks [10, 11], the large-scale non-orthogonal multiple access (NOMA) networks [12], the dual-hop Internet-of-Things (IoT) networks [13], and the ultra-dense networks [14].

In this paper, we mainly investigate the SIR meta distribution in the het-hybrid networks that consist of open tiers and closed tiers, to get a fine-grained analysis on the success transmission probability and analyze the rate control based on the SIR meta distribution.

1.2. Related Work. The meta distribution of the SIR has been applied to different scenarios since it was formally formulated in [2]. For instance, [3] focused on the SIR meta distribution in cellular networks with fractional power control. Some bounds, the analytical expression, the mean local delay, and the beta approximation of the meta distribution were provided. Recently, the joint meta distribution of the SIR at different locations and its applications to physical layer security and cooperative reception were studied in [4]. And for moving networks, the SIR meta distribution was researched in [5], which demonstrated that the moving BSs can reduce the variance of users while keeping the mean success probability constant.

The SIR meta distribution of k-tier downlink HCNs with cell range expansion was researched in [6], where the bth moments of the CSP for each tier and for the entire network were derived, and the metrics including the mean success probability, the variance of the CSP, the mean local delay, and the asymptotic SIR gains of each tier were also obtained. The SIR meta distribution in HCNs with base station cooperation was researched in [7], where the meta distribution in HCNs with downlink coordinated multipoint transmission/-reception (CoMP) was derived. For the HetNet, [8] derived the meta distribution of the downlink SIR in a Poisson cluster process-based model. And for the general cellular networks, [9] provided the AMAPPP (approximate meta distribution analysis using PPP) in the SIR meta distribution analysis to obtain the meta distribution of an arbitrary stationary and ergodic point process from the meta distribution of the Poisson point process (PPP).

The meta distribution of the SINR and the data rate for millimeter wave (mm-wave) D2D networks were derived in [10], where the approximation by using higher moments of the conditional SINR distribution was also proved to be effective. Using stochastic geometry tools, [11] analyzed the meta distributions of the downlink SIR/SNR and data rate of the typical device in a cellular network with coexisting sub-6GHz and mm-wave spectrums. The meta distribution and moments of the conditional success probability (CSP) in large-scale NOMA networks were studied in [12], where a tractable framework was developed to analyze both the uplink NOMA and downlink NOMA. The meta distribution of the downlink SIR attained at a typical device in a dual-hop IoT network was characterized in [13], where the IoT device associates with either a serving macro base station for direct transmissions or associates with a decode and forward relay for dual-hop transmissions. For ultra-dense networks, [14] studied the meta distributions of SIR in a near-optimally short, perfect, Euclidean distance edge-weighted, bipartite matching between two binomial point processes, to obtain a bipartite Euclidean matching and investigate the reliability of communication. For the SIR meta distribution of D2D communication underlaying cellular wireless networks, [15] derived the moments of the conditional SIR distribution to calculate analytical expressions and the mean local delay of the typical receiver.

Another important application of the SIR meta distribution or the rate control was considered in [16]. From [16], the per-link reliability and rate control were proved to be the two facets of the SIR meta distribution.

The work mentioned above considered the SIR meta distribution either in a single tier or a hybrid network, or in a heterogeneous network with only open tiers. The SIR meta distribution of the multi-tier heterogeneous network with hybrid structure has not been covered yet.

1.3. Contributions. In this paper, we focus on the heterogeneous and hybrid network that consists of open tiers and closed tiers and analyze the SIR meta distribution under Rayleigh fading to provide a fine-grained analysis on the network performance. Specifically,

(i) We derive the bth moments of the CSP for both the open tiers and the closed tiers. We find the open tiers and closed tiers make different effects on the bth moment of open tiers, while they impact the bth moment of the closed tiers in the same way.

(ii) We give the beta approximation for the SIR meta distribution by matching the first and second moments of CSP for the open tiers and closed tiers. The simulations show that the beta distribution is an effective approximation in the het-hybrid networks.

(iii) We analyze the mean local delay for the users of open tiers in a het-hybrid network based on the b th moments of the CSP.

(iv) We study the rate control by setting the SIR threshold to meet a target reliability and derive the approximate values of SIR threshold for open tiers and closed tiers. We find that the path loss exponent affects the SIR threshold settings differently in open tiers and closed tiers.

2. System Model and Method

2.1. Network Model. The het-hybrid network is modeled as a K-tier $(K \geq 1)$ wireless network where each tier consists of the transmitting nodes of a particular class. The nodes across tiers may differ in terms of the transmit power, the supported data rate, and their spatial density. We assume that the transmitting nodes of the ith tier are spatially distributed as an independent PPP Φ_i with density λ_i, and transmit power P_i. All transmitting nodes are active, and we do not consider any cooperation between the transmitters.

All tiers are classified into open tiers and closed tiers. The open tiers mean a group of (≥1) tiers of transmitting nodes that can be accessed openly by users through the association rule, also named as open group. In this paper, we consider the downlink performance and the average strongest signal association rule in open tiers. Hence, a transmitter from any open tier can be the signal provider only if it can provide the average strongest signal. There is no need to distinguish the users of each open tier, since they can connect to the arbitrary tier of the open group. The corresponding users of the open tiers are called open users and are assumed to be distributed as an independent PPP.

Each closed tier means a single tier of transmitting nodes. The corresponding users can only associate with the single tier of transmitters, which seems like this tier is closed to other tiers. Moreover, each transmitter is assumed to have a dedicated receiver located at a fixed distance in a random direction in our assumption, such as some direct communications [15, 17]. In such cases, the transmitters and receivers of the closed tier form a Poisson bipolar network. Although in many bipolar networks, the transmitters are active with a probability, here, we only consider the active transmitters at a time slot, or a snapshot of the active nodes. The network can have several closed tiers at the same time, where each tier is closed to other tiers.

In the rest of the paper, we use $\Phi_B = \bigcup \Phi_{i|i\in B}$ to denote the group of open tiers, where $B \subseteq \{1, 2, \cdots K\}$, and $\Phi_{i|i\notin B}$ denote one of the closed tiers. The notation x is used to denote both the location of a transmitter and the transmitter itself, and $|x|$ is used to denote the distance between the transmitter and the origin.

2.2. SIR. Without loss of generality, we conduct analysis on the typical user located at the origin. To analyze the open tiers, we condition on that typical user to be an open user and vice versa when analyzing the closed tier. The fading between a transmitter located at point x and the typical user is denoted by h_x, which is assumed to be i.i.d. exponential (Rayleigh fading). The standard power-law path loss model is $l(r) = r^{-\alpha}$ with exponent $\alpha > 2$. For open tiers, assume the typical user is associated with x_0 in the kth $(k \in B)$ tier, and the fading is h^o_{x0}; the received SIR can be given by

$$\mathrm{SIR}_{o|k} = \frac{P_k h^o_{x0} |x_0|^{-\alpha}}{\sum_{x\in\Phi_k, x\neq x_0} P_k h_x |x|^{-\alpha} + \sum_{i\neq k} \sum_{x\in\Phi_i} P_i h_x |x|^{-\alpha}}, \quad (1)$$

where the numerator is the desired signal and the denominator is the aggregate interference suffered by the user.

Similarly, the SIR received by the typical closed user of tier $j (j \notin B)$ is

$$\mathrm{SIR}_{c|j} = \frac{P_k h^c_{x0} d_j^{-\alpha}}{\sum_{x\in\Phi_j, x\neq x_0} P_j h_x |x|^{-\alpha} + \sum_{i\neq j} \sum_{x\in\Phi_i} P_i h_x |x|^{-\alpha}}, \quad (2)$$

where d_j is the fixed distance between the jth tier transmitters and their receivers, and h^c_{x0} is the fading.

The difference between the open tiers and the closed tiers lies in the association rule, besides the open tiers usually being a group of tiers while each closed tier being a single tier. Since the average strongest association rule in open tiers, node x_0 of the kth tier in fact is the signal provider with a certain probability (equation (6)). Other node in Φ_B can also be the potential signal provider. That means the serving power and the serving distance are not certain until the link is established. Even when the open group has only one tier, the serving distance is uncertain due to the Poisson distribution of nodes. While in a closed tier, the user is definitely served by the transmitter with a certain power and the determined serving distance.

2.3. The Meta Distribution. The SIR meta distribution of the typical user for a threshold θ and a reliability ν is given as follows:

$$\bar{F}(\theta, \nu) = \bar{F}_{P_s}(\theta, \nu) \triangleq \mathbb{P}(P_s(\theta) > \nu), \theta \in R^+, \nu \in [0, 1], \quad (3)$$

where $P_s(\theta)$ is a random variable that represents the link success probability conditioned on the point process $\bigcup_{(i\in\{1\ldots K\})} \Phi_i$. It can be given by

$$P_s(\theta) \triangleq \mathbb{P}\left(\mathrm{SIR} > \theta \mid \bigcup_{i\in\{1\cdots K\}} \Phi_\mathrm{i}\right), \quad (4)$$

where the probability is taken with respect to the fading. The meta distribution is the CCDF of the conditional link success probability $P_s(\theta)$.

The standard success (coverage) probability $p_s(\theta)$ (the SIR distribution) can be obtained from the SIR meta distribution as the mean of the conditional success probability $P_s(\theta)$, i.e., $p_s(\theta) \triangleq \mathbb{P}(SIR > \theta) = \mathbb{E}(P_s(\theta)) = \int_0^1 \bar{F}(\theta, x)dx$. Clearly, the distribution of $P_s(\theta)$ provides much more fine-grained information than merely the average of $P_s(\theta)$.

The exact meta distribution can be expressed by the Gil-Pelaez theorem [18] as

$$\bar{F}(\theta, \nu) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{\mathfrak{J}\left(e^{-jt \log \nu} M_{jt}\right)}{t} dt, \quad (5)$$

where $\mathfrak{J}(z)$ denotes the imaginary parts of $z \in \mathbb{C}$, $j = \sqrt{-1}$, and M_b denotes the bth moment of $P_s(\theta)$, i.e., $M_b = E(P_s(\theta)^b)$, $b \in \mathbb{C}$.

2.4. Method. Based on the above model, we analyze the SIR meta distribution of the typical user for open tiers and closed tiers, respectively. Different from the HCNs [6], the users of the open tiers suffer more interference for the existence of closed tiers. For the similar reason, the receivers of the closed tiers also receive the interference from open tiers. We try to probe the impacts of the tiers on the SIR meta distributions of the open tiers and the closed tiers. The bth moments of the CSP are derived firstly, and then, the mean local delay or the -1st moment of the open tiers is analyzed. The numerical analysis of the SIR meta distribution is mainly approximated by the beta distribution with the first and the second moments. For the per-link rate control of the open tiers and closed tiers, the stochastic interference equivalence [19, 20] is considered to facilitate the analysis. In such a framework, the signal received by the user can be simplified as the serving signal from a tier with density λ_S plus the interference signal from a tier with density λ_I.

For brief expression, we use δ to denote $2/\alpha$, $F_{b,\delta,\theta}$ to denote ${}_2F_1(b,-\delta\,;1-\delta;-\theta)$, and $\Gamma_{b,\delta}$ to denote $(\Gamma(b+\delta))/(\Gamma(b)\Gamma(1+\delta))$ in the rest of the paper.

3. The SIR Meta Distribution in the Het-Hybrid Network

3.1. The bth Moment of the CSP. In this section, we firstly give the bth moment of the CSP for the typical user of the open tiers, and then the bth moment for a closed tier.

For the open tiers, the desired signal received by the typical user is possibly from any transmitter in Φ_B. Assume that the provider of the desired signal x_0 is from the kth ($k\in$ B) tier, and the distance is R_0; the access probability that the typical user is associated with the kth tier is

$$P^{(k)}_{a^{-}R_0} = \prod_{\Phi_{j|j\in B, j\neq k}} \exp\left(-\pi\lambda_j\left(\frac{P_j}{P_k}\right)^{\delta} R_0{}^2\right). \qquad (6)$$

It can be found in Lemma 5 of [6]. By considering the conditional access probability in (6), the bth moment of the CSP for the open tiers can be easily obtained.

Theorem 1. *For the typical open user served by the kth tier in a het-hybrid network, the bth moment of the conditional success probability is given by*

$$M^{open}_{b,(k)} = \frac{1}{\sum_{\Phi_{i|i\in B}}(\lambda_i/\lambda_k)(P_i/P_k)^{\delta}F_{b,\delta,\theta} + \sum_{\Phi_{j|j\notin B}}(\lambda_j/\lambda_k)(P_j/P_k)^{\delta}\theta^{\delta}(\pi\delta/(\sin(\pi\delta)))\Gamma_{b,\delta}}. \qquad (7)$$

Proof. See Appendix A.

Since the open group consists of several tiers, the bth moment of the typical user CSP for the overall open tiers is given by

$$M^{\text{open}}_{b} = \sum_{k\in B}\frac{1}{\sum_{\Phi_{i|i\in B}}(\lambda_i/\lambda_k)(P_i/P_k)^{\delta}F_{b,\delta,\theta} + \sum_{\Phi_{j|j\notin B}}(\lambda_j/\lambda_k)(P_j/P_k)^{\delta}\theta^{\delta}(\pi\delta/\sin(\pi\delta))\Gamma_{b,\delta}}. \qquad (8)$$

Remark 2. From (7), the closed tiers affect the bth moment of the open tiers by $\sum_{\Phi_{j|j\notin B}}(\lambda_j/\lambda_k)(P_j/P_k)^{\delta}\theta^{\delta}(\pi\delta/\sin(\pi\delta))\Gamma_{b,\delta}$ as a term of the denominator. It is different from that of the open tiers $\sum_{\Phi_{i|i\in B}}(\lambda_i/\lambda_k)(P_i/P_k)^{\delta}F_{b,\delta,\theta}$, due to their different association rules. When there is no closed tier or $\Phi_B = \bigcup_{i\in\{1..K\}}\Phi_i$, the bth moment of CSP for the kth tier user can be simplified as

$$M_{b,k} = \frac{1}{\sum_{i=1}^{K}(\lambda_j/\lambda_k)(P_j/P_k)^{\delta}F_{b,\delta,\theta}}, \qquad (9)$$

and the bth moment of the CSP for the overall network is

$$M_b = \frac{1}{F_{b,\delta,\theta}}. \qquad (10)$$

It just describes the bth moment of the CSP for the typical user of HCNs without any range expansion [6].

For the bth moment of the CSP of the typical closed user, we have the following theorem.

Theorem 3. *The typical closed user of jth tier with the distance d_j in a het-hybrid network has the bth moment of the CSP as*

$$M^{closed}_{b,(j)} = \exp\left(-\pi\theta^{\delta}d_j^2(\pi\delta/\sin(\pi\delta)).\Gamma_{b,\delta}\sum_{i=1}^{K}\lambda_i(P_i/P_k)^{\delta}\right). \qquad (11)$$

Proof. See Appendix B.

Remark 4. From Theorem 3, we can see there is no difference among the tiers that affect the bth moment of the closed tier. Adding more tiers or increasing any tier density, whether for an open tier or a closed tier, will only reduce the bth moment. The quantity is also an exponential function of the square of distance d_j.

3.2. Mean Local Delay. Based on the bth moment of the SIR, we further consider the mean local delay which is defined as the mean number of transmission attempts waiting for a packet successfully decoded over a wireless link [2, 15]. Generally, the mean local delay is the −1st moment of the CSP, i.e. M_{-1}. But we find that M_{-1} cannot be derived directly from (7) because the $\Gamma_{b,\delta}$ is not defined when $b=-1$. Fortunately, the distribution of the conditional local delay is geometric with mean M_{-1} and the local delay can be seen as the expectation of the CSP with respect to the static elements of a network [21]. Based on this idea, we derive the mean local delay for the typical user of the open tiers as

$$M^{\text{open}}_{-1,(k)} = \sum_{\Phi_{i|i\in B}}\left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{\delta}F_{1,\delta,\theta} + \sum_{\Phi_{j|j\notin B}}\left(\frac{\lambda_j}{\lambda_k}\right)\left(\frac{P_j}{P_k}\right)^{\delta}\theta^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}, \qquad (12)$$

for $\Gamma_{1,\delta}=1$.

Specially, if there is no closed tier, the mean local delay for the typical user can be derived from (7):

$$M_{-1,k} = \frac{1-\delta}{\sum_{j=1}^{K}(\lambda_j/\lambda_k)(P_j/P_k)^{\delta}(1-\delta(1+\theta))}, \qquad (13)$$

for ${}_2\mathrm{F}_1(-1,b\,;c\,;z)\equiv 1-bz/c$.

The mean local delay is only related to the path loss exponent α and the SIR threshold θ in a single tier Poisson network [2]. But in a multi-tier network, the mean local delay is also related to the ratio of density and power of each tier, as (13). It can be seen that the mean local delay is finite when $\theta<(1/\delta)-1$ from (13). Conversely, if $\theta\geq(1/\delta)-1$, the mean local delay is infinite due to the correlated interference in the system. The value of $\theta = (1/\delta) - 1$ is called the critical value for phase transition.

For the same reason, the -1st moment of the closed tier cannot be obtained from (11) directly. In our assumed model, there is no transmission waiting in the closed tier, because all transmitters are active and each transmitter has a dedicated receiver. Otherwise, the local delay may be infinite if there are more receivers, unless the transmit probability is less than 1 [2], which has been discussed in [15].

3.3. Beta Approximation. Even though the expression of the meta distribution in (5) is exact, it is hard to gain direct insights due to its complexity, and it is not very convenient to evaluate numerically. Fortunately, the beta distribution can be taken as an approximation to analyze the meta distribution.

Beta distribution, as a conjugate prior distribution of the Bernoulli distribution and the binomial distribution, is a natural choice to approximate the distribution of the $P_s(\theta)$. It has been verified that the standard beta distribution can provide an efficient approximation by matching the first and the second moments of the CSP [2, 3, 6, 7, 9, 10, 15, 22]. Specifically,

$$\bar{F}(\theta, \nu) \approx 1 - I_\nu\left(\frac{\beta M_1}{1 - M_1}, \beta\right), \nu \in [0, 1], \tag{14}$$

where $\beta = ((M_1 - M_2)(1 - M_1))/(M_2 - M_1^2)$, and I_ν is the regularized incomplete beta function.

It is worth noting that recently, as shown in [23], the meta distribution can also be directly obtained from the moments by a simple linear transform, which is a more convenient way for efficient calculations. And [24] showed that the entire meta distribution can be reconstructed from its moments using the Fourier-Jacobi expansion.

3.4. Per-Link Rate Control. Another important application of the SIR meta distribution is the rate control. It has been proved in [16] that the SIR meta distribution as a function of the SIR threshold is equivalent to the SIR threshold distribution such that each link is guaranteed a target reliability. For the Poisson bipolar networks, [16] studied the rate control and revealed the trade-off between the SIR threshold (equivalently, the distribution of the transmission rate) and the reliability. In our network model, we also consider the per-link rate control based on the SIR meta distribution. The problem to be solved in this section is "how to set the SIR threshold θ, i.e., the rate control, such that the link can achieve exactly the target reliability ν?".

Firstly, the mean CSP over the fading and the point processes is just the first moment M_1. For the typical link of the k th tier in the open group, the value of the SIR threshold is the solution of θ:

$$\frac{1}{\sum_{\Phi_{i|i\in B}} (\lambda_i/\lambda_k)(P_i/P_k)^\delta F_{1,\delta,\theta} + \sum_{\Phi_{j|j\notin B}} (\lambda_j/\lambda_k)(P_j/P_k)^\delta \theta^\delta (\pi\delta/\sin(\pi\delta))} = \nu, \tag{15}$$

where ν is the target reliability. It is difficult to express θ in an exact closed form. Here, we give an approximate value of θ based on two lemmas.

Lemma 5. *For a Poisson point process Ψ, let r_i denote the distance from node $x_i \in \Psi$, $i = 0, 1, \cdots n$ to the origin, and $r_0 = \min\{r_i\}$; then, the mean of the sum of the r_0/r_i satisfies $\mathbb{E}[\sum_i (r_0/r_i)^\alpha] = 2/(\alpha - 2)$.*

Proof. The $\{r_0/r_i\}_{i=1\cdots n}$ constitutes a relative distance process (RDP), see Definition 2 in [25].

Lemma 6. *For a Poisson point process Ψ, if the distance from the node $x_i \in \Psi$, $i = 0, 1, \cdots n$ to the origin is r_i, and $r_0, r_1, \cdots r_n$ are in ascending order, then $\mathbb{E}(\sum_{i=0}^n r_i^{-\alpha}) = (\lambda\pi)^{\alpha/2}\Gamma(n - (\alpha/2))/\Gamma(n))$.*

Proof. It is derived from the probability density function of r_i in a PPP as [26]:

$$f_{r_i}(X) = \exp\left(-\lambda\pi X^2\right) \cdot \frac{2\left(\lambda\pi X^2\right)^{\mathrm{i}}}{X\Gamma(\mathrm{i})}. \tag{16}$$

Theorem 7. *Given the reliability ν, the SIR threshold of the typical open user connecting to the kth ($k \in B$) tier can be approximated as*

$$\theta \approx \frac{\log(1/\nu)}{(2/(\alpha - 2)\alpha - 2) + \Gamma(1 + (\alpha/2))((\lambda_I/\lambda_s))^{\alpha/2}}, \tag{17}$$

where $\lambda_S = \sum_{\Phi_{i|i\in B}} \lambda_i (P_i/P_k)^\delta$ and $\lambda_I = \sum_{\Phi_{j|j\notin B}} \lambda_j (P_j/P_k)^\delta$ are called service density and interference density, respectively.

Proof. See Appendix C.

Remark 8. From (17), it can be seen that the approximate SIR threshold is related to both the service density and the interference density. Since the serving node can be anyone of the open tiers, the service density is the combined density of all the open tiers as $\lambda_S = \sum_{\Phi_{i|i\in B}} \lambda_i (P_i/P_k)^\delta$. The interference density means that the density of nodes acting only as interference. For the open user, the densities of the closed tiers are combined as the interference density, which is expressed as $\lambda_I = \sum_{\Phi_{j|j\notin B}} \lambda_j (P_j/P_k)^\delta$. From (17), the effect of the closed tiers on the approximate SIR threshold of open tiers is expressed as $\Gamma(1 + (\alpha/2))(\lambda_I/\lambda_s)^{\alpha/2}$ in the denominator. That is to say, the approximate SIR threshold can be maintained if the ratio of the λ_S and λ_I is constant.

A special case is that the network consists of only the open tiers, or just $B = \{1, 2, \cdots, K\}$, the value of the SIR threshold can be approximated as $\theta \approx ((\alpha - 2)\log(1/\nu))/2$, which is the SIR approximate value in a K-tier HCN.

Similarly, we can get the approximate SIR threshold for the closed link. For the closed tier (assume the jth tier), the SIR threshold can be approximated as the following theorem.

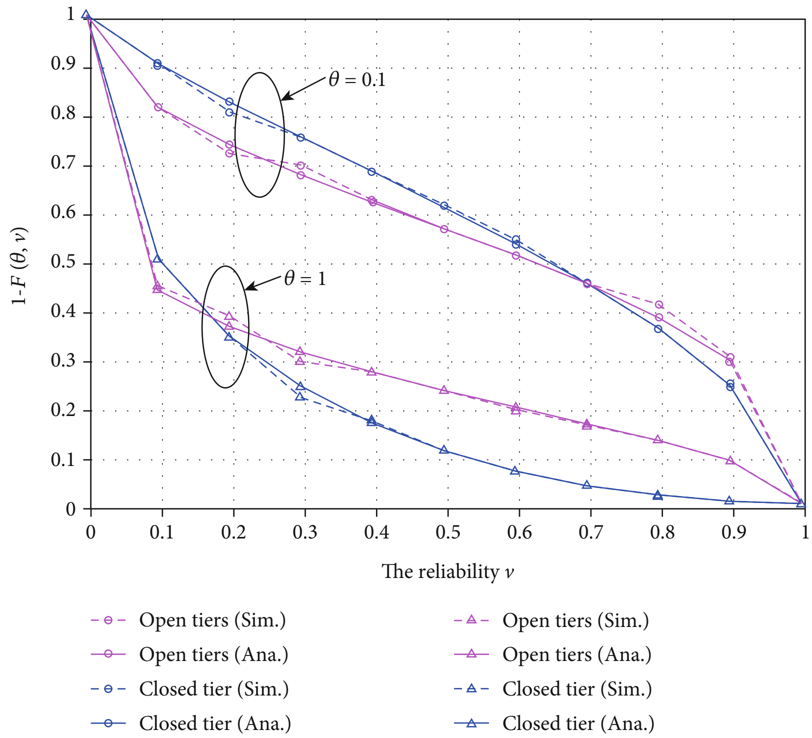

FIGURE 1: The CCDF of the CSP as a function of the reliability.

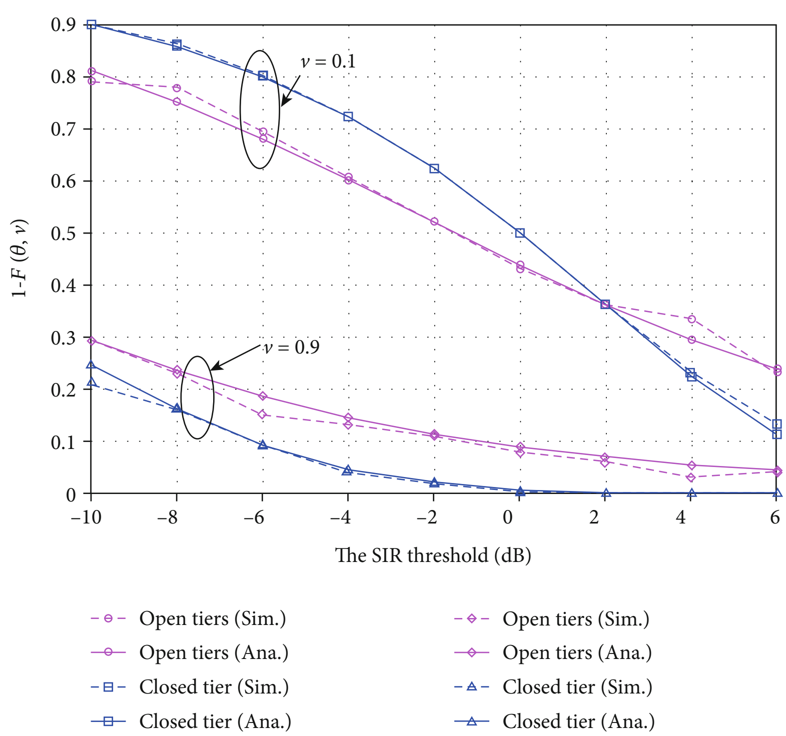

FIGURE 2: The CCDF of the CSP as a function of the SIR threshold.

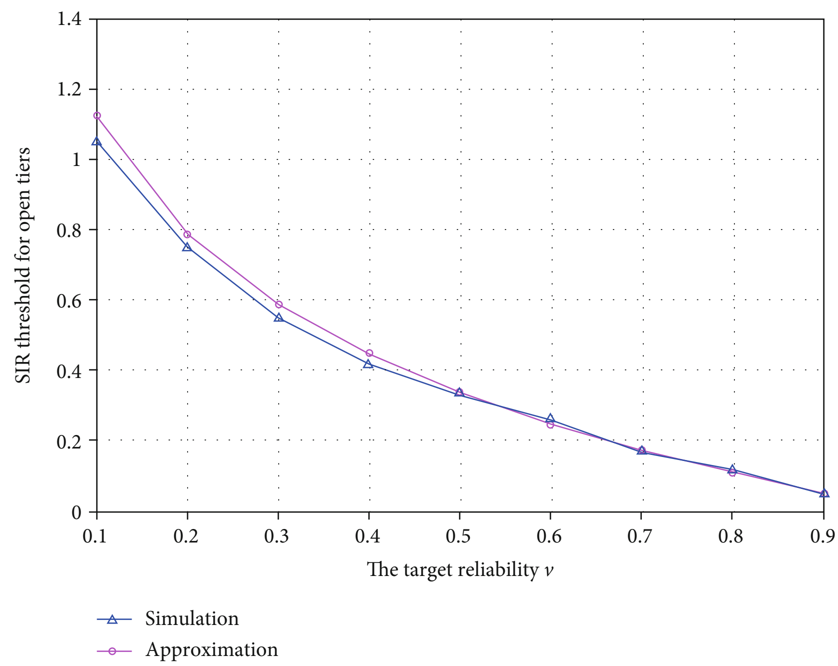

Figure 3: The SIR threshold settings for the open tiers.

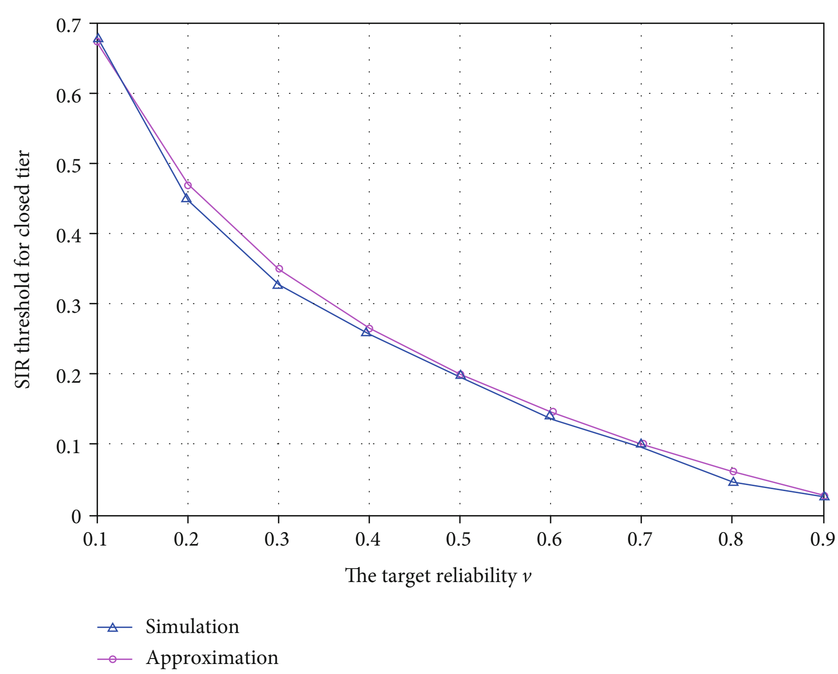

Figure 4: The SIR threshold settings for the closed tier.

Theorem 9. *Given the reliability v, the SIR threshold of the typical closed user can be approximated as*

$$\theta \approx \frac{\log (1/v)}{d_j^{\alpha}\left((\lambda_S \pi)^{\alpha/2} + (\lambda_I \pi)^{\alpha/2}\right)}, \quad (18)$$

where $\lambda_S = \lambda_j$ is the density of this closed tier, and $\lambda_I = \sum_{i\neq j} \lambda_i (P_i/P_k)^{\delta}$ is the density of interference from all other tiers.

Proof. See Appendix D.

Remark 10. For the closed user of jth tier, the signal provider is the only transmitter in a fixed distance d_j with a random orientation. It is obvious that the increase of d_j may lead to a lower SIR threshold setting. Besides the distance, we can see from (18) that the approximate value is inversely proportional to densities by $(\lambda_S \pi)^{\alpha/2} + (\lambda_I \pi)^{\alpha/2}$. That means densities of the open tiers and the closed tiers affect the SIR

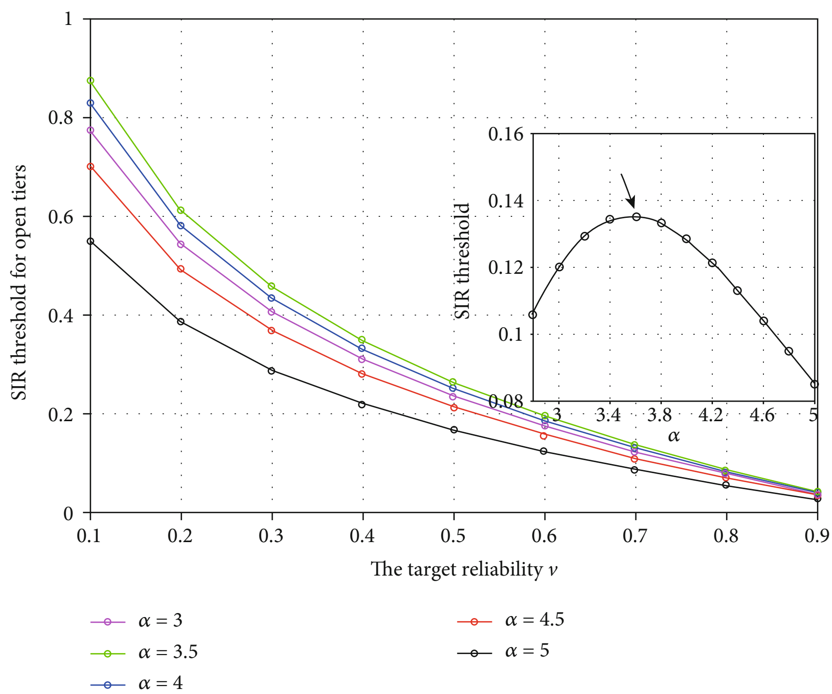

FIGURE 5: The SIR threshold settings for open tiers follow with the path loss exponent.

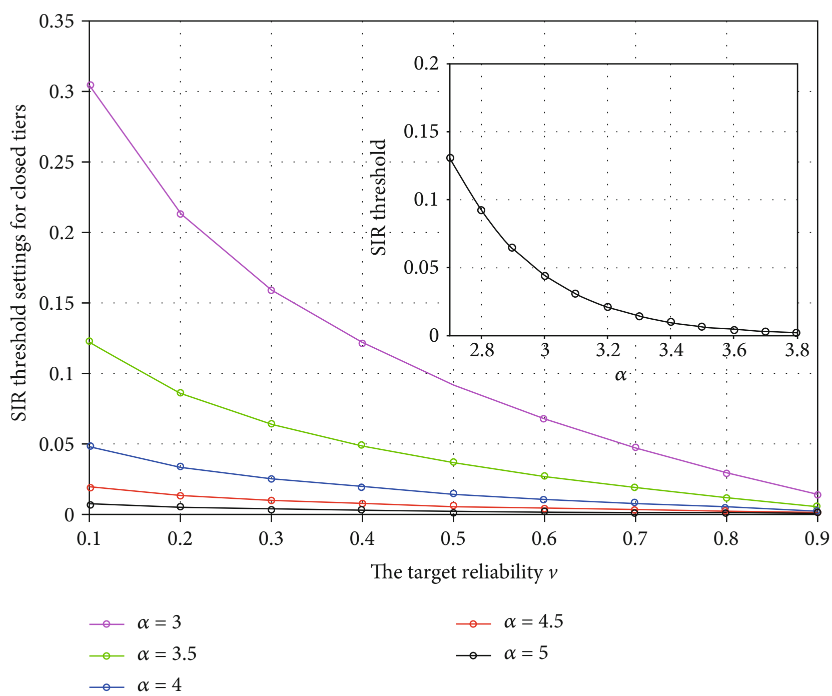

FIGURE 6: The SIR threshold settings for a closed-2 tier follow with the path loss exponent.

threshold in the same way. The reason lies in that all the other nodes of the same tier and all the other tiers are interference definitely. It is different from that in the open tiers; any transmitter can be the serving node with a probability. Consequently, the increase of any node in the interference tier or the serving tier will only cause a lower SIR threshold.

4. Results and Discussion

In this section, we will present the simulation results of the SIR meta distribution, or the distribution of the CSP $(P_s(\theta))$, and the per-link rate control for open tiers and closed tiers in the model mentioned above. The platform we used is MATLAB, and the distributed range of transmitters is $[0,100] \times [0,100]$.

The het-hybrid network consists of 3 tiers, named tier-1, tier-2, and tier-3, where tier-1 and tier-2 form the group of open tiers, and tier-3 is a closed tier. The corresponding transmitters densities are set as $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, and $\lambda_3 = 0.05$, and the transmitting power is set as $P_1 = 10$, $P_2 = 5$, and $P_3 = 3$, respectively. The fading is distributed as exponential random variables with mean 1, and the path loss exponent α is 4 in our assumption. The density of open users is set as 0.05, and the distance from the transmitter to the dedicated user is set as 10 in the closed tier. We repeat 300 transmissions to calculate the success transmission probability and the reliability.

Firstly, the results of the SIR meta distribution for the open tiers and the closed tier are shown in Figures 1 and 2. The numerical results approximated by beta distribution based on the bth moments are also shown in the two figures. For open users, we show the performance for the overall open tiers. Figure 1 is the SIR meta distribution as a function of reliability ν with the given SIR thresholds $\theta = 1$ and $\theta = 0.1$. Figure 2 is the SIR meta distribution as a function of the SIR threshold with the given reliabilities $\nu = 0.1$ and $\nu = 0.9$.

As Figures 1 and 2 illustrate, the beta distribution provides a good match for the distribution of the link success probabilities, which verifies that the approximation of beta distribution is effective in the het-hybrid network. Moreover, we can find some indications from the figures based on our settings. In Figure 1, the users of open tiers have a higher quantity of success transmission than the closed tier when the SIR threshold is set 1, but more percentage of the closed users can successfully transmit when the SIR threshold is deduced to 0.1. That means the meta distribution of the closed tier is more sensitive to the SIR threshold than that of the open tiers. A similar trend can be found in Figure 2. For the same SIR threshold, more percentage of the open users can be covered when the reliability is set a high value ($\nu = 0.9$), while with decreasing the value of ν, the closed users are easier to get a higher success probability. In other words, the meta distribution of the closed tier is also more sensitive to the reliability than that of the open tiers in our settings. The hidden reason is the difference of association rules, that the users of open tiers have more opportunities to associate with a favorable transmitter while the users of closed tier have the fixed transmitter.

For the per-link rate control, it is intuitively that the SIR threshold declines with the increasing reliability, shown in Figures 3 and 4. Figure 3 shows the SIR threshold settings for open tiers to meet the reliability. Figure 4 shows the SIR threshold settings for the closed tier. The path loss exponent α is set 4 in both figures. According to Figures 3 and 4, we can see the approximate values calculated as (17) and (18) are close to the simulations, so the SIR threshold can be set conveniently by the formulas to control the rate to meet the target reliability.

Besides the service density and interference density, we can see from (17) and (18) that the approximate value of SIR threshold is related to the path loss exponent for a target reliability. Figure 5 shows the trend of the SIR threshold following the path loss exponent for open tiers. It is noticed that the trend is not monotonous, and the SIR threshold can get a peak value when the path loss exponent $\alpha = 3.6$ or so. A lower or greater value of α leads to a lower threshold to meet the same target reliability. However, the effect of the path loss exponent in the approximate SIR threshold is different in the closed tier, as Figure 6, where the trend is monotonous and a lower α only leads to a greater SIR threshold setting.

5. Conclusions

The meta distribution is a fine-grained key performance metric of wireless systems. In this paper, we study the SIR meta distribution in the multi-tier heterogeneous and hybrid network characterized by different powers, different densities, and different association rules of each tier. At first, we derive the bth moments of conditional success probability for the users of the open tiers and the closed tiers, respectively. Based on the bth moments, we give the expressions of SIR meta distribution or the CCDF of the conditional success probability and approximate the expressions by beta distribution. The accuracy of the approximation is confirmed by simulations. Then, the mean local delay for users of the open tiers is also analyzed. Furthermore, using another facet of SIR meta distribution, we study the per-link rate control for the open tiers and closed tiers and derive the corresponding approximate value of SIR threshold to control the link rate. The simulations show that the approximate value we derived can be used effectively for setting the SIR threshold to meet the specified reliability.

Appendix

A. Proof of Theorem 1

Conditioned on the typical user associated with the transmitter x_0 of the kth tier in the open group and assume the distance from x_0 to the user is R_0, the CSP is expressed as

$$P_{s,(k)}^{\text{open}} = \mathbb{P}\left(\frac{P_k h_0 R_0^{-\alpha}}{\sum_{x\in\Phi_k, x\neq x_0} P_k h_x |x|^{-\alpha} + \sum_{i\neq k}\sum_{x\in\Phi_i} P_i h_x |x|^{-\alpha}} > \theta \Big| \bigcup \Phi_i\right). \quad \text{(A.1)}$$

By averaging over the fading, we get the conditional bth moment of the CSP, given by

$$\begin{aligned} M_{b,R_0} &= \prod_{x\in\Phi_k, x\neq x_0} \frac{1}{\left(1+\theta(R_0/|x|)^{\alpha}\right)^b} \prod_{i\neq k}\prod_{x\in\Phi_i} \frac{1}{\left(1+\theta\left(P_j/P_k\right)(R_0/|x|)^{\alpha}\right)^b} \\ &= \prod_{x\in\Phi_k, x\neq x_0} \frac{1}{\left(1+\theta(R_0/|x|)^{\alpha}\right)^b} \prod_{i\in B, i\neq k}\prod_{x\in\Phi_i} \frac{1}{\left(1+\theta\left(P_j/P_k\right)(R_0/|x|)^{\alpha}\right)^b} \\ &\quad \prod_{i\notin B}\prod_{x\in\Phi_i} \frac{1}{\left(1+\theta\left(P_j/P_k\right)(R_0/|x|)^{\alpha}\right)^b} \end{aligned} \quad \text{(A.2)}$$

The notation M_{b,R_0} is used to denote that the bth moment conditioned on R_0 and the event that the typical user connects to the k-th tier, which occurs with the probability given in (6). Then, the bth moment of the open tiers can be expressed as

$$M_{b,(k)}^{\text{open}} = \mathbb{E}_{R_0,\Phi_{i|i\in\{1\cdots K\}}}\left[\prod_{\Phi_{i|i\in B,i\neq k}}\exp\left(-\pi\lambda_i\left(\frac{P_i}{P_k}\right)^{\delta}R_0{}^2\right)\prod_{x\in\Phi_k,x\neq x_0}\frac{1}{\left(1+\theta(R_0/|x|)^{\alpha}\right)^b}\prod_{i\in B,i\neq k}\prod_{x\in\Phi_i}\frac{1}{\left(1+\theta(P_i/P_k)(R_0/|x|)^{\alpha}\right)^b}\prod_{i\notin B}\prod_{x\in\Phi_i}\frac{1}{\left(1+\theta(P_i/P_k)(R_0/|x|)^{\alpha}\right)^b}\right]$$

$$\overset{(a)}{=}\mathbb{E}_{R_0}\left[\prod_{\Phi_{i|i\in B,i\neq k}}\exp\left(-\pi\lambda_i\left(\frac{P_i}{P_k}\right)^{\delta}R_0{}^2\right).\exp\cdot\left(-2\pi\lambda_k\int_{R_0}^{\infty}\left(1-\frac{1}{\left(1+\theta(R_0/|x|)^{\alpha}\right)^b}\right)xdx\right).\right.$$

$$\prod_{i\in B,i\neq k}\exp\left(-2\pi\lambda_i\int_{r_i}^{\infty}\left(1-\frac{1}{\left(1+\theta(P_i/P_k)(R_0/|x|)^{\alpha}\right)^b}\right)xdx\right).$$

$$\left.\prod_{j\notin B}\exp\left(-\lambda_j\int_{R^2}\left(1-\frac{1}{\left(1+\theta\left(P_j/P_k\right)(R_0/|x|)^{\alpha}\right)^b}\right)dx\right)\right]$$

$$\overset{(b)}{=}\int_0^{\infty}2\pi\lambda_k r_k e^{-\pi\lambda_k r_k{}^2}\exp\left(\sum_{i\in B,i\neq k}-\pi\lambda_i\left(\frac{P_i}{P_k}\right)^{\delta}r_k{}^2\right).\exp\cdot\left(-2\pi\lambda_k\int_{r_k}^{\infty}\left(1-\frac{1}{\left(1+\theta(R_0/|x|)^{\alpha}\right)^b}\right)xdx\right).\prod_{i\in B,i\neq k}\exp\cdot\left(-2\pi\lambda_i\int_{r_i}^{\infty}\left(1-\frac{1}{\left(1+\theta(P_i/P_k)(r_k/|x|)^{\alpha}\right)^b}\right)xdx\right).\prod_{j\notin B}\exp\cdot\left(-\lambda_j\int_{R^2}\left(1-\frac{1}{\left(1+\theta\left(P_j/P_k\right)(R_0/|x|)^{\alpha}\right)^b}\right)dx\right)$$

$$\overset{(c)}{=}\int_0^{\infty}e^{-z\left(1+\sum_{i\in B,i\neq k}(\lambda_i/\lambda_k)(P_i/P_k)^{\delta}\right)}.\exp\cdot\left(-2z\int_0^1\left(1-\frac{1}{\left(1+\theta u^{\alpha}\right)^b}\right)u^{-3}du\right).\prod_{i\in B}\exp\cdot\left(-2z\frac{\lambda_i}{\lambda_k}\int_0^{r_k/r_i}\left(1-\frac{1}{\left(1+\theta(P_i/P_k)u^{\alpha}\right)^b}\right)u^{-3}du\right).\prod_{j\notin B}\exp\cdot\left(-z\frac{\lambda_j}{\lambda_k}\left(\frac{P_i}{P_k}\right)^{\delta}\theta^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}\Gamma_{b,\delta}\right)dz$$

$$\overset{(d)}{=}\int_0^{\infty}e^{-z}.\exp\left(-z\int_1^{\infty}\left(1-\frac{1}{\left(1+\theta v^{-\alpha/2}\right)^b}\right)dv\right).\exp\cdot\left(-z\left(\sum_{i\in B,i\neq k}\frac{\lambda_i}{\lambda_k}\left(\frac{P_i}{P_k}\right)^{\delta}\int_1^{\infty}\left(1-\frac{1}{\left(1+\theta t^{-\alpha/2}\right)^b}\right)dt\right)\right).\exp\cdot\left(\sum_{j\notin B}-z\theta^{\delta}\left(\frac{\lambda_j}{\lambda_k}\right)\left(\frac{P_j}{P_k}\right)^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}\Gamma_{b,\delta}\right)dz$$

$$\overset{(e)}{=}\int_0^{\infty}\exp\left(-zF_{b,\delta,\theta}\right).\exp\left(-z\sum_{i\in B,i\neq k}\frac{\lambda_i}{\lambda_k}\left(\frac{P_i}{P_k}\right)^{\delta}F_{b,\delta,\theta}\right).\exp\cdot\left(-z\theta^{\delta}\sum_{j\notin B}\left(\frac{\lambda_j}{\lambda_k}\right)\left(\frac{P_j}{P_k}\right)^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}\Gamma_{b,\delta}\right)dz$$
$$=\frac{1}{\sum_{\Phi_{i|i\in B}}(\lambda_i/\lambda_k)(P_i/P_k)^{\delta}F_{b,\delta,\theta}+\sum_{\Phi_{j|j\notin B}}(\lambda_j/\lambda_k)\left(P_j/P_k\right)^{\delta}\theta^{\delta}(\pi\delta/\sin(\pi\delta))\Gamma_{b,\delta}}. \tag{A.3}$$

In the above derivation, (a) is by the probability generating functional (PGFL) of the PPP and the polar coordinate, and (b) is by using the probability density function of R_0. In step (c), we use the variable substitution $\pi\lambda_k r_k{}^2 = z$, and $r_k/|x| = u$ in the $\exp\left(-2\pi\lambda_k\int_{r_k}^{\infty}\left(1-1/\left(1+\theta(R_0/|x|)^{\alpha}\right)^b\right)xdx\right)$ term and the $\prod_{i\in B,i\neq k}\exp\left(-2\pi\lambda_i\int_{r_i}^{\infty}\left(1-1/\left(1+\theta(P_i/P_k)(r_k/|x|)^{\alpha}\right)^b\right)xdx\right)$ term and use

$$\int_{R^2}\left(1-\left(\frac{1}{1+\theta_1|x|^{-\alpha}}\right)^b\right)dx=\pi\theta_1{}^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}\frac{\Gamma(b+\delta)}{\Gamma(b)\Gamma(1+\delta)}, \tag{A.4}$$

in the $\prod_{j\notin B}\exp\left(-\lambda_j\int_{R^2}\left(1-1/\left(1+\theta(P_j/P_k)(R_0/|x|)^{\alpha}\right)^b\right)dx\right)$ term, where $\theta_1=(P_j/P_k)r_k^{\alpha}\theta$. (A.4) can be obtained from [2, 15]. The step (d) is by using the variable substitution $u^{\alpha}=v^{-\alpha/2}$, and $(P_j/P_k)u^{\alpha}=t^{-\alpha/2}$; and step (e) holds for ${}_2F_1(b,-\delta;1-\delta;-\theta)\equiv 1+\int_1^{\infty}\left(1-1/\left(1+\theta t^{-\alpha/2}\right)^b\right)dt$. Thus, $M_{b,(k)}^{\text{open}}$ is derived.

B. Proof of Theorem 3

Assume the jth ($j\notin$ B) tier is the closed tier, the CSP of the typical user is expressed as

$$P_{s,(k)}^{\text{closed}}=\mathbb{P}\left(\frac{P_j h_0 d_j{}^{-\alpha}}{\sum_{x\in\Phi_j,x\neq x_0}P_k h_x|x|^{-\alpha}+\sum_{i\neq j}\sum_{x\in\Phi_i}P_i h_x|x|^{-\alpha}}>\theta\Big|\bigcup\Phi_i\right). \tag{B.1}$$

By averaging over the fading, we get the conditional bth moment as

$$M_{b,(j)}^{\text{closed}}=E\left(\prod_{x\in\Phi_j,x\neq x_0}\frac{1}{\left(1+\theta\left(d_j/|x|\right)^{\alpha}\right)^b}\prod_{j\neq i}\prod_{\Phi_i}\frac{1}{\left(1+\theta\left(P_i/P_j\right)\left(d_j/|x|\right)^{\alpha}\right)^b}\right)$$

$$\overset{(a)}{=} \exp\left(-\lambda_j \int_{R^2}\left(1-\frac{1}{\left(1+\theta\left(d_j/|x|\right)^{\alpha}\right)^{b}}\right)dx\right)\cdot\prod_{i\neq j}\exp \cdot \left(-\lambda_i \int_{R^2}\left(1-\frac{1}{\left(1+\theta\left(P_i/P_j\right)\left(d_j/|x|\right)^{\alpha}\right)^{b}}\right)dx\right)$$

$$\overset{(b)}{=} \exp\left(-\pi\lambda_j \theta^{\delta} d_j^2 \frac{\pi\delta}{\sin(\pi\delta)}\frac{\Gamma(b+\delta)}{\Gamma(b).\Gamma(1+\delta)}\right)\cdot\prod_{i\neq j}\exp \cdot \left(-\pi\lambda_i\left(\frac{P_j}{P_k}\right)^{\delta}\theta^{\delta} d_j^2 \frac{\pi\delta}{\sin(\pi\delta)}\frac{\Gamma(b+\delta)}{\Gamma(b).\Gamma(1+\delta)}\right) = \exp\left(-\pi\sum_{i=1}^{K}\lambda_i\left(\frac{P_i}{P_j}\right)^{\delta}\theta^{\delta} d_j^2\frac{\pi\delta}{\sin(\pi\delta)}\Gamma_{b,\delta}\right), \tag{B.2}$$

where (a) is by the PGFL of the PPP, and (b) holds for equation (A.4).

C. Proof of Theorem 7

Due to the displacement theorem, the stochastic equivalence model has been used in [19, 20]. Here, we make use of the stochastic equivalence to simplify the het-hybrid network as an equivalent two-tier network. One tier is the service-tier Φ_S (all open tiers), and the other is the interference-tier Φ_I (all closed tiers). For a link of the open tier k, the service density can be equivalent to $\lambda_S = \sum_{j\in B}\lambda_j(P_j/P_k)^{\delta}$, and the density of interference is equivalent to $\lambda_I = \sum_{j\notin B}\lambda_j(P_j/P_k)^{\delta}$. Assume the number of nodes in the service tier is n_1, the number of nodes in the interference tier is n_2, and the serving node is x_0 with distance R_0, the interference node is x_i, the CSP can be given by

$$P_{s,(k)}^{\text{open}} = \mathbb{P}\left(\frac{P_k h_0 R_0^{-\alpha}}{\sum_{x_i\in\Phi_k, i\neq 0}P_k h_x|x_i|^{-\alpha}+\sum_{j\neq k}\sum_{x_i\in\Phi_j}P_j h_x|x_i|^{-\alpha}} > \theta \Big| \bigcup \Phi_i\right) = \prod_{j\in B}\prod_{x_i\in\Phi_j, x_i\neq x_0}\frac{1}{1+\theta\left(P_j/P_k\right)\left(R_0/|x_i|\right)^{\alpha}}\prod_{j\notin B}\prod_{x_i\in\Phi_j}\frac{1}{1+\theta\left(P_j/P_k\right)\left(R_0/|x_i|\right)^{\alpha}}$$

$$\overset{(a)}{=} \prod_{\Phi_{S,x_i\neq x0}}\frac{1}{1+\theta(R_0/|x_i|)^{\alpha}}\prod_{\Phi_I}\frac{1}{1+\theta(R_0/|x_i|)^{\alpha}} \geq \frac{1}{\left((1/n_1)\sum_{i=1}^{n_1}(1+\theta(R_0/|x_i|)^{\alpha})\right)^{n_1}\left((1/n_2)\sum_{1}^{n_2}(1+\theta(R_0/|x_i|)^{\alpha})\right)^{n_2}} = \frac{1}{\left(1+\theta(1/n_1)\sum_{i=1}^{n1}(R_0/|x_i|)^{\alpha}\right)^{n1}\left(1+\theta(1/n_2)\sum_{i=1}^{n_2}((R_0/|x_i|))^{\alpha}\right)^{n_2}}, \tag{C.1}$$

where (a) holds for the equivalent network; the "≥" holds for the relation between the geometric mean and the arithmetic mean, as [20], Lemma 5.

For a target reliability ν,

$$\frac{1}{\nu} \leq \left(1+\frac{1}{n_1}\theta\sum_{i=1}^{n_1}\left(\frac{R_0}{|x_i|}\right)^{\alpha}\right)^{n_1}\left(1+\frac{1}{n_2}\theta\sum_{i=1}^{n_2}\left(\frac{R_0}{|x_i|}\right)^{\alpha}\right)^{n_2}$$

$$\overset{(b)}{\approx} \left(1+\frac{1}{n_1}\theta\frac{2}{\alpha-2}\right)^{n_1}\cdot\left(1+\frac{1}{n_2}\theta\frac{\Gamma(1+(\alpha/2))}{(\lambda_S\pi)^{\alpha/2}}\frac{(\lambda_I\pi)^{\alpha/2}\Gamma(n_2-(\alpha/2))}{\Gamma(n_2)}\right)^{n_2} \leq \left(1+\frac{1}{n_1}\theta\frac{2}{\alpha-2}\right)^{n_1}\left(1+\frac{1}{n_2}\theta\frac{(\lambda_I)^{\alpha/2}}{(\lambda_S)^{\alpha/2}}\Gamma\left(1+\frac{\alpha}{2}\right)\right)^{n_2} \approx \lim_{\substack{n_1\to\infty\\ n_2\to\infty}}\left(1+\frac{1}{n_1}\theta\frac{2}{\alpha-2}\right)^{n_1}\left(1+\frac{1}{n_2}\theta\left(\frac{\lambda_I}{\lambda_S}\right)^{\alpha/2}\Gamma\left(1+\frac{\alpha}{2}\right)\right)^{n_2}$$

$$\overset{(c)}{=} \exp\left(\theta\left(\frac{2}{\alpha-2}+\left(\frac{\lambda_I}{\lambda_S}\right)^{\alpha/2}\Gamma\left(1+\frac{\alpha}{2}\right)\right)\right), \tag{C.2}$$

where (b) holds for the $R_0/|x_i|$ in Lemma 5 and Lemma 6, and r_0 is taken a mean value by the probability density function $f_{r_0}(r) = 2\pi\lambda r e^{-\pi\lambda r^2}$ in open tiers. The last "=" (c) is derived from the $\lim_{n\to\infty}(1+(x/n))^n = e^x$. The equation (17) is thus derived.

D. Proof of Theorem 9

We also start from the CSP for the closed user. Similar as the open tiers, the service density is the kth tier density itself $\lambda_S = \lambda_j$, and density of interference from other tiers is $\lambda_I = \sum_{i\neq j}\lambda_i(P_i/P_k)^{\delta}$. Assume the link distance is fixed as d_j, the CSP is

$$P_{s,(j)}^{\text{closed}} = \mathbb{P}\left(P_j h_0 d_j^{-\alpha}/\sum_{x_i\in\Phi_j, i\neq 0}P_j h_x|x_i|^{-\alpha} + \sum_{j\neq k}\sum_{x_i\in\Phi_k}P_k h_x|x_i|^{-\alpha} > \theta \Big| \bigcup\Phi_i\right) = \prod_{x_i\in\Phi_j, x_{i\neq x_0}}\left(1/1+\theta\left(d_j/|x_i|\right)^{\alpha}\right)\prod_{j\neq k}\prod_{x_i\in\Phi_j}\cdot\left(1/1+\theta\left(P_k/P_j\right)\left(d_j/|x_i|\right)^{\alpha}\right)((a)/=)\prod_{\Phi_{j,x_i\neq x0}}\cdot\left(1/1+\theta\left(d_j/|x_i|\right)^{\alpha}\right)\prod_{\Phi_I}\left(1/1+\theta\left(d_j/|x_i|\right)^{\alpha}\right) \geq 1/\left((1/n_1)\sum_{i=1}^{n_1}\left(1+\theta\left(d_j/|x_i|\right)^{\alpha}\right)\right)^{n_1}\cdot\left((1/n_2)\sum_{1}^{n_2}\left(1+\theta\left(d_j/|x_i|\right)^{\alpha}\right)\right)^{n_2}, \tag{D.1}$$

where (a) holds for the interference nodes as an equivalent tier with density λ_I, and the "≥" holds for the relation between the geometric mean and the arithmetic mean, as [20], Lemma 5. Therefore,

$$\frac{1}{\nu} \leq \left(1 + \frac{1}{n_1}\theta \sum_{i=1}^{n_1} \left(\frac{d_j}{|x_i|}\right)^{\alpha}\right)^{n_1} \left(1 + \frac{1}{n_2}\theta \sum_{i=1}^{n_2} \left(\frac{d_j}{|x_i|}\right)^{\alpha}\right)^{n_2}$$

$$\overset{(b)}{\approx} \left(1 + \frac{d_j^{\,\alpha}}{n_1}\theta \frac{(\lambda_S \pi)^{\alpha/2}\Gamma(n_1 - (\alpha/2))}{\Gamma(n_1)}\right)^{n_1} \cdot \left(1 + \frac{d_j^{\,\alpha}}{n_2}\theta \frac{(\lambda_I \pi)^{\alpha/2}\Gamma(n_2 - (\alpha/2))}{\Gamma(n_2)}\right)^{n_2}$$

$$\leq \left(1 + \frac{d_j^{\,\alpha}}{n_1}\theta(\lambda_S\pi)^{\alpha/2}\right)^{n_1} \left(1 + \frac{d_j^{\,\alpha}}{n_2}\theta(\lambda_I\pi)^{\alpha/2}\right)^{n_2}$$

$$\approx \lim_{\substack{n_1\to\infty \\ n_2\to\infty}} \left(1 + \frac{d_j^{\,\alpha}}{n_1}\theta(\lambda_S\pi)^{\alpha/2}\right)^{n_1} \left(1 + \frac{d_j^{\,\alpha}}{n_2}\theta(\lambda_I\pi)^{\alpha/2}\right)^{n_2}$$

$$\overset{(c)}{=} \exp\left(\theta d_j^{\,\alpha}\left((\lambda_S\pi)^{\alpha/2} + (\lambda_I\pi)^{\alpha/2}\right)\right), \tag{D.2}$$

where (b) stems from Lemma 6, and (c) holds for $\lim_{n\to\infty} (1 + (x/n))^n = e^x$. Equation (18) is thus derived.

Data Availability

All data in this paper are fully available without restriction.

Conflicts of Interest

The authors declare that they have no conflicts of interests.

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